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$\hat{\delta}$ – Closed Sets in Ideal Topological Spaces

M. Navaneethakrishnan[1], P. Periyasamy[2], S. Pious Missier[3]
[1] *Department of Mathematics, Kamaraj College, Thoothukudi, Tamilnadu, India.*
[2] *Department of Mathematics, Kamaraj College, Thoothukudi, Tamilnadu, India.*
[3] *Department of Mathematics, V.O.Chidambaram College, Thoothukudi, Tamilnadu, India.*

Abstract: *In this paper we introduce the notion of $\hat{\delta}$ –closed sets and studied some of its basic properties and characterizations. It shows this class lies between δ–closed sets and g–closed sets in particularly lies between δ-I–closed sets and g–closed sets. This new class of sets is independent of closed sets, semi closed and α–closed sets. Also we discuss the relationship with some of the known closed sets.*

Keywords and Phrases: *$\hat{\delta}$ –closed, $\hat{\delta}$ –open.*

I. Introduction

A nonempty collection of subsets of X in a topological space (X,τ) is said to be an ideal I if it satisfies (i) A∈I and B⊂A implies B∈I and (ii) A∈I and B∈I implies A∪B∈I. A topological space (X,τ) with an ideal I is called an ideal topological space or simply ideal space. If P(X) is the set of all subsets of X, a set operator (.)*: P(X) → P(X) is called a local function [10] of a subset A with respect to the topology τ and ideal I is defined as $A^*(X, \tau) = \{x \in X / U \cap A \notin I \text{ for every } U \in \tau(x)\}$ where $\tau(x)=\{U\in\tau / x\in U\}$. A kuratowski closure operator cl*(.) for a topology τ*(I, τ), called the *–topology, finar than τ is defined by $cl^*(A)=A\cup A^*(I, \tau)$ [25]. Levine [13], velicko [27] introduced the notions of generalized closed (briefly g–closed) and δ–closed sets respectively and studied their basic properties. The notion of I_g–closed sets was first introduced by Dontchev [6] in 1999. Navaneetha Krishanan and Joseph [20] further investigated and characterized I_g–closed sets. Julian Dontchev and maximilian Ganster [5], Yuksel, Acikgoz and Noiri [28] introduced and studied the notions of δ–generalized closed (briefly δg–closed) and δ-I–closed sets respectively. The purpose of this paper is to define a new class of sets called $\hat{\delta}$ –closed sets and also study some basic properties and characterizations.

II. Preliminaries

Definition 2.1. A subset A of a topological space (X, τ) is called a
(i) Semi-open set [12] if A⊆cl(int(A))
(ii) Pre-open set [17] if A⊆int(cl(A))
(iii) α - open set [2(a)] if A⊆int(cl(int(A))
(iv) regular open set [24] if A = int(cl(A))

The complement of a semi-open (resp. pre-open, α - open, regular open) set is called Semi-closed (resp. pre-closed, α - closed, regular closed). The semi-closure (resp. pre closure, α-closure) of a subset A of (X, τ) is the intersection of all semi-closed (resp. pre-closed α-closed) sets containing A and is denoted by scl(A) (resp.pcl(A), αcl(A)). The intersection of all semi-open sets of (X, τ) contains A is called semi-kernel of A and is denoted by sker(A).

Definition 2.2. [28]. Let (X, τ, I) be an ideal topological space. A a subset of X and x is a point of X. Then
i) x is called a δ - I – cluster points of A if $A \cap (\text{int } cl^*(U)) \neq \phi$ for each open neighbourhood U of x.
ii) the family of all δ-I-cluster points of A is called the δ-I-closure of A and is denoted by $[A]_{\delta\text{-}I}$.
iii) a subset A is said to be δ-I-closed if $[A]_{\delta\text{-}I} = A$. The complement of a δ-I-closed set of X is said to be δ-I-open.

Remarks 2.3. From the definition [ ] we can write $[A]_{\delta\text{-}I} = \{x\in X : (\text{int}(cl^*(U)) \cap A \neq \phi, \text{ for each } U\in \tau(x)\}$

Notation 2.4. Throughout this paper we use the notation $\sigma cl(A) = [A]_{\delta\text{–}I}$.

Lemma 2.5. [28] Let A and B be subsets of an ideal topological space (x, τ, I). Then, the following properties hold.
(i) $A \subseteq \sigma cl(A)$
(ii) If $A \subset B$, then $\sigma cl(A) \subset \sigma cl(B)$

(iii) $\sigma cl(A) = \cap\{F\subset X / A\subset F$ and F is δ-I-closed$\}$
(iv) If A is δ-I-closed set of X for each $\alpha\in\Delta$, then $\cap\{A\alpha / \alpha\in\Delta\}$ is δ - I - closed
(v) $\sigma cl(A)$ is δ-I- Closed.

Lemma 2.6. [28] Let (X, τ, I) be an ideal topological space and $\tau_{\delta\text{-}I} = \{A\subset X / A$ is δ-I-open set of $(X, \tau, I)\}$. Then $\tau_{\delta\text{-}I}$ is a topology such that $\tau_s\subset\tau_{\delta\text{-}I}\subset\tau$

Remark 2.7. [28] τ_s (resp. $\tau_{\delta\text{-}I}$) is the topology formed by the family of δ-open sets (resp. δ-I - open sets).

Lemma 2.8. Let (X, τ, I) be an ideal topological space and A a subset of X. Then $\sigma cl(A) = \{x\in X : int(cl^*(U))\cap A\neq\phi, U\in\tau(x)\}$ is closed.

Definition 2.9. Let (X, τ) be a topological space. A subset A of X is said to be
(i) a g-closed set [13] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ)
(ii) a generalized semi-closed (briefly gs-closed) set [3] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open set in (X, τ).
(iii) a Semi-generalized closed (briefly sg-clsoed) set [4] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open set in (X, τ).
(iv) a α - generalized closed (briefly αg-closed) set [15] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).
(v) a generalized α-closed (briefly gα-closed) set [14] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α-open in (X, τ).
(vi) a δ-closed set [27] if $A = cl_\delta(A)$, where $\delta cl(A) = cl_\delta(A) = \{x \in X : (int\ cl(U) \cap A \neq \phi, U \in \tau$ and $x\in U\}$.
(vii) a δ - generalized closed set (briefly δg-closed) set [5] if $cl_\delta(A) \subseteq U$ whenever $A \subseteq U$ and U is open
(viii) a α-$\hat{g}$ -closed (briefly $\alpha\hat{g}$ -closed) set [1] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is a $\hat{g}$ -open set in (X, τ).
(ix) a $\delta\hat{g}$ – closed set [11] if $cl_\delta(A) \subseteq U$ whenever $A \subseteq U$ and U is $\hat{g}$ -open set in (X, τ).
(x) a θ-closed set [27] if $\delta cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open set in (X, τ), where $\delta cl(A) = cl_\delta(A)$.
(xi) a θ-g-closed set [7] if $A = \theta cl(A)$ where $cl_\theta(A) = \theta cl(A) = \{x\in X : cl(U) \cap A \neq \phi, U \in \tau$ and $x \in U\}$
(xii) a $\wedge$ - set [16, 18] if $A^\wedge = A$ where $A^\wedge = \cap\{U \in \tau : A \subseteq U\}$
(xiii) a weakly generalized closed (briefly wg-closed) set [19] if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
(xiv) a Regular weakly generalized closed (briefly Rwg-closed) set [9] if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is regular open set in X.
(xv) a $\hat{g}$ (or) ω-closed set [26] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open set in (X, τ). The complement of $\hat{g}$ (or) ω-closed set is $\hat{g}$ (or) ω-open.

Definition 2.10. Let (X, τ, I) be an ideal space. A subset A of X is said to be
(i) Ig – closed set [6] if $A^* \subseteq U$ whenever $A \subseteq U$ and U is open in X.
(ii) θ - I – closed set [2] if $cl^*_\theta(A) = A$ where $cl^*_\theta(A) = \{x \in X : cl^*(U) \cap A \neq \phi$ for all $U \in \tau(x)\}$
(iii) I_*g-closed set [23] if $A^* \subseteq U$ whenever $A \subseteq U$ and U is $\hat{g}$ - open.
(iv) R – I – open set [28] if $Int(cl^*(A)) = A$.

III. $\hat{\delta}$ -closed set

In this section we introduce and study a new class of sets known as $\hat{\delta}$ -closed sets in ideal topological spaces.

Definition 3.1. A subset A of an ideal space (X, τ, I) is called $\hat{\delta}$ -closed if $\sigma cl(A) \subset U$ whenever $A \subset U$ and U is open in (X, τ, I). The complement of $\hat{\delta}$ -closed set in (X, τ, I), is called $\hat{\delta}$ -open set in (X, τ, I).

Example 3.2. Let X = {a, b, c, d}, τ = {X, ϕ, {a}, {b}, {a,b}, {a, b, d} and I = {ϕ, {c}, {d}, {c, d}}. Let A={a, b, c}, then A is $\hat{\delta}$ -closed set.

Theorem 3.3. Every δ-closed set is $\hat{\delta}$ -closed.

Proof: Let A be any δ-closed set and U be any open set in (X, τ, I) such that $A \subseteq U$. Since A is δ-closed and $\sigma cl(A) \subseteq cl_\delta(A)$. Therefore $\sigma cl(A) \subseteq U$. Thus A is $\hat{\delta}$ -closed set.

Remark 3.4. The converse is not always true from the following example.

Example 3.5. Let X = {a, b, c, d}, τ = {X, ϕ, {a}, {b}, {a, b}} and I = {ϕ, {c}, {d}, {c,d}}. Let A = {a, b, d}. Then A is $\hat{\delta}$ -closed set but not δ-closed.

Theorem 3.6. Every δ-I-closed set is $\hat{\delta}$ -closed.

Proof: Let A be any δ-I-closed set and U be any open set in (X, τ, I) such that $A \subseteq U$. Since A is δ-I-closed, $\sigma cl(A) = A \subseteq U$ whenever $A \subseteq U$ and U is open. Therefore A is $\hat{\delta}$ -closed.

Remark 3.7. The reversible implication is not always possible from the following example.

Example 3.8. Let X = {a, b, c, d}, τ = {X, ϕ, {b}, {b, c}} and I = {ϕ}. Let A = {a, b, c}. Then A is $\hat{\delta}$ -closed set but not δ-I-closed.

Theorem 3.9. Every δg-closed set is $\hat{\delta}$ -closed.

Proof: Let A be any δg-closed set and U be any open set. Such that $A \subseteq U$. Then $cl_\delta (A) \subseteq U$, for every subset A of X. Since $\sigma cl(A) \subseteq cl_\delta (A) \subseteq U$, $\sigma cl(A) \subseteq U$ and hence A is $\hat{\delta}$ -closed.

Remark 3.10. A $\hat{\delta}$ -closed set need not be δg-closed as shown in the following example.

Example 3.11. Let X = {a, b, c, d}, τ = {X, ϕ, {b}, {a, b}, {b, c}, {a, b, c}, {a, b, d}} and I = {ϕ, {b}}. Let A = {a, c, d}. Then A is $\hat{\delta}$ -closed but not δg-closed.

Theorem 3.12. Every $\delta\hat{g}$ -closed set is $\hat{\delta}$ -closed.

Proof: Let A be any $\delta\hat{g}$ -closed set and U be any open set containing A. Since every open set is $\hat{g}$ -open, $cl_\delta(A) \subseteq U$, A is $\hat{\delta}$ -closed.

Remark 3.13. A $\hat{\delta}$ -closed set is not always a $\delta\hat{g}$ -closed as shown in the following example.

Example 3.14. Let X = {a, b, c, d}, τ = {X, ϕ, {b}, {a, b}, {b, c}, {a, b, c}, {a, b, d}} and I = {ϕ, {b}}. Let A = {a, d}. Then A is $\hat{\delta}$ -closed but not $\delta\hat{g}$ -closed.

Theorem 3.15. Every θ-closed set is $\hat{\delta}$ -closed.

Proof : Let A be any θ-closed set and U be any open set containing A. Since A is θ-closed and $\sigma cl(A) \subseteq cl_\theta(A)$, $\sigma cl(A) \subseteq U$, whenever $A \subseteq U$ and U is open. Therefore A is $\hat{\delta}$ -closed.

Remark 3.16. A $\hat{\delta}$ -closed set is not always a θ-closed set as it can be seen in the following example.

Example 3.17. Let X = {a, b, c, d}, τ = {X, ϕ, {a}, {b, c}, {a, b, c} and I = {ϕ, {b}}. Let A = {b, c, d}. Then A is $\hat{\delta}$ -closed set but not θ-closed.

Theorem 3.18. Every θ-g-closed set is $\hat{\delta}$ -closed.

Proof: Let A be any θ-g-closed set and U be any open set containing A. Then $cl_\theta(A) \subseteq U$. Since $\sigma cl(A) \subseteq cl_\theta(A)$. A is $\hat{\delta}$ -closed.

Remark 3.19 A $\hat{\delta}$ -closed set is not always a θ-g-closed set as shown in the following example.

Example 3.20. Let X = {a, b, c, d}, τ = {X, ϕ, {b}, {a, b}, {b, c}, {a, b, c} {a, b, d}} and I = {ϕ, {b}}. Let A = {d}. Then A is $\hat{\delta}$ -closed set but not θ-g-closed.

Theorem 3.21. Every θ-I-closed set is $\hat{\delta}$ -closed.

Proof: Let A be any θ-I-closed set and U be any open set such that $A \subseteq U$. Since $\sigma cl(A) \subseteq cl^*_\theta(A)$, A is $\hat{\delta}$ -closed.

Remark 3.22. The reversible implication is not always true from the following example.

Example 3.23. Let X = {a, b, c,d}, τ = {x, ϕ, {a}, {b, c}, {a, b, c}} and I = {ϕ, {b}}. Let A = {a, c, d}. Then A is $\hat{\delta}$ -closed but not θ-I-closed.

Theorem 3.24. In an Ideal Space (X, τ, I), every $\hat{\delta}$ -closed set is (i) g-closed (ii) Ig-closed (iii) gs-closed (iv) αg-closed (v) wg-closed (vi) Rwg-closed.

Proof: (i) Suppose that A is a $\hat{\delta}$ -closed set and U be any open set such that $A \subseteq U$. By hypothesis $\sigma cl(A) \subseteq U$. Then $cl(A) \subseteq U$ and hence A is g-closed.

(ii) Since every g-closed set is Ig-closed set in (X, τ, I). It holds
(iii) It is true that $scl(A) \subseteq \sigma cl(A)$ for every subset A of (X, τ, I).
(iv) It is true that $\alpha cl(A) \subseteq \sigma cl(A)$ for every subset A of (X, τ, I).
(v) Since $cl(int(A) \subseteq \sigma cl(A)$. It holds
(vi) Proof follows from the fact that $cl(int(A)) \subseteq \sigma cl(A)$ and every regular open set is open.

Remark 3.25. The following examples reveals that the reversible implications of (i), (ii), (iii), (iv), (v), (vi) in Theorem 3.24 are not true in general.

Example 3.26. Let X = {a, b, c, d}, τ = {X, ϕ, {b}, {a, b}, {b, c}, {a, b, c}, {a, b, d}} and I = {ϕ, {a}, {c}, {a, c}}. Let A = {a, d}. Then A is g-closed set but not $\hat{\delta}$ -closed.

Example 3.27. Let X = {a, b, c, d}, τ = {X, ϕ, {b}, {a, b}, {b, c}, {a, b, c}, {a, b, d}} and I = (ϕ, {b}}. Let A = {b, c}. Then A is Ig-closed set but not $\hat{\delta}$ -closed.

Example 3.28. Let X = {a, b, c, d}, τ = {X, ϕ, {b}, {c, d}, {b, c, d}}and I = {ϕ,{b}, {c}, {b, c}}. Let A = {c, d}. Then A is gs-closed set but not $\hat{\delta}$ -closed.

Example 3.29. Let X = {a, b, c, d}, τ = {X, ϕ, {a}, {b}, {a, b}} and I = {ϕ, {b}}. Let A = {a}. Then A is αg-closed but not $\hat{\delta}$ -closed.

Example 3.30. Let X = {a, b, c, d}, τ = {X, ϕ, {a}, {b, c}, {a, b, c}} and I = (ϕ, {b}}. Let A = {b}. Then A is wg-closed but not $\hat{\delta}$ -closed.

Example 3.31. Let X = {a, b, c, d}, τ = {X, ϕ, {a}, {b}, {a, b}} and I = {ϕ, {b}}. Let A = {b}. Then A is Rwg-closed but not $\hat{\delta}$ -closed.

Remark 3.32. The following examples shows that, In an Ideal space $\hat{\delta}$-closed set is independent of (i) closed (ii) sg-closed (iii) gα-closed (iv) $\alpha\hat{g}$-closed (v)I_{*g}-closed.

Example 3.33. Let X = {a, b, c, d}, τ = {X, ϕ, {b}, {a, b}, {b, c}, {a, b, c}, (a, b, d}} and I = {ϕ, {a}, {c}, {a, c}}. Let A = {b, c, d} is $\hat{\delta}$-closed but not closed. Let B = {a, d}. Then B is closed but not $\hat{\delta}$-closed.

Example 3.34. Let X = {a, b, c, d}, τ = {X, ϕ, {a}, {b, c}, {a, b, c}} and I = {ϕ, {b}}. Let A = {b, c}. Then A is sg-closed but not $\hat{\delta}$-closed.

Example 3.35. Let X = {a, b, c}, τ = {X, ϕ, {a}} and I = {ϕ, {a}}. Let A = {a, b}. Then A is $\hat{\delta}$-closed but not sg-closed.

Example 3.36. Let X = {a, b, c, d}, τ = {X, ϕ, {b}, {c}, {b, c} and I = {ϕ, {d}}. Let A = {a, b, c}. The A is $\hat{\delta}$-closed but not gα-closed.

Example 3.37. Let X = {a, b, c, d}, τ = {X, ϕ, {c}, {c, d}} and I = {ϕ, {c}, {d}, {c, d}}. Let A = {d}. Then A is gα-closed but not $\hat{\delta}$-closed set.

Example 3.38. Let X = {a, b, c, d}, τ = {X, ϕ, {a}, {b}, (a, b}} and I = {ϕ, {b}}. Let A = {a}. Then A is $\alpha\hat{g}$-closed but not $\hat{\delta}$-closed.

Example 3.39. Let X = {a, b, c}, τ = {X, ϕ, {a}} and I = {ϕ, {a}}. Let A = {a, b}. Then A is $\hat{\delta}$-closed but not $\alpha\hat{g}$-closed.

Example 3.40. Let X = {a, b, c, d}, τ = {X, ϕ, {a}, {b}, {c}, {a, b}, {b, c}, {a, c}, {a, b, c}} and I = {ϕ, {a}, {b}, {a, b}}. Let A = {a, b}. Then A is I_{*g}-closed set but not $\hat{\delta}$-closed.

Example 3.41. Let X = {a, b, c, d}, τ = {X, ϕ, {a}, {b}, (a, b}, {a, b, d}} and I = {ϕ, {c}, {d}, {c, d}}. Let A = {a, b, c}. Then A is $\hat{\delta}$-closed but not I_{*g}-closed.

Theorem 3.42. Let (X, τ, I) be an ideal space and A a subset of X. Then σcl(A) = {x$\in$X / (int(cl*(U)) $\cap$ A $\neq \phi$, for all U $\in \tau$ (x)} is closed.

Proof: If x $\in$ cl (σcl(A)) and U $\in \tau$ (x), then U $\cap$ σcl(A) $\neq \phi$. Then y $\in$ U $\cap$ σcl(A) for some y $\in$ X. Since U $\in$ τ (y) and y $\in$ σcl(A), from the definition of σcl(A) we have int(cl*(U)) $\cap$ A $\neq \phi$. Therefore x $\in$ σcl(A). So cl(σcl(A)) $\subset$ σcl(A) and hence σcl(A) is closed.

IV. Characterizations

Theorem 4.1. If A is a subset of an ideal space (X, τ, I), than the following are equivalent.

(a) A is $\hat{\delta}$-closed
(b) For all x $\in$ σcl(A), cl({x}) $\cap$ A $\neq \phi$
(c) σcl(A) – A contains no non-empty closed set.

Proof: (a) $\Rightarrow$ (b) Suppose x $\in$ σcl(A). If cl({x}) $\cap$ A = ϕ, then A $\subset$ X – cl({x}). Since A is $\hat{\delta}$-closed, σcl(A) $\subset$ X – cl({x}). It is a contradiction to the fact that x$\in$$\sigma$cl(A). This proves (b).
(b) $\Rightarrow$ (c) Suppose F $\subset$ σcl(A)-A, F is closed and x$\in$F. Since F $\subset$ X-A and F is closed cl({x}) $\cap$ A $\subset$ cl(F) $\cap$ A = F $\cap$ A = ϕ. Since x$\in$$\sigma$cl(A), by (b), cl({x}) $\cap$ A $\neq \phi$, a contradiction which proves (c).
(c) $\Rightarrow$(a). Let U be an open set containing A. Since σcl(A) is closed, σcl(A) $\cap$ (X-U) is closed and σcl(A) $\cap$ (X-U) $\subset$ σcl(A) – A. By hypothesis σcl(A) $\cap$ (X-U) = ϕ and hence σcl(A) $\subset$ U. Thus A is $\hat{\delta}$-closed.

If we put I = {ϕ} in the above Theorem 4.1, we get the Corollary 4.2 which gives the characterizations for δg-closed sets. If we put I = P(X) in the above Theorem 4.1, we get corollary 4.3 which gives the characterizations for g-closed sets.

Corollary 4.2. If A is subset of an Ideal topological space (X, τ, I). Then the following are equivalent.
(a) A is δg-closed.
(b) For all $x \in \delta cl(A)$, $cl(\{x\}) \cap A \neq \phi$
(c) δcl(A) – A contains no non-empty closed set.

Corollary 4.3. If A is a subset of a topological space (X, τ). Then the following are equivalent.
(a) A is g-closed
(b) For all $x \in cl(A)$, $cl(\{x\}) \cap A \neq \phi$
(c) cl(A) – A contains no non-empty closed set.

If I = {ϕ}, then δcl(A) = σcl(A) and hence $\hat{\delta}$ -closed sets coincide with δg-closed sets. If I = P(X), then σcl(A) = cl(A) and hence $\hat{\delta}$ -closed sets coincide with g-closed sets.

Corollary 4.4. If (X, τ, I) is an ideal space and A is a $\hat{\delta}$ -closed set, then the following are equivalent.
(a) A is a δ-I-closed set
(b) σcl(A) – A is a closed set

Proof: (a) ⇒ (b). If A is δ-I-closed set, then σcl(A) – A = ϕ and so σcl(A) – A is closed.
(b) ⇒ (c) If σcl(A) – A is closed, since A is $\hat{\delta}$ -closed, by theorem, σcl(A) – A = ϕ and so A is δ-I-closed.

If we put I = {ϕ} in Corollary 4.4, we get Corollary 4.5. If we put I = P(X) in the Corollary 4.4. Then we get Corollary 4.6.

Corollary 4.5. If (X, τ, I) be an Ideal Topological space and A is a δg-closed set, then the following are equivalent.
a) A is a δ-closed set
b) δcl(A) – A is a closed set

Corollary 4.6. If (X, τ) is a topological space and A is a g-closed set, then the following are equivalent.
a) A is closed set
b) cl(A) – A is a closed set

Theorem 4.7. Let (X, τ, I) be an ideal space. Then every subset of X is $\hat{\delta}$ -closed if and only if every open set is δ-I-closed.

Proof: Necessity - Suppose every subset of X is $\hat{\delta}$ -closed. If U is open then U is $\hat{\delta}$ -closed and so σcl(U) ⊂ U. Hence U is δ-I-closed.
Sufficiency - Suppose A ⊂ U and U is open. Since every open set is δ-I-closed, σcl(A) ⊂ σcl(U) = U and so A is $\hat{\delta}$ -closed.

If we put I = {ϕ} in the above Theorem 4.7, we get Corollary 4.8. If we put I = P(X) in the above Theorem 4.7, we get corollary 4.9.

Corollary 4.8. Let (X, τ) be a topological space. Then every subset of X is δg-closed if and only if every open set is δ-closed.

Corollary 4.9. Let (X, τ) be a topological space. Then every subset of X is g-closed if and only if every open set is closed.

Theorem 4.10. If A and B are $\hat{\delta}$ -closed sets in a topological space (X, τ, I), then A ∪ B is $\hat{\delta}$ -closed set in (X, τ, I).

Proof: Suppose that $A \cup B \subseteq U$ where U is any open set in (X, τ, I). Then $A \subseteq U$ and $B \subseteq U$. Since A and B are $\hat{\delta}$-closed sets in (X, τ, I), $\sigma cl(A) \subseteq U$ and $\sigma cl(B) \subseteq U$. Always $\sigma cl(A \cup B) = \sigma cl(A) \cup \sigma cl(B)$. Therefore, $\sigma cl(A \cup B) \subseteq U$. Thus $A \cup B$ is a $\hat{\delta}$-closed set in (X, τ, I).

Remark 4.11. The following example shows that the intersection of two $\hat{\delta}$-closed set is not always $\hat{\delta}$-closed.

Example 4.12. In Example 3.29, Let A = {a, c} and B = {a, d}. Then A and B are $\hat{\delta}$-closed sets, but $A \cap B$ = {a} is not $\hat{\delta}$-closed.

Theorem 4.13. Intersection of a $\hat{\delta}$-closed set and a δ-I-closed set is always $\hat{\delta}$-closed.

Proof: Let A be a $\hat{\delta}$-closed set and F be a δ-I-closed set of an ideal space (X, τ, I). Suppose $A \cap F \subset U$ and U is open set in X. Then $A \subset U \cup (X–F)$. Now, X-F is δ-I-open and hence open. Therefore $U \cup (X–F)$ is an open set containing A. Since A is $\hat{\delta}$-closed, $\sigma cl(A) \subset U \cup (X–F)$. Therefore $\sigma cl(A) \cap F \subset U$ which implies that $\sigma cl(A \cap F) \subset U$. So $A \cap F$ is $\hat{\delta}$-closed.

If we put $I = \{\phi\}$ in the above Theorem 4.13, we get Corollary 4.14 and If we put $I = P(X)$ in Theorem 4.13, we get Corollary 4.15.

Corollary 4.14. Intersection of a δg-closed set and δ-closed set is always δg-closed.

Corollary 4.15. Intersection of a g-closed set and a closed set is always g-closed set.

Theorem 4.16. A subset A of an ideal space (X, τ, I) is $\hat{\delta}$-closed if and only if $\sigma cl(A) \subset A^\wedge$.

Proof: Necessity - Suppose A is $\hat{\delta}$-closed and $x \in \sigma cl(A)$. If $x \notin A^\wedge$, then there exists an open set U such that $A \subset U$, but $x \notin U$. Since A is $\hat{\delta}$-closed, $\sigma cl(A) \subset U$ and so $x \notin \sigma cl(A)$, a contradiction. Therefore $\sigma cl(A) \subset A^\wedge$.
Sufficiency - Suppose that $\sigma cl(A) \subset A^\wedge$. If $A \subset U$ and U is open, then $A^\wedge \subset U$ and so $\sigma cl(A) \subset U$. Therefore A is $\hat{\delta}$-closed.

If we put $I = \{\phi\}$ in the above Theorem 4.16, we get Corollary 4.17. If we put $I = P(X)$ in Theorem 4.16, we get Corollary 4.18.

Corollary 4.17. A subset A of a space (X, τ) is δg-closed if and only if $\delta cl(A) \subset A^\wedge$.

Corollary 4.18. A subset A of a space (X, τ) is g-closed if and only if $cl(A) \subset A^\wedge$.

Theorem 4.19. Let A be a $\wedge$-set of an ideal space (X, τ, I). Then A is $\hat{\delta}$-closed, if and only if A is δ-I-closed.

Proof: Necessity - Suppose A is $\hat{\delta}$-closed. By Theorem 4.16, $\sigma cl(A) = A^\wedge = A$, since A is a $\wedge$-set. Therefore, A is δ-I-closed.
Sufficiency - Proof follows from the fact that every δ-I-closed set is $\hat{\delta}$-closed.

If we put $I = \{\phi\}$ in Theorem 4.19, we get Corollary 4.20. If we put $I = P(X)$ in Theorem 4.19, we get Corollary 4.21.

Corollary 4.20. Let A be a $\wedge$-set of a space (X, τ). Then A is δg-closed if and only if A is δ-closed.

Corollary 4.21. Let A be a $\wedge$-set of a space (X, τ). Then A is g-closed if and only if A is closed.

Theorem 4.22. Let (X, τ, I) be an ideal space and $A \subset X$. If $A^\wedge$ is $\hat{\delta}$-closed, then A is also $\hat{\delta}$-closed.

Proof: Suppose that A^ is a $\hat{\delta}$ -closed set. If $A \subset U$ and U is open then $A^\wedge \subset U$. Since A^ is $\hat{\delta}$ -closed, $\sigma cl(A^\wedge) \subset U$. But $\sigma cl(A) \subset \sigma cl(A^\wedge)$. Therefore, A is $\hat{\delta}$ -closed.

If We put I ={ϕ} in Theorem 4.22, we get Corollary 4.23. If we put I = P(X) in Theorem 4.22, we get Corollary 4.24.

Corollary 4.23. Let (X, τ) be a topological space and $A \subset X$. If A^ is δg-closed, then A is also δg-closed.

Corollary 4.24. Let (X, τ) be a topological space and $A \subset X$. If A^ is g-closed then A is also g-closed.

Theorem 4.25. Let (X, τ, I) be an ideal space. If A is a $\hat{\delta}$ -closed subset of X and $A \subset B \subset \sigma cl(A)$, then B is also $\hat{\delta}$ -closed.

Proof: $\sigma cl(B) - B \subset \sigma cl(A) - A$, and since $\sigma cl(A) - A$ has no non-empty closed subset, $\sigma cl(B) - B$ also has no non-empty closed subset. Then By Theorem 4.1, B is $\hat{\delta}$ -closed.

If we put I = {ϕ} in the above Theorem 4.25, we get Corollary 4.26. If we put I = P(X) in the above Theorem 4.25, we get Corollary 4.27.

Corollary 4.26. Let (X, τ) be a topological space. If A is a δg-closed subset of X and $A \subset B \subset \delta cl(A)$, then B is also δg-closed.

Corollary 4.27. Let (X, τ) be a space. If A is a g-closed subset of X and $A \subset B \subset cl(A)$, then B is also g-closed.

Theorem 4.28. Let (X, τ, I) be an ideal space and $A \subset U$. Then the following are equivalent.

a) A is $\hat{\delta}$ -closed
b) $A \cup (X - \sigma cl(A))$ is $\hat{\delta}$ -closed
c) $\sigma cl(A) - A$ is $\hat{\delta}$ -open

Proof: (a) ⇒ (b) Suppose A is $\hat{\delta}$ -closed. If U is any open set such that $A \cup (X - \sigma cl(A)) \subset U$, then $X - U \subset X - (A \cup (X - \sigma cl(A)) = \sigma cl(A) - A$. Since A is $\hat{\delta}$ -closed, By Theorem 4.1, it follows that $X - U = \phi$ and so X = U. Since X is the only open set containing $A \cup (X - \sigma cl(A)$, $A \cup (X - \sigma cl(A))$ is $\hat{\delta}$ -closed.

(b) ⇒ (a) Suppose $A \cup (X - \sigma cl(A))$ is $\hat{\delta}$ -closed. If F is a closed set contained in $\sigma cl(A) - A$, then $A \cup (X - \sigma cl(A)) \subset X - F$ and X–F is open. Therefore $\sigma cl\ (A \cup X - \sigma cl(A)) \subset X - F$, which implies that $\sigma cl(A) \cup \sigma cl(X - \sigma cl(A) \subset X - F$ and so $X \subset X - F$ it follows that $F = \phi$. Hence A is $\hat{\delta}$ -closed.

The equivalence of (b) and (c) follows from the fact $X - (\sigma cl(A) - A) = A \cup (X - \sigma cl(A))$.

If we put I={ ϕ} in the above Theorem 4.28, we get Corollary 4.29, If we put I=P(X) in the above Theorem 4.28, we get Corollary 4.30.

Corollary 4.29. Let (x, τ) be a topological space, and $A \subset U$. Then the following are equivalent.

a) A is δg – closed
b) $A \cup (X - \delta cl(A))$ is σg closed.
c) $\delta cl\ (A) - A$ is δg – open

Corollary 4.30. Let (X, τ) be a space and $A \subset U$. Then the following are equivalent.

a) A is g – closed
b) AU(X–cl (A)) is g – closed
c) Cl(A) – A is g – open.

Theorem 4.31. For an ideal space (X, τ, I), the following are equivalent.

a) Every $\hat{\delta}$ - closed set is δ-I-closed
b) Every singleton of X is closed or δ-I-open.

Proof : (a)⇒(b) Let $x \in X$. If $\{x\}$ is not closed, then $A = X-\{x\}$ is not open and then A is trivially $\hat{\delta}$ -closed, Since the only open set containing A is X. Therefore by (A), A is δ-I-closed. Hence $\{x\}$ is δ-I-open.
(b)⇒(a) Let A be a $\hat{\delta}$ -closed set and let $x \in \sigma cl(A)$. We have the following cases.

Case (i). $\{x\}$ is closed. By Theorem 4.1, σcl (A) – A does not contain a non-empty closed subset. This shows that $\{x\} \in A$,

Case (ii) $\{x\}$ is δ - I – Open, then $\{x\} \cap A \neq \phi$. Hence $\{x\} \in A$.
Thus in both cases $\{x\} \in A$ and so A= σcl (A). That is, A is δ-I-closed, which proves (a).

If we put I = {ϕ} in the above Theorem 4.31, we get Corollary 4.32. If we put I=P(x) in the above Theorem 4.31, we get Corollary 4.33.

Corollary 4.32. For a topological space (X, τ), the following are equivalent.
a. Every δg-closed set is δ-closed.
b. Every singleton of X is closed or δ-open.

Corollary 4.32. For a topological space (X, τ), the following are equivalent.
a. Every g – closed set is closed
b. Every singleton of X is closed or open.

Theorem 4.33. Let (X, τ, I) be an ideal space and $A \subset X$. Then A is $\hat{\delta}$ - closed if and only if A=F–N, where F is δ-I-closed and N contains no non empty closed set.

Proof: Necessity - If A is $\hat{\delta}$ - closed, then By Theorem 4.1, N = σcl(A)–A contains no nonempty closed set. If F = σcl(A), then F is δ-I- closed, such that $F-N=\sigma cl(A) - (\sigma cl(A) - A) = \sigma cl(A) \cap (X-(\sigma cl(A) - A))=A$.
Sufficiency - Suppose A=F–N, Where F is δ-I-closed and N contains no non – empty closed set. Let U be an open set such that $A \subset U$. Then $F-N \subset U$ which implies that $F \cap (X-U) \subset N$. Now, $A \subset F$ and F is δ-I-closed implies that $\sigma cl(A) \cap (X-U) \subset \sigma cl(F) \cap (X-U) \subset F \cap (X-U) \subset N$. Since δ-I-closed sets are closed, $\sigma cl(A) \cap (x-U)$ is closed. By hypothesis $\sigma cl(A) \cap (X-U)=\phi$ and so $\sigma cl(A) \subset U$ which implies that A is $\hat{\delta}$ -closed.

If we put I = {ϕ}, in Theorem 4.34, we get Corollary 4.35, if we put I=P(X) in Theorem 4.34, we get corollary 4.36.

Corollary 4.35. Let (X, τ) be a topological space and $A \subset X$. Then A is δg-closed if and only if A = F – N, where F is δ-closed and N contains no nonempty closed set.

Corollary 4.36. Let (X, τ) be a topological space and $A \subset X$. Then A is g-closed if and only if A = F – N, where F is closed and N contains no nonempty closed set.

Theorem 4.37. Let (X, τ, I) be an ideal space and $A \subseteq X$. If $A \subseteq B \subseteq \sigma cl(A)$ then σcl(A)= σcl(B).

Proof : Since $A \subseteq B$, then $\sigma cl(A) \subseteq \sigma cl(B)$ and Since $B \subseteq \sigma cl(A)$, then $\sigma cl(B) \subseteq \sigma cl(\sigma cl(A)) = \sigma cl(A)$. Therefore σcl(A) = σcl(B).

Theorem 4.38. If (X, τ, I) is an ideal space, then σcl(A) is always $\hat{\delta}$ -closed for every subset A of X.

Proof : Let $\sigma cl(A) \subseteq U$ where U is open. Since σcl(σcl(A))= σcl(A) we have $\sigma cl(\sigma cl(A)) \subseteq U$ whenever $\sigma cl(A) \subseteq U$ and U is open. Hence σcl(A) is $\hat{\delta}$ -closed.

Theorem 4.39. Let (X, τ, I) be an Ideal space. Then every $\hat{\delta}$ -closed open set is δ-I-closed set.

Proof : Assume that A is $\hat{\delta}$ -closed and open set. Then $\sigma cl(A) \subset A$ whenever $A \subseteq A$ and A is open. Thus A is δ-I-closed.

Theorem 4.40. If A is both semi-open and pre-closed set in an ideal space (X, τ, I), then A is $\hat{\delta}$ -closed in (X, τ, I).

Proof : If is clear that if A is both semi-open and pre-closed then A is regular closed and hence it is δ-closed in (X, τ, I) therefore it is $\hat{\delta}$ -closed in (X, τ, I).

Corollary 4.41. If A is both open and pre-closed set in an ideal space (X, τ, I), then A is $\hat{\delta}$ -closed in (X, τ, I).

Theorem 4.42. In an Ideal Space (X, τ, I), for each x∈X, either {x} is closed or $\{x\}^c$ is $\hat{\delta}$ -closed. That is $X = X_c \cup X_{\hat{\delta}}$.

Proof: Suppose that {x} is not a closed set in (X, τ, I). Then $\{x\}^c$ is not an open set and the only open set containing $\{x\}^c$ is X. Therefore $\sigma cl(\{x\}^c) \subseteq X$ and hence $\{x\}^c$ is $\hat{\delta}$ -closed set in (X, τ, I).

Theorem 4.43. In an Ideal Space (X, τ, I), $X_2 \cap \sigma cl(A) \subseteq A^\wedge$ for any subset A of (X, τ, I), where $X_2 = \{x \in X : \{x\} \subset int(cl^*(\{x\}))\}$.

Proof: Suppose that $x \in X_2 \cap \sigma cl(A)$ and $x \notin A^\wedge$. Since $x \in X_2$ and $x \notin X_1$ implies that $int(cl^*(\{x\})) \neq \phi$. Since $x \in \sigma cl(A)$, $A \cap int(cl^*(U)) \neq \phi$ for any open set U containing x. Choose $U = int(cl^*(\{x\}))$. Then $A \cap int(cl^*(\{x\})) \neq \phi$. Choose $y \in A \cap int(cl^*(\{x\}))$. Since $x \notin A^\wedge$, there exist a open set v in (X, τ, I) set such that $x \in F \subseteq X - A$. Also $int(cl^*(\{x\})) \subseteq int(cl^*(F)) \subseteq F$ and hence $y \in A \cap F$, a contradiction. Thus $x \in A^\wedge$.

Definition 4.44. A partition space is a topological space where every open set is closed.

Theorem 4.45. In an Ideal space (X, τ, I) the following conditions are equivalent.

a) X is a partition space

b) Every subset of X is $\hat{\delta}$ -closed

Proof: (a) ⇒ (b). Let $A \subset U$, where U is open and A is an arbitrary subset of X. Since X is a partition space, U is clopen. Thus $\sigma cl(A) \subseteq \sigma cl(U) = U$.
(b) ⇒ (a). If $U \subseteq X$ is open, then by (b) $\sigma cl(U) = U$ or equivalently U is δ-I-closed and hence closed.

Definition 4.46. A proper nonempty $\hat{\delta}$ -closed subset A of an ideal space (X, τ, I) is said to be maximal $\hat{\delta}$ -closed if any $\hat{\delta}$ -closed set containing A is either X or A.

Example 4.47. Let X = {a, b, c, d}, τ = {X, ϕ, {a}, {a, b}} and I = {ϕ} Let A = {a, b, c}. Then A is maximal $\hat{\delta}$ -closed set.

Remark 4.48. Every maximal $\hat{\delta}$ -closed set is a $\hat{\delta}$ -closed set but not conversely as shown in the following example.

Example 4.49. In the above Example 4.47. A = {b, c} is $\hat{\delta}$ -closed but not maximal $\hat{\delta}$ -closed.

Theorem 4.50. In an ideal space (X, τ, I), the following statements are true.

(i) Let F be a maximal $\hat{\delta}$ -closed set and G be a $\hat{\delta}$ -closed set. Then $F \cup G = X$ or $G \subset F$.

(ii) Let F and G be maximal $\hat{\delta}$ -closed sets. Then $F \cup G = X$ or $F = G$.

Proof: (i) Let F be a maximal $\hat{\delta}$ -closed set and G be a $\hat{\delta}$ -closed set. If $F \cup G = X$, then there is nothing to prove. Assume that $F \cup G \neq X$. Now, $F \subset F \cup G$. By Theorem 4.10. $F \cup G$ is a $\hat{\delta}$ -closed set. Since F is a maximal $\hat{\delta}$ -closed set, we have $F \cup G = X$ or $F \cup G = F$. Hence $F \cup G = F$ and so $G \subset F$.

(ii) Let F and G be maximal $\hat{\delta}$ -closed sets. If $F \cup G = X$, then there is nothing to prove. Assume that $F \cup G \neq X$. Then by (i) $F \subset G$ and $G \subset G$ and $G \subset F$ which implies that F = G.

Theorem 4.51. Let (X, τ, I) be an ideal space and A a subset of X.Then X - σcl(X–A) = σint(A).

Theorem 4.53. If A is an $\hat{\delta}$ -open set of an ideal space (X, τ, I) and $\sigma int(A) \subseteq B \subseteq A$. Then B is also an $\hat{\delta}$ -open set of (X, τ, I).

Proof: Suppose $F \subseteq B$ where F is closed set of (X, τ, I). Then $F \subseteq A$. Since A is $\hat{\delta}$ -open, $F \subseteq \sigma int(A)$. Since $\sigma int(A) \subseteq \sigma int(B)$, we have $F \subseteq \sigma int(B)$. By the above Theorem 4.52, B is $\hat{\delta}$ -open.

Theorem 4.52. A subset A of an ideal space (X, τ, I) is $\hat{\delta}$ -open if and only if $F \subset \sigma int(A)$ whenever F is closed and $F \subseteq A$.

Proof: Necessity - Suppose A is $\hat{\delta}$ -open and F be a closed set contained in A. Then $X–A \subset X–F$ and hence $\sigma cl(X–A) \subseteq X–F$. Thus $F \subseteq X–\sigma cl(X–A) = \sigma int(A)$.
Sufficiency - Suppose $X–A \subseteq U$ where U is open. Then $X–U \subseteq A$ and X–U is closed. Then $X–U \subseteq \sigma int(A)$ which implies $\sigma cl(X–A) \subseteq U$. Consequently X–A is $\hat{\delta}$ -closed and so A is $\hat{\delta}$ -open.

Theorem 4.53. Let x be any point in an ideal topoligical space (X, τ, I). Then either int(cl*({x})) = ϕ or $\{x\} \subset$ int(cl*({x}) in (X, τ, I). Also $X = X_1 \cup X_2$, where $X_1 = \{x \in X : int\ (cl^*\{x\}) = \phi\}$ and $X_2 = \{x \in X : \{x\} \subset int(cl^*(\{x\})$.

Proof. Let x be any point in a an ideal space (X, τ, I).

Case (i) If $U \subset cl^*(\{x\})$ for some $U \in \tau$ (x), then $x \in U \subset int(cl^*(\{x\}))$

Case (ii) If $U \not\subset cl^*(\{x\})$ for all $U \in \tau$ (x). Let V be any open set if $x \in V$ then $V \not\subset cl^*(\{x\})$. If $x \notin V$, then for any $y \in V$, $\{x\} \cap V = \phi \in I$. Therefore $y \notin cl^*(\{x\})$. Therefore $V \not\subset cl^*(\{x\})$. Therefore $V \not\subset cl^*(\{x\})$ for any open set V. Therefore int(cl*({x})) = ϕ.

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