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Dynamic Programming approach in Two Echelon Inventory System with Repairable items

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Abstract: *The problem we address is to decide optimal ordering policy in a two echelon inventory system with repairable items. The system consists of a set of operating sites which are base station, depot. Each operating site has an inventory spare items to run the system properly without any delay of supplying items against failure demand. Replenishment of stock at base station is done from depot. The base station contains an inspection center which is used to inspect the failed items. The arrival of a failed items follows Poisson process with parameter λ (>0) and the inspection time follows an exponential distribution with rate μ (>0). After inspection the repairable items are sent to depot for repair, otherwise the item is treated as condemned and it would be removed from the system. The repaired items which can be used are included in the depot stock. The system is designed under the continuous review Markov process model to find the steady distribution and applying the decision rule to obtain the optimal average cost of the system and optimal ordering policy by using Markov decision process concept. Numerical examples are provided to illustrate the problem.*
Keywords: *Repairable items, Poison arrival, Inspection time, Transition probability, Value iteration.*

I. Introduction

Inventory management in the context of recoverable items has drawn a lot of attention from researchers. During the time of war, we can not manage all the weapons at the spot. So, we keep the base inventory at the spot for replacement of failed items. At the same time, maintenance of inventory at depot is important to supply the items to the base inventory. The fact is that all the failed items are not fully recoverable. So, early diagnosis is reduces the service time, service cost and transport cost of the failed items.

Sherbrooke [11] pointed out that the US Air Force spent 15 billion dollars for recoverable items from the total investment. Schrady [10] studied a single-facility recoverable item model in which the demand of failed items were deterministic and item condemnations also occured. Phelps [8] concentrated on the decision making as when to repair and when to procure multi-product recoverable items and he assumed that the repairing time and procurement time were zero. Allen and D'Esopo [1] obtained approximate steady-state results for a single-facility model of recoverable parts subject to condemnation; they assumed stochastic demand and deterministic (positive) repair and procurement lead-times.

Simon [12] considered a two-echelon model for consumable or repairable parts in which the repair and transportation times are assumed to be deterministic, and the failure processes that generate demands are assumed to be a Poisson. Hau L. Lee and Kamran Moinzadeh [4] considered an inventory system for repairable items where the stocking point also serves as a repair center for the failed items. Operating characteristics of the inventory system were analysed and optimal re-order points and order quantities were also obtained.

Axsäter [2] developed a recursive procedure to determine average holding and shortage costs and discussed the determination of optimal inventory base stock levels. Hopp et al. [5] considered that the depots were replenished by a control warehouse that followed an (r,Q) inventory policy. Their focus was to minimize system-wide inventory holding costs while keeping the average total delay at each facility below a threshold level. Jun Xie Hongwei Wang [7] calculated the performance measures and optimal stock of the two echelon inventory system by using simulation algorithm and genetic algorithms.

In this paper, we considered a two echelon inventory system with repairable items which is treated as a Markov process with continuous time review model and the steady state probability distribution is obtained in each state of the system. Besides, the problem is modelled as a Markov decision model with specific cost structure at each state of the system. Transportation time, transportation cost, repairing cost and item cost are considered to formulate the cost function for inventory maintenance.

The paper is organized as follows. Section 2 presents the notations and assumptions which are given to describe the system. The models are formulated in section 3. Section 4 presents numerical examples followed by a conclusion in section 5.

II. Model Description

2.1 Notations

The following notations are used in describing the system.

$B(t)$ $\rightarrow$ *Base Inventory level at time t.*

$D(t)$ $\rightarrow$ *Depot Inventory level at time t.*

λ $\rightarrow$ *Arrival rate of the failed item to the base station (Poisson arrival).*

μ $\rightarrow$ *Inspection rate of the failed item.*

δ $\rightarrow$ *Service rate of the failed item.*

γ $\rightarrow$ *Transportation rate from depot to base.*

c $\rightarrow$ *Item cost per unit.*

c_f $\rightarrow$ *Transportation cost from supplier to depot.*

c_r $\rightarrow$ *Holding cost of each item at depot.*

c_t $\rightarrow$ *Transportation cost from depot to base.*

2.2 Assumptions:

The arrival process of failed item to inspection centre is a Poison with rate λ. However, the failed items arrive at the base, the replacement is done from the base stock and an order is placed at the depot for replenishment under (S-1,S) policy. The failed item is inspected at the base station and the inspection time is exponentially distributed with parameter $\mu(>0)$.

The inspection reveals either the item is repairable or condemn with certain probabilities p and (1-p) respectively. If the item is condemned, it is removed from the system. The rate of repairable item sent to Central depot is μp. The transportation time of a failed item from base to depot is assumed to be negligible.

Assume that ample servers are available at depot to avoid the queues ($M/M/\infty$). So that the repairable item from inspection center arrives to the depot for repair, the service is started immediately. The service time of an item is exponentially distributed with parameter $\delta(>0)$. After completion of service the item is included in the depot stock and the holding cost is incurred for each item. The lead time for replenishment at base is also exponentially distributed with rate $\gamma(>0)$. The reordering policy at depot is 'Order up to S', that is, an order is placed to pull inventory from 's' to the max level 'S' (s+Q=S, where Q is the re-order quantity).

III. Analysis and Formulation of the system

3.1 Analysis of the steady state probability distribution of the system

Let $B(t)$ and $D(t)$ denote the inventory levels of the base station and depot at time 't'. It is considered as a two dimensional continuous time Markov chain $\{(B(t), D(t)), t \geq 0\}$ on the state space $I = \{(q,i);\ 0 \leq q \leq S_1, 0 \leq i \leq S_2\}$, where S_1 and S_2 are the maximum inventory level of base and depot and assume that $S_1 < S_2$.

Changes of state occur when the environment changes, when a failed item arrives or when inspection is completed or when a service is completed. The possible transitions, and the corresponding instantaneous rates are given in the table beow.

From	To	Rate
(q,i)	$(q+1,i)$	λ
(q,i)	$(q+1,i-1)$	γ
(q,i)	$(q-1,i)$	$\mu(1-p)$
(q,i)	$(q-1,i+1)$	μp
(q,i)	$(q,i+1)$	δ

In order to display the infinitesimal generator Q of the system, it is neccessary to define a linear ordering of the states. The lexicographical ordering of the states is

$\{(0,1),(0,2),\ldots,(0,i),(1,1),(1,2),\ldots,(1,i),\ldots(q,1),(q,2),\ldots,(q,i),\ldots\}$.

and the corresponding infinitesimal generator Q is given by

$$Q=\begin{pmatrix} A & B & 0 & 0 & 0 & \cdots \\ C & A_1 & B & 0 & 0 & \cdots \\ 0 & C & A_1 & B & 0 & \cdots \\ 0 & 0 & C & A_1 & B & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & B_1 \end{pmatrix},$$

Where

$$A=\begin{pmatrix} -\delta-\lambda & \delta & 0 & 0 & \cdots \\ 0 & -\delta-\gamma-\lambda & \delta & 0 & \cdots \\ 0 & 0 & -\delta-\gamma-\lambda & \delta & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix}$$

$$A_1=\begin{pmatrix} -\mu-\delta-\lambda & \delta & 0 & 0 & \cdots \\ 0 & -\mu-\delta-\gamma-\lambda & \delta & 0 & \cdots \\ 0 & 0 & -\mu-\delta-\gamma-\lambda & \delta & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & -\mu-\delta-\gamma-\lambda & \delta \\ 0 & 0 & 0 & 0 & -(1-p)\mu-\gamma-\lambda \end{pmatrix}$$

$$B=\begin{pmatrix} \lambda & 0 & 0 & 0 & \cdots \\ \gamma & \lambda & 0 & 0 & \cdots \\ 0 & \gamma & \lambda & 0 & \cdots \\ 0 & 0 & \gamma & \lambda & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix}$$

$$B_1=\begin{pmatrix} -\mu-\delta & \delta & 0 & 0 & \cdots & 0 \\ 0 & -\mu-\delta & \delta & 0 & \cdots & 0 \\ 0 & 0 & -\mu-\delta & \delta & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \delta \\ 0 & 0 & 0 & 0 & \cdots & -(1-p)\mu \end{pmatrix}$$

$$C=\begin{pmatrix} (1-p)\mu & p\mu & 0 & 0 & \cdots \\ 0 & (1-p)\mu & p\mu & 0 & \cdots \\ 0 & 0 & (1-p)\mu & p\mu & \cdots \\ 0 & 0 & 0 & (1-p)\mu & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix}$$

The Embedded Markov chain method is used to find the steady state probability distribution π of a continuous – time Markov chain Q (refer [3],[6]). The Embedded Markov chain of Q is a regular discrete – time Markov chain T with the transition probability from state (q,i) to (r,j), as given by

$$t_{(q,i)(r,j)} = \begin{cases} \dfrac{p_{(q,i)(r,j)}}{\sum_{(q,i)\neq(s,k)} p_{(q,i)(s,k)}}, & \text{if} \quad (q,i) \neq (r,j) \\ 0 & \text{otherwise} \end{cases}$$

where $t_{(q,i)(r,j)} \varepsilon T$ and $p_{(i,j)(k,l)} \varepsilon Q$.

3.2 Formulation of the system by using Markov decision concept

The two dimensional Markov process $\{(B(t), D(t)), t \geq 0\}$ can be converted as a two dimensional Embedded Markov chain with state space $I = \{(q,i);\ 0 \leq q \leq S_1, 0 \leq i \leq S_2\}$ which is reviewed at equidistant points of time $t = 0,1,2,\cdots$. The inventory levels of base and depot are reviewed periodically. At each review the system is classified into one of a possible number of states and subsequently a decision has to be made. For each state $(q,i)\varepsilon I,$ a set $A(i)$ of decisions or actions is given. The state space I and the action sets $A(i)$ are assumed to be finite. At the beginning of each period, the inventory level may be in any one of the state (q,i).

The economic consequences of the decisions taken at the review times (decision epochs) are reflected in costs and it satisfies the Markovian property. If at a decision epoch the action 'a' is chosen in state (q,i), then regardless of the past history of the system, the following happens:

(a) an immediate cost $c_{(q,i)}(a)$ is incurred.

(b) at the next decision epoch the system will be in state (r,j) with probability $P_{(q,i)(r,j)}(a)$, where $\sum_{(r,j)\varepsilon I} P_{(q,i)(r,j)}(a) = 1, \quad (q,i)\varepsilon I.$ refer Tijms [13].

At each state of the system the decision can be taken from the action set A. The possible actions are denoted by,

$$a = \begin{cases} 0 & - \quad \textit{No order is placed.} \\ 1 & - \quad \textit{Safety purchase is done to fulfill the depot inventory levels.} \\ 2 & - \quad \textit{Compulsory purchase is done when the depot stock comes zero.} \end{cases}$$

where $a \in A$.

Then for action a=0 in state (q,i), the transition probability is of the form, $P_{(q,i)(r,j)}(0) = t_{(q,i)(r,j)}$, where $t_{(q,i)(r,j)}\varepsilon T.$

The corresponding one-step cost function in state (q,i) is given by,

$$c_s(0) = \begin{cases} c_t q + c_r i & ,1 \leq q < S_1 \quad ;1 \leq i \leq S_2 \\ c_t q & ,1 \leq q < S_1 \quad ;i = S_2 \\ c_r i & ,q = S_1 \quad ;1 \leq i \leq S_2 - 1 \\ 0 & ,q = S_1 \quad ;i = S_2 \end{cases}$$

where c_t is the transportation cost from base to depot and c_r is the repairing cost.

For action a=1 in state (q,i), $P_{(q,i)(q,S_2)}(1) = 1.$

While the time of service, the repairable item is available in depot stock ,but it is not in use. In this situation, we take action a=1 to fulfil the depot stock by the remaining quantity and avoid the backlogging order at the depot from the base station. The corresponding one step-cost function in state (q,i) is,

$$c_{(q,i)}(1) = \quad c_f + |S_2 - i|\, c \quad ,0 \leq q \leq S_1 - 1 \quad ;1 \leq i \leq S_2 - 1.$$

where c_f is the transportation cost for replenishment of depot inventory from the supplier.

For action a=2 in state $(q,0)$, $P_{(q,0)(q,S_2)}(2)=1$ and the corresponding cost function in state $(q,0)$ is,

$c_{(q,i)}(2) = \quad c_f + S_2 c \quad ,0 \le q \le S_1 \quad ;i = 0.$

Assume that the set of all state space is a weak unichain (Tijms [13]) and the minimal average cost per time unit is independent of the initial state. Let g^* denote the minimal long run average cost per unit time. The value iteration algorithm computes recursively for $n = 1,2,\ldots$, the optimality equation is

$$v_n(q,i) = \min_{a \in A(q,i)} \left\{ c_{(q,i)}(a) + \sum_{(r,j) \in I} p_{(q,i)(r,j)}(a) v_{n-1}(r,j) \right\}, (q,i) \in I \qquad (2)$$

where $v_n(q,i)$ is the value function and it is starting with an arbitrary chosen function $v_0(q,i)$, $(q,i) \in I$. The quantity $v_n(q,i)$ can be interpreted as the minimal total expected costs with n periods left to the time horizon when the currentstate is (q,i) and a terminal cost $v_0(r,j)$ is incurred when the system ends up at state (r,j).

Value iteration algorithm is used to obtain the one-step difference $v_n(q,i) - v_{n-1}(q,i)$, which is very close to the minimal average cost per time unit and the actions can be taken in each state of the system to minimize the right hand side of the value function. It is very close to the minimal average maintenance cost of the system.

Theorem 1: Suppose that the weak unichain assumption is satisfied. Let $v = (v_{(q,i)})$ be given, and define the stationary policy $R(v)$ as a policy which adds to each state $(q,i) \in I$ an action $a = R_{(q,i)}(v)$ that minimizes the right-hand side of (2). Then

$$\min_{(q,i) \in I} \left\{ (Tv)_{(q,i)} - v_{(q,i)} \right\} \le g^* \le g_{(s,t)}(R(v)) \le \max_{(q,i) \in I} \left\{ (Tv)_{(q,i)} - v_{(q,i)} \right\} \qquad (3)$$

for any $(s,t) \in I$, where g^* is the minimal long-run average cost per time unit and $g_{(s,t)}(R(v))$ denotes the long-run average cost per time unit under policy $R(v)$ when the initial state is (s,t).

Theorem2: In the standard value iteration algorithm, the lower and upper bounds satisfy $m_{k+1} \ge m_k$ *and* $M_{k+1} \le M_k$ *for all* $k \ge 1.$ where

$$m_k = \min_{(q,i) \in I} \{ v_k(q,i) - v_{k-1}(q,i) \}; \qquad M_k = \max_{(q,i) \in I} \{ v_k(q,i) - v_{k-1}(q,i) \}.$$

The proof of these theorems could be found in our earlier work (Satheesh kumar and Elango [9]). Based on the above theorems we can get the following value iteration algorithm.

Algorithm:

Step 0 : (initialization).Choose $v_0(q,i), (q,i) \in I$ with $0 \le v_0(q,i) \le min_a c_{(q,i)}(a)$.

Let n:=1.

Step 1: (value-iteration step).For each state $(q,i) \in I$, compute

$$v_n(q,i) = \min_{a \in A(q,i)} \left\{ c_{(q,i)}(a) + \sum_{(r,j) \in I} p_{(q,i)(r,j)}(a) v_{n-1}(r,j) \right\}, \qquad (q,i),(r,j) \in I$$

Let $R(n)$ be any stationary policy such that the action $a = R_{(q,i)}(n)$ minimizes the right-hand side of the equation for $v_n(q,i)$ for each state (q,i).

Step 2: (bounds on the minimal costs).Compute the bounds

$$m_n = \min_{(q,i) \in I} \{ v_n(q,i) - v_{n-1}(q,i) \}; \qquad M_n = \max_{(q,i) \in I} \{ v_n(q,i) - v_{n-1}(q,i) \}.$$

Step 3: (stopping test). If $0 < M_n - m_n \le \varepsilon m_n$ with $\varepsilon > 0$ a prespecified accuracy number $(e.g.\varepsilon = 0.001)$, stop with policy $R(n)$.

Step 4: (continuation). $n := n+1$ and repeat step 1. By the theorem, we have

$$0 \leq \frac{g_{(q,i)}(R(n)) - g^*}{g^*} \leq \frac{M_n - m_n}{m_n} \leq \varepsilon, \ (q,i) \in I.$$

where the algorithm is stopped after the n^{th} iteration with policy $R(n)$. In other words, the average cost of policy $R(n)$ cannot deviate more than ε from the theoretically minimal average reordering cost when the bounds m_n and M_n satisfy $0 < M_n - m_n \leq \varepsilon m_n$. In practical applications one is usually satisfied with a policy whose average cost is sufficiently close to the theoretically minimal average cost.

IV. Numerical examples and discussions

The system is designed as a two dimensional continuous time Markov chain process $\{(B(t), D(t)), t \geq 0\}$ on the state space $I = \{(q,i);\ 0 \leq q \leq S_1, 0 \leq i \leq S_2\}$, where S_1 and S_2 are the maximum inventory level of base and depot and assume that $S_1 < S_2$. Let S_1=3 and S_2=5. Form the infinitesimal generator Q and using embbed Markov chain concept to obatined the steady state probability distribution of each state of the system for the different values of λ, γ, μ, δ and p=0.6.

S.No	π	λ=4 μ=6 δ=3 γ=5	λ=4 γ=5 μ=6 δ=3	λ=4 γ=5 μ=6 δ=3	λ=4 γ=5 μ=6 δ=3	λ=4 γ=5 μ=6 δ=3
1	π_1	0.0000	0.0000	0.0000	0.0000	0.0000
2	π_2	0.0001	0.0000	0.0000	0.0001	0.0001
3	π_3	0.0011	0.0002	0.0002	0.0010	0.0006
4	π_4	0.0293	0.0046	0.0052	0.0235	0.0067
5	π_5	0.0186	0.0020	0.0024	0.0116	0.0029
6	π_6	0.0000	0.0001	0.0000	0.0001	0.0002
7	π_7	0.0006	0.0003	0.0002	0.0006	0.0012
8	π_8	0.0090	0.0032	0.0027	0.0075	0.0061
9	π_9	0.0303	0.0090	0.0084	0.0271	0.0148
10	π_{10}	0.1075	0.0405	0.0442	0.0959	0.0463
11	π_{11}	0.0002	0.0003	0.0002	0.0002	0.0013
12	π_{12}	0.0023	0.0026	0.0016	0.0022	0.0069
13	π_{13}	0.0116	0.0118	0.0081	0.0118	0.0238
14	π_{14}	0.0759	0.0723	0.0624	0.0689	0.0956
15	π_{15}	0.3335	0.3407	0.3556	0.3617	0.3119
16	π_{16}	0.0004	0.0011	0.0006	0.0004	0.0032
17	π_{17}	0.0027	0.0057	0.0033	0.0027	0.0124
18	π_{18}	0.0167	0.0330	0.0239	0.0156	0.0485
19	π_{19}	0.1371	0.2186	0.2063	0.1179	0.2081
20	π_{20}	0.2230	0.2540	0.2746	0.2514	0.2092

Table 1. Steady state distribution of each states.

It is seen from Table 1 that some of the states having probability values as zero in a long run. It reveals that the inventory position of the system would not come to the particular states due to the arrival rate, inspection time, service time and the tranportation time.

Applying the decision rule the system is considered as a Markov decision process model with action set A. Form the value (cost) function associated with the following costs. Let c be the item cost per unit. Let c_t be

the transportation cost from depot to base and c_h be the holding cost for each item. Let c_f be the transportation cost for replenishment of inventory at depot from the Manufacturer.
The Optimal average cost of inventory maintenance is given below for the different values of λ , γ , μ , δ , c, c_t , c_h , c_f and p.

For p = 0.4

Case(i):

Table 2. Optimal Average cost & Stationary Policies for $c = 1$; $c_t = 2$; $c_h = 1.5$; $c_f = 3$.

S. No	λ	μ	δ	γ	Number of iterations	Optimal Average cost	Stationary Policy
1	4	5	6	3	27	09.9031	[0 0 0 0 0 0 0 0 1 0 0 0 0 1 0 0 0 0 1 0 2 2 2 2]
2	5	3	4	7	33	10.2597	[0 0 0 0 0 0 0 0 1 0 0 0 0 1 0 0 0 0 1 0 2 2 2 2]
3	5	3	5	6	28	10.3262	[0 0 0 0 0 0 0 0 1 0 0 0 0 1 0 0 0 0 1 0 2 2 2 2]
4	7	6	4	3	34	10.3750	[0 0 0 0 0 0 0 0 1 0 0 0 0 1 0 0 0 0 1 0 2 2 2 2]
5	6	4	3	8	50	10.1862	[0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 1 0 2 2 2 2]

Case(ii):

Table 3. Optimal Average cost & Stationary Policies for $c = 2$; $c_t = 2$; $c_h = 3$; $c_f = 5$.

S. No	λ	μ	δ	γ	Number of iterations	Optimal Average Cost	Stationary Policy
1	4	5	6	3	27	16.4337	[0 0 0 0 0 0 0 0 1 0 0 0 0 1 0 0 0 0 1 0 2 2 2 2]
2	5	3	4	7	35	16.6751	[0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 1 0 2 2 2 2]
3	5	3	5	6	29	16.8064	[0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 1 0 2 2 2 2]
4	7	6	4	3	35	17.0736	[0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 1 0 2 2 2 2]
5	6	4	3	8	53	16.4406	[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 2 2 2 2]

For p = 0.5

Case(i):

Table 4. Optimal Average cost & Stationary Policies for $c = 1$; $c_t = 2$; $c_h = 1.5$; $c_f = 3$.

S. No	λ	μ	δ	γ	Number of iterations	Optimal Average cost	Stationary Policy
1	4	5	6	3	25	10.0993	[0 0 0 0 0 0 0 0 1 0 0 0 0 1 0 0 0 0 1 0 2 2 2 2]
2	5	3	4	7	31	10.3242	[0 0 0 0 0 0 0 0 1 0 0 0 0 1 0 0 0 0 1 0 2 2 2 2]
3	5	3	5	6	26	10.3951	[0 0 0 0 0 0 0 0 1 0 0 0 0 1 0 0 0 0 1 0 2 2 2 2]
4	7	6	4	3	31	10.5700	[0 0 0 0 0 0 0 0 1 0 0 0 0 1 0 0 0 0 1 0 2 2 2 2]
5	6	4	3	8	43	10.2859	[0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 1 0 2 2 2 2]

Table 5. Optimal Average cost & Stationary Policies for $c = 2$; $c_t = 2$; $c_h = 3$; $c_f = 5$.

Case(ii):

S. No	λ	μ	δ	γ	Number of iterations	Optimal Average cost	Stationary Policy
1	4	5	6	3	25	16.6744	[0 0 0 0 0 0 0 0 1 0 0 0 0 1 0 0 0 0 1 0 2 2 2 2]
2	5	3	4	7	32	16.7615	[0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 1 0 2 2 2 2]
3	5	3	5	6	27	16.8925	[0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 1 0 2 2 2 2]
4	7	6	4	3	31	17.3365	[0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 1 0 2 2 2 2]
5	6	4	3	8	47	16.6402	[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 2 2 2 2]

For p = 0.6

Case(i):

Table 6. Optimal Average cost & Stationary Policies for $c = 1$; $c_t = 2$; $c_h = 1.5$; $c_f = 3$

S. No	λ	μ	δ	γ	Number of iterations	Optimal Average cost	Stationary Policy
1	4	5	6	3	24	10.3001	[0 0 0 0 0 0 0 0 1 0 0 0 0 1 0 0 0 0 1 0 2 2 2 2]
2	5	3	4	7	28	10.3841	[0 0 0 0 0 0 0 0 1 0 0 0 0 1 0 0 0 0 1 0 2 2 2 2]
3	5	3	5	6	25	10.4618	[0 0 0 0 0 0 0 0 1 0 0 0 0 1 0 0 0 0 1 0 2 2 2 2]
4	7	6	4	3	27	10.7597	[0 0 0 0 0 0 0 0 1 0 0 0 0 1 0 0 0 0 1 0 2 2 2 2]
5	6	4	3	8	38	10.3672	[0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 1 0 2 2 2 2]

Case(ii):

Table 7. Optimal Average cost & Stationary Policies for $c = 2\,;\, c_t = 2\,;\, c_h = 3\,;\, c_f = 5$.

S. No	λ	μ	δ	γ	Number of iterations	Optimal Average cost	Stationary Policy
1	4	5	6	3	24	16.9163	[0 0 0 0 0 0 0 0 1 0 0 0 0 1 0 0 0 0 1 0 2 2 2 2]
2	5	3	4	7	30	16.8404	[0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 1 0 2 2 2 2]
3	5	3	5	6	26	16.9731	[0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 1 0 2 2 2 2]
4	7	6	4	3	28	17.5758	[0 0 0 0 0 0 0 0 1 0 0 0 0 1 0 0 0 0 1 0 2 2 2 2]
5	6	4	3	8	41	16.7963	[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 2 2 2 2]

From Table 2 to 7, the optimal average cost and optimal reordering policies are obtained for the repairable rate p= 0.4, 0.5, 0.6 based on the values of λ, γ, μ, δ, c, c_t, c_h, c_f and p. From this result, we see that if the repairable rate increases, then the average cost of maintenance is also increase. The outcome of the result is illustrated from the graph Figure 1, given below.

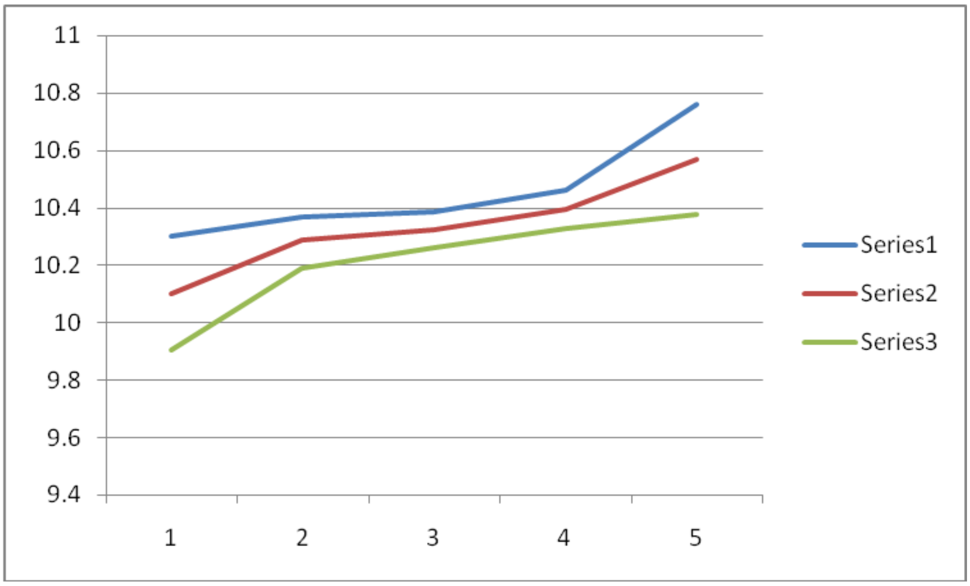

Fig. 1

V. Conclusion

Management of inventory in the context of repairable items has drawn attention of the researchers. But the analysis of the failed item which is recoverable or not is extremely difficult. Most of the papers available in literature address the problem for a single echelon. The literature of Multi echelon systems with repairable item is very limited. In this paper, we considered a two-echelon inventory system with repairable items under generalized conditions, and developed a infinitesimal generator Q. The Embedded Markov chain method is implemented to find the steady state probability of the each state of the system. The same system is treated as a Markov decision model problem associated with action set A. This model is solved by value iteration procedure to obtain the optimal average cost to maintain the inventory in the two-echelon inventory system. The sensitive analysis of the cost funtion in section 4 reveals that, if the repairing rate is increased, the average cost of inventory maintenance is also increased. The future investigation of the two echelon inventory system is to find the optimal average maintenance cost for the system consisting of one depot and finite number of base stations.

This study may be furthered to include Multi-echelon inventory system holding perishable inventory. For example,

1. Perishable items supply from depot to base stations.
2. Maintenance of blood units in the district head quarter and rural areas.

The complexity level would increase when the requirements of the base stations differ from one another. If we allow formation of queue in the model the study would become more challenging and complex. We may apply a decision rule in each station to replenish and try developing a model using Markov decision process.

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