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On Pairs of Pythagorean Triangles –I

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Abstract: *We search for pairs of distinct Pythagorean triangles suchthat the difference between their perimeters is represented by (i)* $k\alpha^2$ *(ii)* $k\alpha^n, n > 2$ *(iii)* $3P_{2N}^{3}$ *(iv)* $12Pt_{2N}$

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I. Introduction:

Number is the essence of mathematical calculation. Varieties of numbers have variety of range and richness, many of which can be explained very easily but extremely difficult to prove. One of the varieties of numbers that have fascinated mathematicians and the lovers of maths is the pythagorean number as they provide limitless supply of exciting and interesting properties. In other word, the method of obtaining three non-zero integers x,y and z under certain relations satisfying the equation $x^2 + y^2 = z^2$ has been a matter of interest to various mathematicians. For an elaborate review of various properties are may refer [1-17]. In [18], the author proves existence of an infinite family of pairs of dissimilar Pythagorean triangles that are pseudo smarandache related. For problems on pairs of Pythagorean triangles one may refer [1,5].

In this communication concerns with the problem of determining pairs of Pythagorean triangles wherein each pair the difference between the perimeters is represented by (i) $k\alpha^2$ (ii) $k\alpha^n, n > 2$ (iii) $3P_{2N}^{3}$ (iv) $12Pt_{2N}$

II. Method Of Analysis:

Let $PT_1(m^2 - q^2, 2mq, m^2 + q^2)$ and $PT_2(p^2 - q^2, 2pq, p^2 + q^2)$ be two distinct pythagorean triangles whose perimeters are $P_1 = 2m(m+q), P_2 = 2p(p+q)$ (1)

where $m, p, q > 0; p, m > q$ (1a)

Assume $P_1 - P_2 = k\alpha^2, k > 0$ (2)

Using (1) in (2), it is written as

$$(2m+q)^2 - (2p+q)^2 = 2k\alpha^2 \quad (3)$$

Choice (i) : Let k be suchthat 2k is not a perfect square. Then the solutions of (3) are given by

$$\alpha = 2rs$$
$$2p + q = 2kr^2 - s^2 \quad (4)$$
$$2m + q = 2kr^2 + s^2$$

Solving the system of equation (4), one obtains

$$m = s^2 + p$$
$$q = 2kr^2 - s^2 - 2p$$

Since m, q and p has to satisfy (1a), it is seen that the values of r,s,k and p has to satisfy the condition

$$2p < 2kr^2 - s^2 < 3p \quad (5)$$

Thus, knowing the values of m,p and q it is seen that $P_1 - P_2 = k\alpha^2$

The above process is illustrated through the following examples.

k	r	s	p	q	m	P_1	P_2	$P_1 - P_2 = k\alpha^2$
1	3	2	5	4	9	234	90	$1(12)^2$
2	2	1	6	3	7	140	108	$2(4)^2$
3	2	2	7	6	11	374	182	$3(8)^2$
4	2	1	12	7	13	520	456	$4(4)^2$
1	5	3	14	13	23	1656	756	$1(30)^2$

Choice (2): Let k be such that 2k is a perfect square. Then the solutions of (3) are given by

$$2p + q = 2rs, \beta = r^2 - s^2, 2m + q = r^2 + s^2$$

Following the procedure presented in choice(1), then the values of r,s,k and p has to satisfy the condition

$6p < 6rs < 6p + r^2 + s^2$; r,s are of the same parity, which is satisfied by $P_1 - P_2 = k\alpha^2$

The above process is illustrated through the following examples.

k	r	s	p	q	m	P_1	P_2	$P_1 - P_2 = k\alpha^2$
2	1	3	2	2	4	48	16	$2(4^2)$
8	2	4	5	6	7	182	110	$8(3^2)$
2	6	4	16	16	18	1224	1024	$4(10^2)$
18	5	1	2	6	10	320	32	$18(4^2)$
50	7	3	14	14	22	1584	784	$50(4^2)$

Note: For k=6, P1-P2 is Nasty number and thus, it represents area of Pythagorean triangle.

For simplicity, we exhibit the connections between pairs of Pythagorean triangles through the difference of their perimeters and special numbers in the following table respectively.

$P_1 - P_2$	Values of generators m,q interms of p	Conditions for sides to be integers
$k\alpha^n, k > 0, n > 2$	$\beta > 0, m = 2k\beta + p$ $q = 2^{n-2}(\beta^{n-1}) - 2p - 2k\beta$	$2p < 2^{n-2}(\beta^{n-1}) - 2k\beta < 3p$
$3P^3_{2N}$	$m = P + N + 1$ $q = 2N^2 - 2P - 1$	$2p < 2N^2 - 1 < 3P$
$12 Pt_{2N}$	$m = 2N^2 + N + p$ $q = 4N - 2P + 3$	$2p < 4N + 3 < 3P$

III. Conclusion:

In this paper, we have obtained infinitely many pairs of Pythagorean triangles where each pair connects the difference between the perimeters with special polygonal numbers. To conclude, one may search for the connections between special numbers and the other characterizations of Pythagorean triangle.

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