IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 11, Issue 1 Ver. IV (Jan - Feb. 2015), PP 56-69
www.iosrjournals.org

Modelling of repairable items for production inventory with random deterioration

Dr.Ravish Kumar Yadav[1], RajeevKumar[2]
[1,]Associate Professor ,Department of mathematics ,Hindu College, Moradbad,
[2]Devta Mahavidhyal, Bijnoor

Abstract: *Keeping in view the concern about environmental protection, the study incorporate the concept of repairing in a production inventory model consisting of production system and repairing system over infinite planning horizon. This study presents a forward production and reverse repairing system inventory model with a time dependent random deterioration function and increasing exponentially demand with the finite production rate is proportional to the demand rate at any instant. The shortages allow and excess demand is backlogged. Expressions for optimal parameter are obtained .We also obtained Production and repairing scheduling period, maximum inventory level and total average cost. Using calculus, optimum production policy is derived, which minimizes the total cost incurred*

I. Introduction

An inventory system the effect of deterioration plays an important role. Deterioration is derived as decay or damage such that the item cannot be used for its original propose. Foods, pharmaceuticals, chemicals, blood, drugs are a few examples of such items in which sufficient deterioration can take place during the storage period of the units and the importance of this loss must be taken into account when analyzing the system.

When describing optimum policies for deteriorating items **Ghare and Schrader (1963)** proposed a constant rate of deterioration and constant rate demand. In recent year, inventory problem for deterioration items have been widely studied after **Ghare and Schrader (1963), Covert and Philip (1973)** formulated the model for variable deterioration rate with two parameters Weibull disturbation **Goswami and Chaudhuri (1991), Bose et al (1995)** assumed either instantaneous or finite production with different assumption on the pattern of deterioration.

Balkhi and Benkheroot (1996) considered a production a production lot size inventory model with arbitrary production and demand rate depends on the time function.

Bhunia and Maiti's (1977) model to formulate a production inventory model. **Chang and Deve (1999)** investigated an EOQ model allow shortage and backlogging. It is assumed that the backlogging rate is variable and dependent on the length of waiting time for the next replenishment. Recently, many researchers have modified inventory policies by considering the " time proportional partial backlogging rate" such as **Wang (2002), Perumal (2002), Teng et al (2003), Skouri** and **Papachristos (2003)** etc.

Schrady (1967) first studied the problem on optimal lot sizes for production/procurement and recovery. For issues in the greening process, **Nahmias and Rivera (1979)** studied an EPQ variant of Schrady's model (1967) with a finite recovery rate. **Richter (1996a, 1996b, 1997)** and **Richter and Dobos (1999)** investigated a waste disposal model by considering the returned rate as a decision variable. Dobos and Richter **(2003, 2004)** investigated a production/remanufacturing system with constant demand that is satisfied by noninstantaneous production and remanufacturing for single and multiple remanufacturing and production cycle**. Dobos and Richter (2006)** extended their previous model and assumed that the quality of collected returned items is not always suitable for further repairing**. Konstantaras and Skouri (2010)** presented a model by considering a general cycle pattern in which a variable number of reproduction lots of equal size were followed by a variable number of manufacturing lots of equal size. They also studied a special case where shortages were allowed in each manufacturing and reproduction cycle and similar sufficient conditions, as the non-shortages case, are given.**El Saadany and Jaber (2010)** extended the models developed by **Dobos and Richter (2003, 2004)** by assuming that the collection rate of returned items is dependent on the purchasing price and the acceptance quality level of these returns. That is, the flow of used/returned items increases as the purchasing price increases, and decreases as the corresponding acceptance quality level increases**. Alamri (2010)** developed a general reverse logistics inventory model. **Chung and Wee (2011)** developed an inventory model on short life-cycle deteriorating product remanufacturing in a green supply chain model

In this paper we present a realistic inventory model in which the production rate depends on the demand and demand is an exponentially increasing function time and deterioration is random function says that deterioration of an item depends upon the fluctuation of humidity, temperature, etc. It would be more reasonable and realistic if we assume the deterioration function θ to depend upon a parameter "α" in addition to time t

.This model is developed for deteriorating item by assuming that the deterioration rate is uniform and the finite production rate is proportional to the demand rate & the demand rate increasing exponentially. Repairable Items are collected at time of production run and repairs at time of no production no shortage completely. These repaired items as good as new and consumed at time of shortage. When shortages is maximum production start and items consumed from both the channels forward production and repaired items as well. We derive an expressions for different cost associated in the model and total average cost .We derive equations, solution of these equations gives the optimal cycle and optimal cost of repairable items.

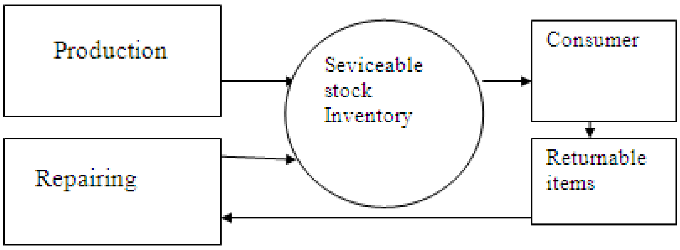


Fig. 1. Flow of inventory in the integrated supply system

II. Assumption and Notation

The mathematical model of the production inventory problem with repairable system considered herein is developed on the basic of the following assumptions-:

a. A single item is considered over a prescribed period of T units of time, which is subject to a time dependent Random deterioration rate.
b. Deteriorate D (t) is known and increasing exponentially $D(t) = Ae^{\lambda t}$, $t \geq 0$, A is initial demand, λ is a constant governing the increasing rate of demand.
c. Production rate P(t) at any instant depends on the demand that is, at time t, t > 0, $P(t) = a + bD(t)$, a > 0, $0 \leq b < 1$ and P(t) > D(t).
d. Deterioration of the units is considered only after they have been received into the inventory.
e. Items are returnable and are repaired. Repaired items are as good as new ones and they are used during the shortage period of forward production.
f. The time horizon of the inventory system is infinite. Only a typical planning schedule of length T is considered, all remaining cycles are identical.
g. Shortages are allowed and backlogged.
h. The production time interval for forward production coincides with the collection time interval for reverse repairing system.

Notations for production system and repairing system:

(1) I(t) = Inventory level at any time t, $t \geq 0$
(2) $\theta = \theta_0(\alpha)t =$ the items deterioration rate is random.
(3) I_m = Maximum inventory level.
(4) I_b = Maximum shortages level.
(5) C = Setup cost for new cycle.
(6) C_S = Shortage cost per unit.
(7) K = The total average cost of system.
(8) C_{HP}=Holding cost per unit per unit of time during the production.
(9) C_{DP}=Deteriorating cost per unit per unit of time during the production.
(10) P_{cp} = Production cost per item.
(11) C_{HC}= Holding cost per unit per unit of time during the collecting and consuming process for the repairing system.
(12) C_{DC}= Deteriorating cost per unit per unit of time during the collecting and consuming process for the repairing system.
(13) C_{HR} = Deteriorating cost per unit per unit of time during the repairing process for the repairing system.
(14) C_{DR} =Holding cost per unit per unit of time during the repairing process for the repairing system.
(15) $I_c(t)$ = Inventory level during the collecting process for the returnable items.
(16) $I_1(t)$= Inventory level during the repairing process for the returnable items.
(17) z =Fraction of the production lot size 0 <z<1.
(18) R_c =Rate of collection of returnable items.
(19) M =Rate of repair of returnable items to be repaired.

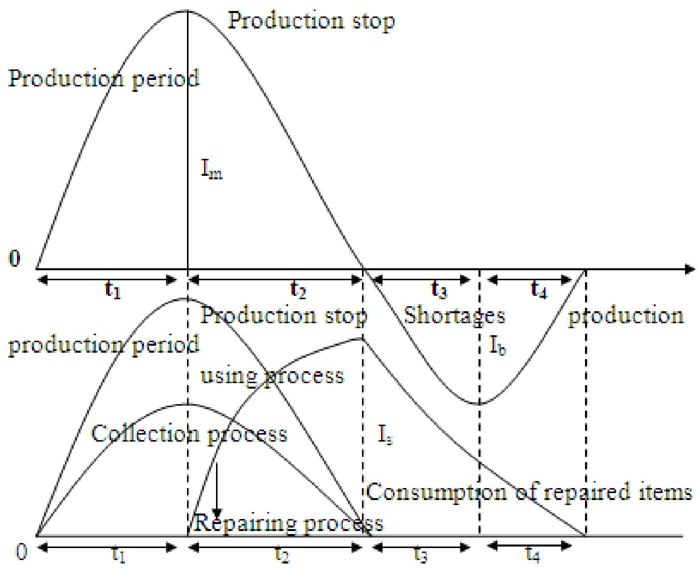
(20) t_1 = Time when production stops and also the time when collecting process for returnable items stops. At this very time repairing of collected items start.
(21) t_2 = Time period when repairing of returnable items stops and also the time when accumulated inventory of production system vanishes.
(22) t_3 = Time when shortages is maximum.(t= t_1+ t_2+ t_3)
(23) t_4 = Period of time when production starts again during the period of shortage.
(24) T = $(t_1 + t_2 + t_3 + t_4)$ is the cycle time.
(25) I_S = Maximum inventory level of repaired items.
(26) P_{cc}= Cost of purchasing the returnable items per unit.
(27) P_{cr} =Repair cost of repaired items per unit.

III. Mathematical Model :

Initially, the inventory level is start with zero. The forward production inventory level starts at time t=0 and it reaches at maximum inventory level I_m unit after t_1 time. At that time production is stopped and the inventory level is decreasing continuously and reaches zero at time t_2,at this time shortages start developing at time t_3 it reaches to maximum shortage level I_b . This time fresh production start to remove backlog by the time t_4 .

At the beginning of each cycle, the inventory is zero. The production starts at the very beginning of the cycle. As production progresses the inventory of finished goods piles up even after meeting the market demand, deterioration. At the beginning of each cycle, the process of collecting returnable items in a separate store also begins. At a point where the production from the forward production system stops; the collection process of returnable items also stops at the same point .It is assumed there is no collection of used items once the repairing of collected items starts. At this very point the repairing of reusable items begin at a constant rate. The accumulated inventory produced from the advanced production system in the meanwhile starts getting consumed and ultimately becomes nil. The accumulated inventory of repairing products, which are assumed to be as good as the newly produced products, is consumed when the shortages from the forward production system begin to surface. Thereafter, production starts when shortages is maximum in forward inventory system and shortages are gradually cleared after meeting demand by repairable items and produced item from forward system simultaneously and the cycle ends with zero inventories. Here our aim is to find out the optimal values of t_1, t_2, t_3 , t_4, I_m & I_b that minimize the total average cost (K) over the time planning horizon cycle(0,T).

Forward production system



Repairing System

Fig. 2. Inventory of production and repairing system

The differential equation governing the stock status during the period $0 \le t \le T$ can be written as

$$\frac{dI(t)}{dt} = a + (b-1)Ae^{\lambda t} - \theta_0(\alpha)tI(t), \quad I(0) = 0, (I_1) = I_m, \quad 0 \le t \le t_1 \qquad \text{…(1)}$$

$\frac{dI(t)}{dt} = -Ae^{\lambda t} - \theta_0(\alpha)tI(t)$, $I(t_1) = I_m$, $I(t_2) = 0$, $0 \le t \le t_2$ … (2)

$\frac{dI(t)}{dt} = -Ae^{\lambda t}$, $I(0) = o$, $I(t_3) = I_b$, $0 \le t \le t_3$, ….(3)

$\frac{dI(t)}{dt} = a + (b-1)Ae^{\lambda t}$, $I(0) = I_b$, $I(t_4)=0$, $0 \le t \le t_4$ ….(4)

Differential equations representing repairing system in collecting time & consuming time

$\frac{dI_c(t)}{dt} = R_c - \theta_0(\alpha)\, t\, I_c(t)$, $I_c(0) = 0$ $0 \le t \le t_1$ … (5)

$\frac{dI_c(t)}{dt} = M - \theta_0(\alpha)\, t\, I_c(t)$, $I_c(t_1) = Bz$, $0 \le t \le t_2$ … (6)

Where

B=Production lot size during production system=Production- Deterioration

$$= \int_0^{t_1} Pdt - \int_0^{t_1} \theta_0(\alpha) tPdt = \int_0^{t_1} (1 - \theta_0(\alpha)t)(\mathrm{a} + \mathrm{bAe}^{\lambda t})dt$$

$$= \int_0^{t_1} \{\mathrm{a} + \mathrm{bAe}^{\lambda t} - \theta_0(\alpha)t(\mathrm{a} + \mathrm{bAe}^{\lambda t})\}dt$$

$$= -\{\frac{\mathrm{bA}}{\lambda}(1 + \frac{\theta_0(\alpha)}{\lambda}) + \frac{a\theta_0(\alpha)t_1^{\ 2}}{2} + \} + \frac{\mathrm{bAe}^{\lambda t_1}}{\lambda}\{1 - \theta_0(\alpha)t_1) + \frac{\theta_0(\alpha)}{\lambda}\}$$

Differential equations representing inventory of repaired items.

$\frac{dI_1(t)}{dt} = M - \theta_0(\alpha)\, t\, I_1(t)$, $I_1(0) = 0$, $I_1(t_2) = I_s$, $0 \le t \le t_2$, ….(7)

$\frac{dI(t)}{dt} = -Ae^{\lambda t} - \theta_0(\alpha)tI(t)$,$I(0) = I_s$,$I(t_3 + t_4) = 0$, $I(t_3) = I_{b1}$, $0 \le t \le t_3 + t_4$ … (8)

Solution of equation (1) , (2) , .3) and (4) by adjusting the constant of integration using boundary condition are given by

$$I(t) = a\left\{t + \frac{\theta_0(\alpha)t^3}{6}\right\}e^{-\theta_0(\alpha)t^2/2} + \frac{(b-1)}{\lambda}Ae^{\left\{\lambda - \frac{\theta_0(\alpha)t}{2}\right\}t}\left[1 - \frac{\theta_0(\alpha)t}{\lambda} + \frac{\theta_0(\alpha)t^2}{2} - \frac{\theta_0(\alpha)e^{-\lambda t}}{\lambda^2} + \frac{\theta_0(\alpha)}{\lambda^2} - e^{-\lambda t}\right], \quad 0 \le t \le t_1 \qquad \text{… (9)}$$

$$I(t) = \frac{A}{\lambda}\, e^{-\theta_0(\alpha)t^2/2}\left[\frac{e^{\lambda t_2}}{\lambda}\left\{1 + \frac{\theta_0(\alpha)t^2}{2} - \frac{\theta_0(\alpha)t_2}{\lambda} + \frac{\theta_0(\alpha)}{\lambda^2}\right\} - e^{\lambda t}\left\{1 + \frac{\theta_0(\alpha)t^2}{2} - \frac{\theta_0(\alpha)t}{\lambda} + \frac{\theta_0(\alpha)}{\lambda^2}\right\}\right], \quad 0 \le t \le t_2 \qquad \text{… (10)}$$

$$I(t) = \frac{A}{\lambda}(1 - e^{\lambda t}) \quad ,0 \le t \le t_3 \qquad \text{… (11)}$$

$$I(t) = a(t - t_4) + \frac{A(b-1)}{\lambda}(e^{\lambda t} - e^{\lambda t_4}) , \quad 0 \le t \le t_4 \qquad \text{… (12)}$$

Solving (5) and (6)

$$I_c(t) = R_c\left\{t + \frac{\theta_0(\alpha)t^3}{6}\right\}e^{-\theta_0(\alpha)t^2/2}, \quad 0 \le t \le t_1 \qquad \ldots (13)$$

$$I_c(t_1) = R_c\left\{t_1 + \frac{\theta_0(\alpha)t_1^3}{6}\right\}e^{-\theta_0(\alpha)t_1^2/2} = \text{Bz},$$

$$R_C = \frac{Bze^{-\theta_0(\alpha)t_1^2/2}}{\left\{t_1 + \frac{\theta_0(\alpha)t_1^3}{6}\right\}}$$

$$I_c(t) = Bz + M\left\{t + \frac{\theta_0(\alpha)t^3}{6}\right\}e^{-\theta_0(\alpha)t^2/2} - M\left\{t_1 + \frac{\theta_0(\alpha)t_1^3}{6}\right\}e^{-\theta_0(\alpha)t_1^2/2}, \ 0 \le t \le t_2 \qquad \ldots (14)$$

Solution of equation (7) and (8) by adjusting the constant of integration using boundary condition are given by

$$I_1(t) = M\left\{t + \frac{\theta_0(\alpha)t^3}{6}\right\}e^{-\theta_0(\alpha)t^2/2}, \quad 0 \le t \le t_2 \qquad \ldots (15)$$

$$I(t) = \frac{A}{\lambda}e^{\left\{\lambda - \frac{\theta_0(\alpha)t}{2}\right\}t}\left[1 - \frac{\theta_0(\alpha)t}{\lambda} + \frac{\theta_0(\alpha)t^2}{2} - \frac{\theta_0(\alpha)e^{-\lambda t}}{\lambda^2} + \frac{\theta_0(\alpha)}{\lambda^2} - e^{-\lambda t}\right], \ 0 \le t \le t_3 + t_4 \qquad \ldots (16)$$

$$I_{b1} = \frac{A}{\lambda}e^{\left\{\lambda - \frac{\theta_0(\alpha)t_3}{2}\right\}t_3}\left[1 - \frac{\theta_0(\alpha)t_3}{\lambda} + \frac{\theta_0(\alpha)t_3^2}{2} - \frac{\theta_0(\alpha)e^{-\lambda t_3}}{\lambda^2} + \frac{\theta_0(\alpha)}{\lambda^2} - e^{-\lambda t_3}\right], \qquad \ldots (16)$$

The inventory level of production start initially at time unit t = 0 to t = t_1 at maximum level I_m is obtained by equation (9)

$$I_m = a\left\{t_1 + \frac{\theta_0(\alpha)t_1^3}{6}\right\}e^{-\theta_0(\alpha)t_1^2/2} + \frac{(b-1)}{\lambda}Ae^{\left\{\lambda - \frac{\theta_0(\alpha)t_1}{2}\right\}t_1}\left\{1 - \frac{\theta_0(\alpha)t_1}{\lambda} + \frac{\theta_0(\alpha)t_1^2}{2} - \frac{\theta_0(\alpha)e^{-\lambda t_1}}{\lambda^2} + \frac{\theta_0(\alpha)}{\lambda^2} - e^{-\lambda t_1}\right\}$$

$$= \frac{A}{\lambda}\left[e^{\lambda t_2}\left\{1 + \frac{\theta_0(\alpha)t_1^2}{2} - \frac{\theta_0(\alpha)t_2}{\lambda} + \frac{\theta_0(\alpha)}{\lambda^2}\right\} - 1\right] \qquad \ldots (17)$$

and after time t_1 the production is stopped and stock level is decreasing continuously and become zero at time $t=t_2$ at that time shortages are develop and reaching to I_b at time $t = t_3$ is obtained .

$$I_b - I_{b1} = \frac{A}{\lambda}(1 - e^{\lambda t_3}) = -at_4 + \frac{A(b-1)}{\lambda}(1 - e^{\lambda t_4}) \qquad \ldots (.18)$$

Thus by equation (17) we observed that t_1 and t_2 are dependent so they are related by the equation

$$t_2 = R(t_1) \qquad \ldots (19)$$

and by equation (18) t_3 and t_4 are dependent to each other so related by the equation

$$t_3 = R_1(t_4) \qquad \ldots (20)$$

Total amount of deteriorated units (I_{DP}) of production inventory $(0, \mathrm{T})$ is given by

$$I_{DP} = \int_0^{t_1}\theta_0(\alpha)tI(t)dt + \int_0^{t_2}\theta_0(\alpha)tI(t)dt$$

$$=\left[\left\{a\theta_0(\alpha)\left(\frac{t_1^2}{2}+\frac{\theta_0(\alpha)t_1^5}{30}-\frac{\theta_0(\alpha)t_1^4}{8}-\frac{\theta_0^2(\alpha)t_1^7}{72}\right)\right\}\right]$$

$$+\frac{\theta_0(\alpha)(b-1)Ae^{\lambda t_1}}{\lambda}\left[\left[\theta_0(\alpha)\left(\frac{(6+\lambda)t_1}{\lambda^3}-\frac{t_1^2(\lambda+5)}{2\lambda^2}+\frac{t_1^3}{2\lambda}\right)\right.\right.$$

$$\left.\left.+\theta_0^2(\alpha)\left(-\frac{42t_1}{\lambda^5}-\frac{(3-12\lambda)t_1^2}{2\lambda^5}-\frac{7t_1^3}{2\lambda^3}+\frac{3t_1}{4\lambda^2}\right)\right]\right]$$

$$+\left[\frac{\theta_0(\alpha)(b-1)A}{\lambda}\left(\frac{6\lambda^2t_1+6\theta_0(\alpha)t+\theta_0(\alpha)\lambda^2t_1^3+\theta_0^2(\alpha)\lambda t_1^3}{6\lambda^2}\right)\right]$$

$$+\left[\frac{Ae^{\lambda t_2}}{\lambda^2}\left(1+\frac{\theta_0(\alpha)t_2^2}{2}-\frac{\theta_0(\alpha)t_2}{\lambda}-\frac{\theta_0(\alpha)}{\lambda}\right)\left(1-e^{-\frac{\theta_0(\alpha)t_2^2}{2}}\right)\right]$$

$$+\frac{A}{\lambda}\left(1+\frac{\theta_0(\alpha)}{\lambda^2}\right)\left[\frac{\theta_0(\alpha)t_2e^{\lambda t_2}}{\lambda}-\frac{e^{\lambda t_2}}{\lambda^2}\right.$$

$$\left.-\frac{\theta_0^2(\alpha)e^{\lambda t_2}}{2}\left(\frac{t_2^3}{\lambda}-\frac{3t_2^2}{\lambda^2}+\frac{6t_2}{\lambda^3}-\frac{6}{\lambda^4}\right)-\frac{3\theta_0^2(\alpha)}{\lambda^4}+\frac{1}{\lambda^2}\right]$$

$$+\frac{A\theta_0^2(\alpha)}{\lambda}\left[e^{\lambda t_2}\left(\frac{t_2^3}{\lambda}-\frac{3t_2^2}{\lambda^2}+\frac{6t_2}{\lambda^3}-\frac{6}{\lambda^4}\right)\right.$$

$$-\frac{\theta_0(\alpha)e^{\lambda t_2}}{2}\left(\frac{t_2^5}{\lambda}-\frac{5t_2^4}{\lambda^2}+\frac{20t_2^3}{\lambda^3}-\frac{60t_2^2}{\lambda^4}-\frac{120t_2}{\lambda^5}-\frac{120}{\lambda^6}\right)$$

$$\left.+\left(\frac{6}{\lambda^4}-\frac{60\theta_0(\alpha)}{\lambda^6}\right)\right]-\frac{A\theta_0^2(\alpha)}{\lambda}\left[e^{\lambda t_2}\left(\frac{t_2^2}{\lambda}-\frac{2t_2}{\lambda^2}+\frac{2}{\lambda^3}-\frac{t_2^4}{\lambda}\right.\right.$$

$$\left.\left.+\frac{4t_2^3}{\lambda^2}-\frac{12t_2^3}{\lambda^3}+\frac{24t_2}{\lambda^4}-\frac{24}{\lambda^5}\right)-\frac{2}{\lambda^3}+\frac{24}{\lambda^5}\right] \qquad \dots (21)$$

Total amount of deteriorated units (I_{DC}) of collected items of repairable inventory channel in $(0, t_2)$ is given by

$$I_{DC}=\int_0^{t_1}\theta_0(\alpha)tI_C(t)dt+\int_0^{t_2}\theta_0(\alpha)tI_C(t)dt$$

$$=\int_0^{t_1}\theta_0(\alpha)tR_c\left\{t+\frac{\theta_0(\alpha)t^3}{6}\right\}e^{-\theta_0(\alpha)t^2/2}dt+$$

$$\int_0^{t_2}\theta_0(\alpha)t\left[Bz+M\left\{t+\frac{\theta_0(\alpha)t^3}{6}\right\}e^{-\theta_0(\alpha)t^2/2}-M\left\{t_1+\frac{\theta_0(\alpha)t_1^3}{6}\right\}e^{-\theta_0(\alpha)t_1^2/2}\right]dt \qquad \dots (22)$$

$$=R_c\left(\frac{\theta_0(\alpha)t_1^3}{3}-\frac{\theta_0(\alpha)^2t_1^5}{15}\right)+\frac{\theta_0(\alpha)Bzt_2^2}{2}+M\left(\frac{\theta_0(\alpha)t_2^3}{3}-\frac{\theta_0(\alpha)^2t_2^5}{15}\right)-$$

$$\frac{\theta_0(\alpha)t_2^2M}{2}\left\{t_1+\frac{\theta_0(\alpha)t_1^3}{6}\right\}e^{-\theta_0(\alpha)t_1^2/2}$$

Total amount of deteriorated units (I_{DR}) of Repaired items of repairable inventory channel is given by

$$I_{DR}=\int_0^{t_2}\theta_0(\alpha)tI_1(t)dt+\int_0^{t_3+t_4}\theta_0(\alpha)tI(t)dt$$

$$=\int_0^{t_2}\theta_0(\alpha)tM\left\{t+\frac{\theta_0(\alpha)t^3}{6}\right\}e^{-\theta_0(\alpha)t^2/2}dt+$$

$$\int_0^{t_3+t_4}\theta_0(\alpha)t\frac{A}{\lambda}e^{\left\{\lambda-\frac{\theta_0(\alpha)t}{2}\right\}t}\left[1-\frac{\theta_0(\alpha)t}{\lambda}+\frac{\theta_0(\alpha)t^2}{2}-\frac{\theta_0(\alpha)e^{-\lambda t}}{\lambda^2}+\frac{\theta_0(\alpha)}{\lambda^2}-e^{-\lambda t}\right]dt$$

$$=M\left(\frac{\theta_0(\alpha)t_2^{\;3}}{3}-\frac{\theta_0(\alpha)^2t_2^{\;5}}{15}\right)+A\theta_0(\alpha)\left(\frac{(t_3+t_4)^2}{2}+\frac{\lambda(t_3+t_4)^3}{3}-\frac{\theta_0(\alpha)(t_3+t_4)^4}{8}\right) \quad \ldots (23)$$

During period $(0,T)$ total inventory of produced items (I_{HP}) in forward production channel can be obtained as

$$I_{HP}=\int_0^{t_1}I(t)dt+\int_0^{t_2}I(t)dt$$

$$=\Bigg[a\left(t_1-\frac{\theta_0(\alpha)t_1^3}{6}-\frac{a\theta_0^2(\alpha)t_1^6}{72}\right)+\frac{(b-1)A}{\lambda}\left(\frac{e^{\lambda t_1}}{\lambda}-t_1-\frac{1}{\lambda}\right)$$

$$-e^{\lambda t_1}\theta_0(\alpha)\left(\frac{t_1}{\lambda^2}-\frac{2}{\lambda^3}\right)+e^{\lambda t_1}\theta_0^2(\alpha)\Bigg(\frac{4t_1}{\lambda^4}-\frac{5t_1^2}{\lambda^3}+\frac{3t_1^3}{2\lambda^2}-\frac{t_1^4}{4\lambda}-\frac{1}{\lambda^4}$$

$$+\frac{6}{\lambda^5}\Bigg)-\theta_0(\alpha)\left(\frac{t_1}{\lambda^2}-\frac{t_1^3}{6}+\frac{2}{\lambda^3}\right)+\theta_0^2(\alpha)\left(\frac{t_1^3}{6\lambda^3}+\frac{6t_1}{\lambda^3}+\frac{2}{\lambda^4}+\frac{7}{\lambda^5}\right)$$

$$+\frac{A}{\lambda}\left\{\theta_0(\alpha)\left(-\frac{2}{\lambda^3}+\frac{e^{\lambda t_2}}{\lambda^2}-\frac{t_2e^{\lambda t_2}}{\lambda^2}\right)\right\}+\theta_0^2(\alpha)\Bigg\{-\frac{t_2^4e^{\lambda t_2}}{4\lambda}+\frac{t_2^3e^{\lambda t_2}}{2\lambda}$$

$$-\frac{t_2^2e^{\lambda t_2}}{2\lambda^2}-\frac{3t_2^2e^{\lambda t_2}}{2\lambda^3}+\frac{t_2e^{\lambda t_2}}{\lambda^2}-\frac{3t_2e^{\lambda t_2}}{\lambda^4}+\frac{e^{\lambda t_2}}{\lambda^4}-\frac{1}{\lambda^4}+\frac{3e^{\lambda t_2}}{\lambda^5}-\frac{3}{\lambda^5}\Bigg\}$$

$$+\left(\frac{e^{\lambda t_2}}{\lambda}-\frac{1}{\lambda}\right)\Bigg]+\frac{Ae^{\lambda t_2}}{\lambda^2}\Bigg[t_2+\frac{1}{3}\theta_0(\alpha)t_2^3-\frac{\theta_0^2(\alpha)t_2^3}{6\lambda^2}+\frac{\theta_0(\alpha)t_2}{\lambda^2}-\frac{\theta_0(\alpha)t_2^2}{\lambda}$$

$$-\frac{\theta_0^3(\alpha)t_2^5}{12}-\frac{\theta_0(\alpha)t_2^4}{6\lambda}\Bigg] \quad \ldots (24)$$

During period $(0,T)$ total inventory of collected items (I_{HC}) can be obtained as

$$I_{HC}=\int_0^{t_1}I_C(t)dt+\int_0^{t_2}I_C(t)dt$$

$$=\int_0^{t_1}R_c\left\{t+\frac{\theta_0(\alpha)t^3}{6}\right\}e^{-\theta_0(\alpha)t^2/2}dt+$$

$$\int_0^{t_2}\left[Bz+M\left\{t+\frac{\theta_0(\alpha)t^3}{6}\right\}e^{-\theta_0(\alpha)t^2/2}-M\left\{t_1+\frac{\theta_0(\alpha)t_1^{\;3}}{6}\right\}e^{-\theta_0(\alpha)t_1^{\;2}/2}\right]dt \quad \ldots (25)$$

$$=R_c\left(\frac{\theta_0(\alpha)t_1^{\;2}}{2}-\frac{\theta_0(\alpha)^2t_1^{\;4}}{12}\right)+\theta_0(\alpha)Bzt_2+M\left(\frac{\theta_0(\alpha)t_2^{\;2}}{2}-\frac{\theta_0(\alpha)^2t_2^{\;4}}{12}\right)-$$

$$\theta_0(\alpha)t_2M\left\{t_1+\frac{\theta_0(\alpha)t_1^{\;3}}{6}\right\}e^{-\theta_0(\alpha)t_1^{\;2}/2}$$

During period $(0,T)$ total inventory of repaired items (I_{HR}) can be obtained as

$$I_{HR} = \int_0^{t_2} I_1(t)dt + \int_0^{t_3+t_4} I(t)dt$$

$$= \int_0^{t_2} \theta_0(\alpha) M \left\{ t + \frac{\theta_0(\alpha)t^3}{6} \right\} e^{-\theta_0(\alpha)t^2/2} dt +$$

$$\int_0^{t_3+t_4} \theta_0(\alpha)\frac{A}{\lambda} e^{\left\{\lambda - \frac{\theta_0(\alpha)t}{2}\right\}t} \left[1 - \frac{\theta_0(\alpha)t}{\lambda} + \frac{\theta_0(\alpha)t^2}{2} - \frac{\theta_0(\alpha)e^{-\lambda t}}{\lambda^2} + \frac{\theta_0(\alpha)}{\lambda^2} - e^{-\lambda t} \right] dt$$

$$= M\left(\frac{\theta_0(\alpha){t_2}^2}{2} - \frac{\theta_0(\alpha)^2 {t_2}^4}{12} \right) + A\theta_0(\alpha)\left((t_3+t_4) + \frac{\lambda(t_3+t_4)^2}{2} - \frac{\theta_0(\alpha)(t_3+t_4)^3}{6} \right)$$

…(26)

Total amount of shortage units (I_s) during the period (0, T) is given by

$$I_S = -\int_0^{t_3} I(t)dt + \int_0^{t_4} I(t)dt = \left[\frac{A}{\lambda}\left\{ \left(\frac{e^{\lambda t_3}}{\lambda} - t_3 \right) - \frac{1}{\lambda} \right\} - \frac{a t_4^2}{2} + \frac{A(b-1)}{\lambda}\left\{ \left(\frac{e^{\lambda t_4}}{\lambda} - \frac{1}{\lambda} \right) - t_4 e^{\lambda t_4} \right\} \right]$$

… (27)

P=Production cost +Collection cost +Repaired cost

$$P = P_{CP}\int_0^{t_1}(a + bAe^{\lambda t})dt + P_{CC}\int_0^{t_1} R_c dt + P_{CR}\int_0^{t_2} M dt$$

$$= P_{CP}(at_1 + \frac{bAe^{\lambda t_1}}{\lambda}) + P_{CC}R_c t_1 + P_{CR}Mt_2$$

Hence the total average cost of the inventory system is
K = setup cost +production cost+ deterioration cost + inventory carrying
cost + shortage cost

$$= \frac{1}{T}\left[C + P + C_{DP}I_{DP} + C_{DC}I_{DC} + C_{DR} + I_{DR} + C_{HP}I_{HP} + C_{HC}I_{HC} + C_{HR}I_{HR} + C_S I_S \right]$$

… (28)

and putting the value of $I_{DP,}$ $I_{DC,}$ $I_{DR,}I_{HP,}$ $I_{HC,}$ I_{HR} and I_S we getting the total average cost of the inventory system.

IV. The Approximation Solution Procedure

In many cases λ and $\theta_0(\alpha)$ are extremely small hence to use Maclaurin series for approximation

$$e^{-\lambda t} = 1 - \lambda t + \frac{\lambda^2 t^2}{2}$$ … (29)

By using equation (4.29) the total average cost of system

$$K = \frac{1}{T}[C + P_{CP}(at_1 + \frac{bAe^{\lambda t_1}}{\lambda}) + P_{CC}R_c t_1 + P_{CR}Mt_2] + \frac{C_{DP}}{T}$$

$$\left[\frac{\theta_0(\alpha)at_1^2}{2} + \frac{\theta_0(\alpha)(b-1)At_1}{\lambda} + \frac{A\theta_0(\alpha)}{2\lambda^4}\left(\lambda^3 t_2^3 + \lambda^2 t_2^2 + 2\lambda^4 + 2\lambda^4 t_2 + 2\lambda^2 - 2 \right) \right] +$$

$$\frac{C_{DC}}{T}\left(\begin{array}{l} R_c\left(\frac{\theta_0(\alpha){t_1}^3}{3} - \frac{\theta_0(\alpha)^2 {t_1}^5}{15} \right) + \frac{\theta_0(\alpha)Bz{t_2}^2}{2} + M\left(\frac{\theta_0(\alpha){t_2}^3}{3} - \frac{\theta_0(\alpha)^2 {t_2}^5}{15} \right) - \\ \frac{\theta_0(\alpha){t_2}^2 M}{2}\left\{ t_1 + \frac{\theta_0(\alpha){t_1}^3}{6} \right\} e^{-\theta_0(\alpha){t_1}^2/2} \end{array} \right)$$

$$+\frac{C_{DR}}{T}\left(M\left(\frac{\theta_0(\alpha)t_2^{\,3}}{3}-\frac{\theta_0(\alpha)^2t_2^{\,5}}{15}\right)+A\theta_0(\alpha)\left(\frac{(t_3+t_4)^2}{2}+\frac{\lambda(t_3+t_4)^3}{3}-\frac{\theta_0(\alpha)(t_3+t_4)^4}{8}\right)\right)$$

$$+\frac{C_{HP}}{T}\left[\left\{at_1-\frac{a\theta_0(\alpha)t_1^3}{6}+\frac{(b-1)A}{\lambda}\left(\lambda t_1^2-\frac{\theta_0(\alpha)t_1^3}{3}\right)+\frac{A}{\lambda}\left\{-\frac{2\theta_0(\alpha)}{\lambda^3}\right.\right.\right.$$

$$\left.+t_2\left(2+\frac{\theta_0(\alpha)}{\lambda}-\frac{\theta_0(\alpha)}{\lambda^2}\right)-\frac{\theta_0(\alpha)t_2^2}{\lambda^2}+\frac{\theta_0(\alpha)}{\lambda^2}\right\}$$

$$\left.+\frac{A}{\lambda^2}\left\{\left(1+\frac{\theta_0(\alpha)}{\lambda^2}\right)t_2+\lambda t_2^2-\frac{2\theta_0(\alpha)t_2^3}{3}+\left(\frac{2\lambda^2\theta_0(\alpha)-\theta_0(\alpha)}{6\lambda}\right)t_2^4-\frac{\theta_0(\alpha)}{6}t_2^5\right\}\right]+$$

$$\frac{C_{HC}}{T}\left(\begin{array}{l}R_c\left(\frac{\theta_0(\alpha)t_1^{\,2}}{2}-\frac{\theta_0(\alpha)^2t_1^{\,4}}{12}\right)+\theta_0(\alpha)Bzt_2+M\left(\frac{\theta_0(\alpha)t_2^{\,2}}{2}-\frac{\theta_0(\alpha)^2t_2^{\,4}}{12}\right)-\\ \theta_0(\alpha)t_2M\left\{t_1+\frac{\theta_0(\alpha)t_1^{\,3}}{6}\right\}e^{-\theta_0(\alpha)t_1^{\,2}/2}\end{array}\right)+$$

$$\frac{H_{HR}}{T}\left(M\left(\frac{\theta_0(\alpha)t_2^{\,2}}{2}-\frac{\theta_0(\alpha)^2t_2^{\,4}}{12}\right)+A\theta_0(\alpha)\left((t_3+t_4)+\frac{\lambda(t_3+t_4)^2}{2}-\frac{\theta_0(\alpha)(t_3+t_4)^3}{6}\right)\right)$$

$$\frac{C_S}{T}\left\{\frac{At_3^2}{2}-\frac{at_4^2}{2}-\frac{A(b-1)}{2}\lambda t_4^2\right\} \qquad \dots (30)$$

And

$$K=\frac{1}{T}[C+P_{CP}(at_1+\frac{bAe^{\lambda t_1}}{\lambda})+P_{CC}R_ct_1+P_{CR}MR(t_1)]+\frac{C_{DP}}{T}$$

$$\left[\frac{\theta_0(\alpha)at_1^2}{2}+\frac{\theta_0(\alpha)(b-1)At_1}{\lambda}+\frac{A\theta_0(\alpha)}{2\lambda^4}\left(\lambda^3R(t_1)^3+\lambda^2R(t_1)^2+2\lambda^4+2\lambda^4R(t_1)+2\lambda^2-2\right)\right]+$$

$$\frac{C_{DC}}{T}\left(\begin{array}{l}R_c\left(\frac{\theta_0(\alpha)t_1^{\,3}}{3}-\frac{\theta_0(\alpha)^2t_1^{\,5}}{15}\right)+\frac{\theta_0(\alpha)BzR(t_1)^2}{2}+M\left(\frac{\theta_0(\alpha)R(t_1)^3}{3}-\frac{\theta_0(\alpha)^2R(t_1)^5}{15}\right)-\\ \frac{\theta_0(\alpha)R(t_1)^2M}{2}\left\{t_1+\frac{\theta_0(\alpha)t_1^{\,3}}{6}\right\}e^{-\theta_0(\alpha)t_1^{\,2}/2}\end{array}\right)$$

$$+\frac{C_{DR}}{T}\left(M\left(\frac{\theta_0(\alpha)R(t_1)^3}{3}-\frac{\theta_0(\alpha)^2R(t_1)^5}{15}\right)+A\theta_0(\alpha)\left(\begin{array}{l}\frac{(R_1(t_4)+t_4)^2}{2}+\frac{\lambda(R_1(t_4)+t_4)^3}{3}\\ -\frac{\theta_0(\alpha)(R_1(t_4)+t_4)^4}{8}\end{array}\right)\right)$$

$$+\frac{C_{HP}}{T}\left[\left\{at_1-\frac{a\theta_0(\alpha)t_1^3}{6}+\frac{(b-1)A}{\lambda}\left(\lambda t_1^2-\frac{\theta_0(\alpha)t_1^3}{3}\right)+\frac{A}{\lambda}\left\{-\frac{2\theta_0(\alpha)}{\lambda^3}\right.\right.\right.$$

$$\left.+R(t_1)\left(2+\frac{\theta_0(\alpha)}{\lambda}-\frac{\theta_0(\alpha)}{\lambda^2}\right)-\frac{\theta_0(\alpha)R(t_1)^2}{\lambda^2}+\frac{\theta_0(\alpha)}{\lambda^2}\right\}$$

$$+\frac{A}{\lambda^2}\left\{\left(1+\frac{\theta_0(\alpha)}{\lambda^2}\right)R(t_1)+\lambda R(t_1)^2-\frac{2\theta_0(\alpha)R(t_1)^3}{3}+\left(\frac{2\lambda^2\theta_0(\alpha)-\theta_0(\alpha)}{6\lambda}\right)R(t_1)^4\right.$$

$$\left.\left.-\frac{\theta_0(\alpha)}{6}R(\mathrm{t}_1)^5\right\}\right]+$$

$$\frac{C_{HC}}{T}\left(\begin{array}{l}R_c\left(\frac{\theta_0(\alpha)t_1^2}{2}-\frac{\theta_0(\alpha)^2t_1^4}{12}\right)+\theta_0(\alpha)Bzt_2+M\left(\frac{\theta_0(\alpha)R(t_1)^2}{2}-\frac{\theta_0(\alpha)^2R(t_1)^4}{12}\right)-\\ \theta_0(\alpha)R(t_1)M\left\{t_1+\frac{\theta_0(\alpha)t_1^3}{6}\right\}e^{-\theta_0(\alpha)t_1^2/2}\end{array}\right)+$$

$$\frac{H_{HR}}{T}\left(M\left(\frac{\theta_0(\alpha)R(t_1)^2}{2}-\frac{\theta_0(\alpha)^2R(t_1)^4}{12}\right)+A\theta_0(\alpha)\left(\begin{array}{l}(R_1(t_4)+t_4)+\frac{\lambda(R_1(t_4)+t_4)^2}{2}-\\ \frac{\theta_0(\alpha)(R_1(t_4)+t_4)^3}{6}\end{array}\right)\right)$$

$$\frac{C_S}{T}\left\{\frac{AR_1(t_4)^2}{2}-\frac{at_4^2}{2}-\frac{A(b-1)}{2}\lambda t_4^2\right\} \quad \ldots (31)$$

According to equation (30) contain four variables t_1, t_2, t_3 and t_4 and these are dependent variable and related by equation (19) and (20). Also we have K > 0, hence the optimum value of t_1 and t_4 which minimize total average cost are the solutions of the equations

$$\frac{\partial K}{\partial t_1}=0 \text{ and } \frac{\partial K}{\partial t_4}=0 \quad \ldots (.32)$$

Provided that these values of t_1 satisfy the conditions

$$\frac{\partial^2 K}{\partial t_1^2}>0,\ \frac{\partial^2 K}{\partial t_4^2}>0 \text{ and } \frac{\partial^2 K}{\partial t_1^2}.\frac{\partial^2 K}{\partial t_4^2}-\left(\frac{\partial^2 K}{\partial t_1\partial t_4}\right)^2>0$$

Now differentiating (31) with respect to t_1 and t_4, we get

$$\frac{\partial K}{\partial t_1}=\frac{-1}{T^2}[C+P_{CP}(at_1+\frac{bAe^{\lambda t_1}}{\lambda})+P_{CC}R_ct_1+P_{CR}MR(t_1)]\left(1+R'(t_1)\right)-\frac{C_{DP}}{T^2}$$

$$\left[\frac{\theta_0(\alpha)at_1^2}{2}+\frac{\theta_0(\alpha)(b-1)At_1}{\lambda}+\frac{A\theta_0(\alpha)}{2\lambda^4}\left(\lambda^3R(t_1)^3+\lambda^2R(t_1)^2+2\lambda^4+2\lambda^4R(t_1)+2\lambda^2-2\right)\right]$$

$$\left(1+R'(t_1)\right)-$$

$$\frac{C_{DC}}{T^2}\left(\begin{array}{l}R_c\left(\frac{\theta_0(\alpha)t_1^3}{3}-\frac{\theta_0(\alpha)^2t_1^5}{15}\right)+\frac{\theta_0(\alpha)BzR(t_1)^2}{2}+M\left(\frac{\theta_0(\alpha)R(t_1)^3}{3}-\frac{\theta_0(\alpha)^2R(t_1)^5}{15}\right)-\\ \frac{\theta_0(\alpha)R(t_1)^2M}{2}\left\{t_1+\frac{\theta_0(\alpha)t_1^3}{6}\right\}e^{-\theta_0(\alpha)t_1^2/2}\end{array}\right)$$

$$\left(1+R'(t_1)\right)$$

$$+\frac{C_{DR}}{T^2}\left(\begin{array}{l}M\left(\frac{\theta_0(\alpha)R(t_1)^3}{3}-\frac{\theta_0(\alpha)^2R(t_1)^5}{15}\right)+\\ A\theta_0(\alpha)\left(\begin{array}{l}\frac{(R_1(t_4)+t_4)^2}{2}+\frac{\lambda(R_1(t_4)+t_4)^3}{3}\\ -\frac{\theta_0(\alpha)(R_1(t_4)+t_4)^4}{8}\end{array}\right)\end{array}\right)\left(1+R'(t_1)\right)$$

$$-\frac{C_{HP}}{T^2}\left[\left\{at_1-\frac{a\theta_0(\alpha)t_1^3}{6}+\frac{(b-1)A}{\lambda}\left(\lambda t_1^2-\frac{\theta_0(\alpha)t_1^3}{3}\right)+\frac{A}{\lambda}\left\{-\frac{2\theta_0(\alpha)}{\lambda^3}\right.\right.\right.$$

$$\left.+R(t_1)\left(2+\frac{\theta_0(\alpha)}{\lambda}-\frac{\theta_0(\alpha)}{\lambda^2}\right)-\frac{\theta_0(\alpha)R(t_1)^2}{\lambda^2}+\frac{\theta_0(\alpha)}{\lambda^2}\right\}$$

$$+\frac{A}{\lambda^2}\left\{\left(1+\frac{\theta_0(\alpha)}{\lambda^2}\right)R(t_1)+\lambda R(t_1)^2-\frac{2\theta_0(\alpha)R(t_1)^3}{3}+\left(\frac{2\lambda^2\theta_0(\alpha)-\theta_0(\alpha)}{6\lambda}\right)R(t_1)^4\right.$$

$$\left.\left.-\frac{\theta_0(\alpha)}{6}R(t_1)^5\right\}\right]\left(1+R'(t_1)\right)--\frac{C_{HC}}{T^2}\begin{pmatrix}R_c\left(\frac{\theta_0(\alpha)t_1^{\,2}}{2}-\frac{\theta_0(\alpha)^2t_1^{\,4}}{12}\right)+\theta_0(\alpha)BzR(t_1)+\\ M\left(\frac{\theta_0(\alpha)R(t_1)^2}{2}-\frac{\theta_0(\alpha)^2R(t_1)^4}{12}\right)-\\ \theta_0(\alpha)t_2M\left\{t_1+\frac{\theta_0(\alpha)t_1^{\,3}}{6}\right\}e^{-\theta_0(\alpha)t_1^{\,2}/2}\end{pmatrix}\left(1+R'(t_1)\right)$$

$$-\frac{H_{HR}}{T^2}\left(M\left(\frac{\theta_0(\alpha)R(t_1)^2}{2}-\frac{\theta_0(\alpha)^2R(t_1)^4}{12}\right)+A\theta_0(\alpha)\begin{pmatrix}(R_1(t_4)+t_4)+\frac{\lambda(R_1(t_4)+t_4)^2}{2}\\ -\frac{\theta_0(\alpha)(R_1(t_4)+t_4)^3}{6}\end{pmatrix}\right)\left(1+R'(t_1)\right)$$

$$-\frac{C_S}{T^2}\left\{\frac{AR_1(t_4)^2}{2}-\frac{at_4^2}{2}-\frac{A(b-1)}{2}\lambda t_4^2\right\}\left(1+R'(t_1)\right)$$

$$+\frac{1}{T}[P_{CP}(a+bAe^{\lambda t_1})+P_{CC}R_c+P_{CR}MR'(t_1)]+\frac{C_{DP}}{T}$$

$$\left[\theta_0(\alpha)at_1+\frac{\theta_0(\alpha)(b-1)A}{\lambda}+\frac{A\theta_0(\alpha)}{2\lambda^4}\left(3\lambda^3R(t_1)^2R'(t_1)+2\lambda^2R(t_1)R'(t_1)+2\lambda^4R'(t_1)\right]+\right.$$

$$\frac{C_{DC}}{T}\begin{pmatrix}R_c\left(\frac{3\theta_0(\alpha)t_1^{\,2}}{3}-\frac{\theta_0(\alpha)^2t_1^{\,4}}{3}\right)+\frac{2\theta_0(\alpha)BzR(t_1)\;R'(t_1)}{2}+\\ M\left(\frac{3\theta_0(\alpha)R(t_1)^2R'(t_1)}{3}-\frac{\theta_0(\alpha)^2R(t_1)^4R'(t_1)}{3}\right)-\\ \frac{2\theta_0(\alpha)R(t_1)R'(t_1)M}{2}\left\{t_1+\frac{\theta_0(\alpha)t_1^{\,3}}{6}\right\}e^{-\theta_0(\alpha)t_1^{\,2}/2}+\frac{\theta_0(\alpha)R(t_1)^2M}{2}\left\{1+\frac{\theta_0(\alpha)t_1^{\,2}}{2}\right\}e^{-\theta_0(\alpha)t_1^{\,2}/2}+\\ -\frac{\theta^2{}_0(\alpha)\,t_1\,R(t_1)^2M}{2}\left\{t_1+\frac{\theta_0(\alpha)t_1^{\,3}}{6}\right\}e^{-\theta_0(\alpha)t_1^{\,2}/2}\end{pmatrix}$$

$$+\frac{C_{DR}}{T}\left(M\left(\frac{3\theta_0(\alpha)R(t_1)^2R'(t_1)}{3}-\frac{\theta_0(\alpha)^2R(t_1)^4R'(t_1)}{3}\right)\right)$$

$$+\frac{C_{HP}}{T}\left[\left\{a-\frac{a\theta_0(\alpha)t_1^2}{2}+\frac{(b-1)A}{\lambda}\left(2\lambda t_1-\frac{\theta_0(\alpha)t_1}{1}\right)+\frac{A}{\lambda}\right\{\right.$$

$$\left.+R'(t_1)\left(2+\frac{\theta_0(\alpha)}{\lambda}-\frac{\theta_0(\alpha)}{\lambda^2}\right)-\frac{2\theta_0(\alpha)R(t_1)R'(t_1)}{\lambda^2}\right\}$$

$$+\frac{A}{\lambda^2}\left\{\left(1+\frac{\theta_0(\alpha)}{\lambda^2}\right)R'(t_1)+2\lambda R(t_1)R'(t_1)-\frac{2\theta_0(\alpha)R(t_1)^2R'(t_1)}{1}+\left(\frac{2\lambda^2\theta_0(\alpha)-\theta_0(\alpha)}{3\lambda}\right)2R(t_1)^3R'(t_1)\right.$$

$$\left.\left.-\frac{5\theta_0(\alpha)}{6}R(\mathrm{t}_1)^4R'(t_1)\right\}\right]+$$

$$\frac{C_{HC}}{T}\left(\begin{array}{l}R_c\left(\frac{2\theta_0(\alpha)t_1}{2}-\frac{\theta_0(\alpha)^2t_1^3}{3}\right)+\theta_0(\alpha)BzR'(t_1)+M\left(\frac{2\theta_0(\alpha)R(t_1)R'(t_1)}{2}-\frac{\theta_0(\alpha)^2R(t_1)^3}{3}\right)-\\ \theta_0(\alpha)R'(t_1)M\left\{t_1+\frac{\theta_0(\alpha)t_1^3}{6}\right\}e^{-\theta_0(\alpha)t_1^2/2}+\theta_0(\alpha)R(t_1)M\left\{1+\frac{\theta_0(\alpha)t_1^2}{2}\right\}e^{-\theta_0(\alpha)t_1^2/2}\\ -\frac{\theta^2{}_0(\alpha)\,\mathrm{t}_1\,R(t_1)M}{2}\left\{1+\frac{\theta_0(\alpha)t_1^2}{2}\right\}e^{-\theta_0(\alpha)t_1^2/2}\end{array}\right)+$$

$$\frac{H_{HR}}{T}\left(M\left(\frac{2\theta_0(\alpha)R(t_1)R'(t_1)}{2}-\frac{\theta_0(\alpha)^2R(t_1)^3R'(t_1)}{3}\right)\right)=0 \qquad \ldots (33)$$

$$\frac{\partial K}{\partial t_4}=\frac{-1}{T^2}[C+P_{CP}(at_1+\frac{bAe^{\lambda t_1}}{\lambda})+P_{CC}R_ct_1+P_{CR}MR(t_1)]\left(1+R_1'(t_4)\right)-\frac{C_{DP}}{T^2}$$

$$\left[\begin{array}{l}\frac{\theta_0(\alpha)at_1^2}{2}+\frac{\theta_0(\alpha)(b-1)At_1}{\lambda}+\frac{A\theta_0(\alpha)}{2\lambda^4}\\ \left(\lambda^3R(t_1)^3+\lambda^2R(t_1)^2+2\lambda^4+2\lambda^4R(t_1)+2\lambda^2-2\right)\end{array}\right]\left(1+R_1'(t_4)\right)-$$

$$\frac{C_{DC}}{T^2}\left(\begin{array}{l}R_c\left(\frac{\theta_0(\alpha)t_1^3}{3}-\frac{\theta_0(\alpha)^2t_1^5}{15}\right)+\frac{\theta_0(\alpha)BzR(t_1)^2}{2}+\\ M\left(\frac{\theta_0(\alpha)R(t_1)^3}{3}-\frac{\theta_0(\alpha)^2R(t_1)^5}{15}\right)-\\ \frac{\theta_0(\alpha)R(t_1)^2M}{2}\left\{t_1+\frac{\theta_0(\alpha)t_1^3}{6}\right\}e^{-\theta_0(\alpha)t_1^2/2}\end{array}\right)\left(1+R_1'(t_4)\right)$$

$$+\frac{C_{DR}}{T^2}\left(\begin{array}{l}M\left(\frac{\theta_0(\alpha)R(t_1)^3}{3}-\frac{\theta_0(\alpha)^2R(t_1)^5}{15}\right)+\\ A\theta_0(\alpha)\left(\begin{array}{l}\frac{(R_1(t_4)+t_4)^2}{2}+\frac{\lambda(R_1(t_4)+t_4)^3}{3}\\ -\frac{\theta_0(\alpha)(R_1(t_4)+t_4)^4}{8}\end{array}\right)\end{array}\right)\left(1+R_1'(t_4)\right)$$

$$-\frac{C_{HP}}{T^2}\Bigg[\Bigg\{at_1-\frac{a\theta_0(\alpha)t_1^3}{6}+\frac{(b-1)A}{\lambda}\left(\lambda t_1^2-\frac{\theta_0(\alpha)t_1^3}{3}\right)+\frac{A}{\lambda}\Bigg\{-\frac{2\theta_0(\alpha)}{\lambda^3}$$

$$+R(t_1)\left(2+\frac{\theta_0(\alpha)}{\lambda}-\frac{\theta_0(\alpha)}{\lambda^2}\right)-\frac{\theta_0(\alpha)R(t_1)^2}{\lambda^2}+\frac{\theta_0(\alpha)}{\lambda^2}\Bigg\}$$

$$+\frac{A}{\lambda^2}\Bigg\{\left(1+\frac{\theta_0(\alpha)}{\lambda^2}\right)R(t_1)+\lambda R(t_1)^2-\frac{2\theta_0(\alpha)R(t_1)^3}{3}+\left(\frac{2\lambda^2\theta_0(\alpha)-\theta_0(\alpha)}{6\lambda}\right)R(t_1)^4$$

$$-\frac{\theta_0(\alpha)}{6}R(t_1)^5\Bigg\}\Bigg]\left(1+R_1'(t_4)\right)-$$

$$-\frac{C_{HC}}{T^2}\begin{pmatrix} R_c\left(\frac{\theta_0(\alpha)t_1^2}{2}-\frac{\theta_0(\alpha)^2t_1^4}{12}\right)+\theta_0(\alpha)BzR(t_1)+M\left(\frac{\theta_0(\alpha)R(t_1)^2}{2}-\frac{\theta_0(\alpha)^2R(t_1)^4}{12}\right)- \\ \theta_0(\alpha)t_2M\left\{t_1+\frac{\theta_0(\alpha)t_1^3}{6}\right\}e^{-\theta_0(\alpha)t_1^2/2} \end{pmatrix}$$

$$\left(1+R_1'(t_4)\right)$$

$$-\frac{H_{HR}}{T^2}\begin{pmatrix} M\left(\frac{\theta_0(\alpha)R(t_1)^2}{2}-\frac{\theta_0(\alpha)^2R(t_1)^4}{12}\right)+ \\ A\theta_0(\alpha)\begin{pmatrix}(R_1(t_4)+t_4)+\frac{\lambda(R_1(t_4)+t_4)^2}{2} \\ -\frac{\theta_0(\alpha)(R_1(t_4)+t_4)^3}{6}\end{pmatrix}\end{pmatrix}\left(1+R_1'(t_4)\right)$$

$$-\frac{C_S}{T^2}\left\{\frac{AR_1(t_4)^2}{2}-\frac{at_4^2}{2}-\frac{A(b-1)}{2}\lambda t_4^2\right\}\left(1+R_1'(t_4)\right)$$

$$+\frac{C_{DR}}{T}\left(A\theta_0(\alpha)\begin{pmatrix}\frac{(R_1(t_4)+t)(R_1'(t_4)+1)}{1}+\frac{\lambda(R_1(t_4)+t_4)^2(R_1'(t_4)+1)}{1}- \\ \frac{\theta_0(\alpha)(R_1(t_4)+t_4)^3(R_1'(t_4)+1)}{2}\end{pmatrix}\right)$$

$$+\frac{H_{HR}}{T}\left(A\theta_0(\alpha)\left((R_1'(t_4)+1)+\frac{\lambda(R_1(t_4)+t_4)(R_1'(t_4)+1)}{1}-\frac{\theta_0(\alpha)(R_1(t_4)+t_4)^2(R_1'(t_4)+1)}{2}\right)\right)$$

$$\frac{C_S}{T}\left\{AR_1(t_4)R_1'(t_4)-at_4-A(b-1)\lambda t_4\right\}=0$$

… (34)

Here we obtain two simultaneous non-linear equation in of t_1 and t_4 can be find out optimal value by using some suitable computational numerical method and the optimum value of t_2, t_3, I_m, I_b and minimum total average cost 'K' can be obtained from equations.

V. Special Cases:

Case I : If b = 0 then the discussed model convert to production inventory model in which production rate is constant and independent on the demand.

Case II : If $\theta_0(\alpha)=0$ then the discussed model reduces to production inventory model with out deterioration

Case III : If $\lambda=0$, $b=0$ the model reduce to uniform production rate and constant demand.

VI. Conclusion

In the proposed model a production inventory model is formulated for random deteriorating item with a increasing market demand rate with time and production rate is dependent on the demand. Result in this study can provide a valuable reference for decision markers in planning the production and controlling the inventory. The model proposed here in is resolved by using maclaurin series and cost minimization technique is used to get the approximate expression for total average cost and other parameters & some special cases of model are also discussed. We derive an expressions for different cost associated in the model. We derive equations, solution of these equations gives the optimal cycle and optimal cost of repairable items. A future study will incorporate more realistic assumption in the proposed model.

References

[1]. **Alamri, A.A. (2010).** Theory and methodology on the global optimal solution to a Reverse logistics inventory model for deteriorating items and time varying rates. Computer & Industrial Engineering,(60), 236-247.

[2]. **Balkhi, Z. T. and Benkherouf, L.** (1996 b)On the optimal replenishment schedule for an inventoy system with deteriorating items and time varying demand and production rates.Computers & Industrial Engineering, 30 ; 823-829..

[3]. **Bhunia, A. K. and Maiti, M.** (1998) A two warehouse inventory model for deteriorating items with a linear trend in demand and shortages.Jour of Opl. Res. Soc., 49 : 287-292.

[4]. **Chang, H. J. and Dye, C. Y.** (1999) An EOQ model for deteriorating items with time varying demand and partial backlogging.**Jour, of Opl. Res. Soc.,** 50 : 1176-1182.

[5]. **Chung, C. J., & Wee, H. M. (2011).** Short life-cycle deteriorating product remanufacturing in a green supply chain inventory control system. International Journal of Production Economics, 129(1), 195-203.

[6]. **Covert, R.P., and Philip, G.C**.,(1973), An EOQ Model for Item with Weibull Distribution Deterioration, AIIE Transactions, 5, (4), 323-326,

[7]. **Dobos, I., & Richter, K. (2003),** A production/recycling model with stationary demand and return rates.Central European journal of Operations Research, 11(1), 35-46.

[8]. **Dobos, I., & Richter, K. (2004),** An extended production/recycling model with stationary demand and return rates. International Journal of production Economics, 90(3), 311-323.

[9]. **Dobos, I., & Richter, K. (2006),** A production/recycling model with quality considerations. International Journal of Production Economics, 104(2), 571-579.

[10]. **El Saadany A. M. A., & Jaber M. Y. (2010**)., A production/ remanufacturing inventory model with price and quality dependant return rate. Computers and Industrial Engineering, 58(3), 352–362.

[11]. **Ghare, P.M., and Schrader, G.F.**,(1963), A Model for An Exponentially Decaying Inventory, The Journal of Industrial Engineering, 14, (5), 238-243.

[12]. **Goswami, A and Chaudhuri, K. S.** (1991) EOQ model for an inventory with a linear trend in demand and finite rate of replenishment considering shortages.int. Jour. Syst ScL, 22, 181-187.

[13]. **Konstantaras, I., & Skouri, K. (2010),** Lot sizing for a single product recovery system with variable set up numbers. European Journal of Operations Research, 203(2), 326-335.

[14]. **Perumal, V. and Arivarignan, G.** (2002)A production inventory model with two rates of production and backorders.**Int. Jour of Mgt. & Syst.,** 18 (1) : 109-119.

[15]. **Pyke, D.** (1990)"Priority Repair and Dispatch Policies for Reparable-Item Logistics Systems."Naval Research Logistics v37 n1 1-30 Feb.

[16]. **Raafat, F., Wolfe, P.M. and Eldin, H.K.** An Inventory Model for Deteriorating Item, Computers & Industrial Engineering, 20, (1), 89-94, 1991.

[17]. **Richter, K, (1996b).** The extended EOQ repair and waste disposal model. International Journal of Production Economics, 45(1-3), 443-447.

[18]. **Richter, K. (1996),** The EOQ repair and waste disposal model with variable setup numbers. European Journal of Operational Research, 95(2), 313-324.

[19]. **Richter, K., & Dobos, I. (1999).** Analysis of the EOQ repair and waste disposal model with integer set up numbers. International journal of production economics, 59(1-3), 463-467.

[20]. **Schrady, D.A. (1967).** A deterministic inventory model for repairable items. Naval Research Logistics Quarterly, 14, 391–398.

[21]. **Teng, J. T. & Chang, C. T. (2005).** Economic production quantity models for deteriorating items with price and stock-dependent demand. Computational Operations Research, 32 (2), 297-308.

[22]. **Teng, J. T. (1996)** A deterministic replenishment model with linear trend in demand. Opns. Res. Lett. 19 : 33-41.