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# MHD Flow past a Vertical Oscillating Plate with Radiation and Chemical Reaction in Porous Medium

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***Abstract:*** *An analysis is performed to study the effect of Thermal radiation and first order Chemical reaction on unsteady natural Convective flow of a viscous incompressible Conducting fluid past over an infinite isothermal vertical oscillating plate in Porous medium. The dimensionless governing equations are solved using the Laplace transform technique. The velocity, temperature and concentration are studied for different parameters like the magnetic field parameter, radiation parameter, chemical reaction parameter, thermal Grashof number, Schmidt number, phase angle and time. It is observed that the velocity increases with decreasing magnetic field parameter or radiation parameter. It is also observed that the velocity increases with decreasing magnetic field parameter, radiation parameter and phase angle.*

***Keywords:*** *Chemical reaction, magnetic field, oscillating vertical plate, porous medium, radiation.*

## I. Introduction

MHD flow has application in metrology, solar physics and in motion of earth core. Also it has applications in the field of stellar and planetary magnetospheres, aeronautics, chemical engineering and electronics. Magneto-convection plays an important role in agriculture, petroleum industries, geophysics and in astrophysics. Heat transfer with radiation effects also has mathematical as well as physical importance and many researchers found interest it as a subject of investigation. Mass transfer with chemical reaction is another most commonly encountered circumstance in chemical industry as well as in physical and biological sciences. There are many situations where convection heat transfer phenomena are accompanied by mass transfer as well as radiation also. When mass transfer takes place in a fluid at rest, the mass is transferred purely by molecular diffusion resulting from concentration gradients. For low concentration of the mass in the fluid and low mass transfer rates, the convective heat and mass transfer processes are similar in nature. Studies in porous medium are other important areas in heat transfer processes.

Due to these important industrial and engineering applications, magneto-convection with radiation and chemical reaction has been gaining considerable attention amongst researchers. Hence a study combining all these aspects will surely enhance the already developed areas further for more complex studies.

A number of investigations have also been made by considering some or all these aspects but under different physical and initial conditions. Exact solutions of free convection flow past a vertical oscillating plate in free convective flow was first obtained by Soundalgekar [1] and the same problem with mass transfer effect was considered by Soundalgekar and Akolkar [2]. Das et. al. [3] studied the effects of mass transfer on free convection flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction. They also studied the transient free convection flow past an infinite vertical plate with periodic temperature [4]. Effect of mass transfer on the flow past an infinite vertical oscillating plate with constant heat flux was studied by Soundalgekar et. al. [5].

The effects of mass transfer on free convection flow past a semi-infinite vertical isothermal plate was first studied by Gebhart and Pera [6] and the effects of mass transfer on the flow past an impulsively started infinite vertical plate with variable temperature was studied by Soundalgekar et. al. [7]. Muthucumaraswamy et. al. [8] considered the effects of mass transfer on impulsively started infinite vertical plate with variable temperature and uniform mass flux. All of them considered the fact that free convection current caused by temperature differences is also caused by the differences in concentration or material constitution as suggested by Gebhart [9]. MHD Flow Past a Vertical Oscillating Plate with Radiation and chemical Reaction in Porous MediumFurther, in many cases in the process of free convection, chemical reaction also takes place due to the presence of foreign masses as impurities in fluid. It is found that in many chemical engineering processes, chemical reaction takes place between foreign masses, present in the form of ingredients and the fluid. This type of chemical reaction may change the temperature and the heat content of the fluid and may affect the free convection process. However, if the presence of such foreign mass is very low then we can assume the first order chemical reaction so that heat generation due to chemical reaction can be considered to be very negligible. Das et. al. [10] considered the effects of mass transfer on flow past an impulsively started vertical plate and Muthucumaraswamy and Meenakshisundaram [11] studied the chemical reaction effects on vertical oscillating plate with variable temperature and chemical reaction. Deka and Neog [12] considered the combined effects of thermal radiation and chemical reaction on free convection flow past a vertical plate in porous medium. Chaudhary and Jain [13] studied the magneto-hydrodynamic transient heat and mass transfer flow by free convection past a vertical plate, when the temperature of the plate oscillates in time about a constant mean temperature and the plate is embedded in a porous medium. They extended the work of Das et.al. [14], which include the effects of mass transfer, magnetic field and porous medium. Recently, Neog [15] studied the unsteady MHD flow past a vertical oscillating plate with variable temperature and chemical reaction and Deka and Neog [16] studied the MHD flow past a vertical oscillating plate with thermal radiation and variable mass diffusion.

Although different authors studied mass transfer with or without chemical reaction in flow past oscillating vertical plate by considering different surface conditions but the study on the effects of magnetic field on free convection heat and mass transfer in the presence of chemical reaction and oscillating plate has not been found in literature and hence the motivation to undertake this study. It is therefore proposed to study the effects of chemical reaction on hydro magnetic flow past an oscillating vertical plate under the assumption of first order chemical reaction.

## II. Mathematical Analysis

We have considered here an unsteady natural convection flow of a viscous incompressible electrically conducting fluid past an infinite vertical plate in porous medium. To visualize the flow pattern a Cartesian co-ordinate system is considered where x'-axis is taken along the infinite vertical plate, y'-axis is normal to the plate and fluid fills the region $y' \geq 0$. Initially, the fluid and the plate are kept at the same constant temperature $T'_\infty$ and species concentration $C'_\infty$. At time $t' > 0$, the plate is given an oscillatory motion in its own plane with a velocity $U_0\cos\omega' t'$. At the same time the plate temperature is raised to $T'_w$ and concentration is raised to $C'_w$ and a magnetic field of uniform strength $B_0$ is applied normal to the plate. It is assumed that the magnetic Reynolds number is very small and the induced magnetic field is negligible in comparison to the transverse magnetic field. It is also assumed that the effect of viscous dissipation is negligible in the energy equation and the level of species concentration is very low so the Soret and Dufour effects are negligible.

As the plate is infinite in extent so the derivatives of all the flow variables with respect to x' vanish and they can be assumed to be functions of y' and t' only as a result the motion becomes one dimensional with only non-zero vertical velocity component u', varying with y' and t' only. Due to one dimensional nature, the equation of continuity is trivially satisfied.

Under the above assumptions and following Boussinesq approximation, the unsteady flow field is governed by the following set of equations:

$$\frac{\partial u'}{\partial t'} = g\beta(T'-T'_\infty) + g\beta^*(C'-C'_\infty) + \nu\frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2}{\rho}u' - \frac{\nu}{\kappa^*}u' \tag{1}$$

$$\rho C_p\frac{\partial T'}{\partial t'} = k\frac{\partial^2 T'}{\partial y'^2} - \frac{\partial q_r}{\partial y'} \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2} - K_1 C' \tag{3}$$

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Along with the following initial and boundary conditions:

$$\left.\begin{array}{llll} u'=0, & T'=T'_\infty, & C'=C'_\infty & \text{for all } y' \text{ and } t'\le 0 \\ u'=U_0\cos\omega' t', & T'=T'_w, & C'=C'_w & \text{at } y'=0 \\ u'\to 0, & T'\to T'_\infty, & C'\to C'_\infty & \text{as } y'\to\infty \end{array}\right\} t'>0 \tag{4}$$

Now to reduce the above equations in non-dimensional form we introduce the following non-dimensional quantities.

$$\left.\begin{array}{l} u=\dfrac{u'}{U_0},\quad t=\dfrac{t' U_0^2}{\nu},\quad y=\dfrac{y' U_0}{\nu},\quad \theta=\dfrac{T'-T'_\infty}{T'_w-T'_\infty},\quad \phi=\dfrac{C'-C'_\infty}{C'_w-C'_\infty}, \\ Pr=\dfrac{\mu C_p}{\kappa},\quad Gr=\dfrac{g\beta\nu(T'_w-T'_\infty)}{U_0^3},\quad Gm=\dfrac{g\beta^*\nu(C'_w-C'_\infty)}{U_0^3},,\ K=\dfrac{K^* U_0^2}{\nu^2} \\ R=\dfrac{\nu K^*}{U_0^2},\quad Sc=\dfrac{\nu}{D},\quad \omega=\dfrac{\omega'\upsilon}{U_0^2},\quad M=\dfrac{\sigma B_0^2\upsilon}{\rho U_0^2},\quad F=\dfrac{4I_1\nu^2}{KU_0^2},\quad \dfrac{\partial q_r}{\partial y'}=(4I_1\Delta T')\theta \end{array}\right\} \tag{5}$$

Thus with the help of these non-dimensional quantities, equations (1), (2) and (3) reduce to:

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial y^2}+Gr\theta+Gm\phi-M'u \tag{6}$$

$$\frac{\partial \theta}{\partial t}=\frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2}-\frac{F}{Pr}\theta \tag{7}$$

$$\frac{\partial \phi}{\partial t}=\frac{1}{Sc}\frac{\partial^2 \phi}{\partial y^2}-R\phi \tag{8}$$

And the initial and boundary conditions are as follows:

$$\left.\begin{array}{l} u=0, \quad \theta=0, \quad \phi=0, \text{ for all } y \text{ and } t\le 0 \\ \left.\begin{array}{l} u=\cos\omega t, \quad \theta=1, \quad \phi=1 \text{ at } y=0 \\ u\to 0, \quad \theta\to 0, \quad \phi\to 0 \text{ as } y\to\infty \end{array}\right\}, \ t>0 \end{array}\right\} \tag{9}$$

Solutions of the equations (6), (7) and (8) subject to the initial and boundary conditions (9) are obtained with the help of Abramowtz and Stegum [16] and Hetnarski's [17] algorithm. They are obtained as follows:

$$\theta(y,t)=\frac{1}{2}\left\{e^{-y\sqrt{a\,Pr}}erfc\left(\frac{y\sqrt{Pr}}{2\sqrt{t}}-\sqrt{at}\right)+e^{y\sqrt{ScR}}erfc\left(\frac{y\sqrt{Pr}}{2\sqrt{t}}+\sqrt{at}\right)\right\} \tag{10}$$

$$\phi(y,t)=\frac{1}{2}\left\{e^{-y\sqrt{ScR}}erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}-\sqrt{Rt}\right)+e^{y\sqrt{ScR}}erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}+\sqrt{Rt}\right)\right\} \tag{11}$$

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$$\begin{aligned} u(y,t) &= \frac{1}{4}\left[e^{i\omega t}\left\{e^{-y\sqrt{e}}erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{et}\right)+e^{y\sqrt{e}}erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{et}\right)\right\}+\text{cc}\right] \\ &+\frac{G_1}{2b}\left[\begin{array}{l} e^{bt}\left(\left\{e^{-y\sqrt{f}}erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{ft}\right)+e^{y\sqrt{f}}erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{ft}\right)\right\}\right. \\ \left.-\left\{e^{-y\sqrt{c\Pr}}erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}}-\sqrt{ct}\right)-e^{y\sqrt{c\Pr}}erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}}+\sqrt{ct}\right)\right\}\right) \\ +\left\{e^{-y\sqrt{a\Pr}}erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}}-\sqrt{at}\right)+e^{y\sqrt{a\Pr}}erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}}+\sqrt{at}\right)\right\} \end{array}\right] \\ &+\frac{G_2}{2d}\left[\begin{array}{l} e^{dt}\left(\left\{e^{-y\sqrt{g}}erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{gt}\right)+e^{y\sqrt{g}}erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{gt}\right)\right\}\right. \\ \left.-\left\{e^{-y\sqrt{hSc}}erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}-\sqrt{ht}\right)+e^{y\sqrt{hSc}}erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}+\sqrt{ht}\right)\right\}\right) \\ +\left\{e^{-y\sqrt{ScR}}erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}-\sqrt{Rt}\right)+e^{y\sqrt{ScR}}erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}+\sqrt{Rt}\right)\right\} \end{array}\right] \end{aligned} \tag{12}$$

Here, the following symbols are used in the above solutions:

$$\left.\begin{array}{l} a=\frac{F}{Pr}, \quad b=\frac{M'-a\,Pr}{Pr-1}, \quad c=a+b, \quad d=\frac{RSc-M'}{1-Sc}, \quad e=M'+i\omega, \quad f=M'+b, \\ g=M'+d, \quad h=R+d, \quad G_1=\frac{Gr}{Pr-1}, \quad G_2=\frac{Gm}{Sc-1}, \quad G_3=\frac{G_2}{d}+\frac{G_1}{b}, \ M'=M+\frac{1}{Da} \end{array}\right\} \tag{13}$$

## III. Results And Discussion

The numerical values of the velocity, temperature and concentration fields are computed for different parameters like magnetic field parameter, radiation parameter, chemical reaction parameter, Schmidt number, Prandtl number ,thermal Grashof number and mass Grashof number and phase angle and they are presented graphically in figure.

Figure 1 represents the temperature profiles for different values of Pr (0.71, 7) and F (0.5, 5). From this figure it is clear that temperature decreases with the increase of Pr and F.

In figures 2 concentration profiles are presented for different values of Sc (0.6, 3.5) and R (2, 5). It is observed that increase of Schmidt number and chemical reaction parameter lead to decrease in concentration.

Velocity profiles for different values of parameters are shown in figures 3-5. Influence of R (2, 5) and K (1, 5, 10) are shown in figure 3 for some fixed values of the other parameters. Effect of Gr (5, 10), Gm (2, 5) and M (0.5, 1) are presented in figure 4 for some fixed values of other parameters and in figure 5 velocity

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profiles are presented for different values of $\omega t$ ($\pi/6$, $\pi/4$, $\pi/3$, $\pi/2$). It is clear from these figures that velocity increases with the increase of Gr, Gm and K. Further, velocity decreases with the increase of M, F, Sc, R and $\omega t$.

## IV. Figures

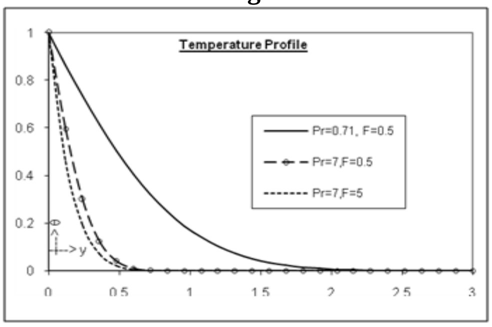


**Figure-1** Temperature Profile showing the effect of Pr and F

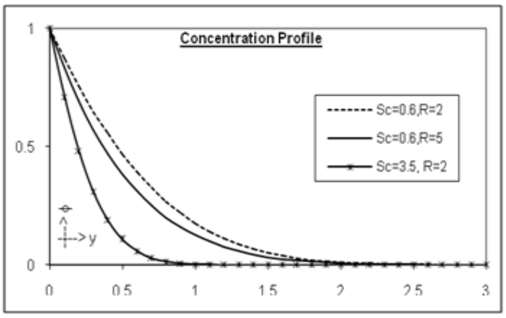


**Figure-2** Concentration Profile showing the effect of Sc and R

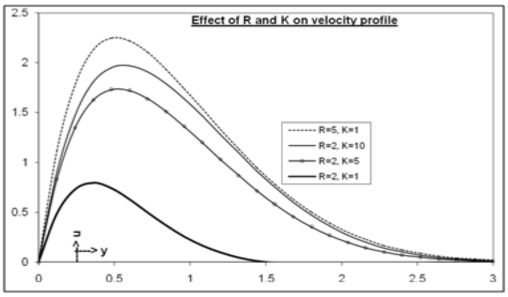


**Figure-3** Velocity Profile showing the effect of R and K.

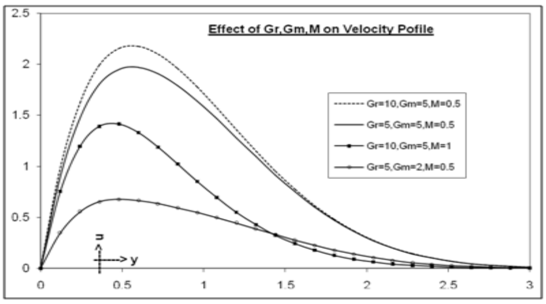


**Figure-4** Velocity Profile showing the effect of Gr, Gm and M

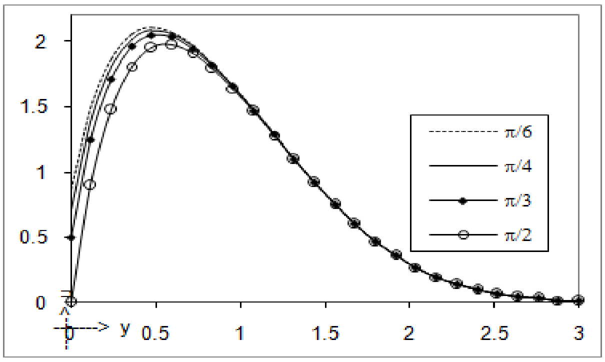


**Figure-5** Velocity Profile showing the effect of phase angle.

## IV. Conclusions

An exact analysis in closed form is performed to study the influence of chemically reacting hydro magnetic flow past a vertical oscillating plate in porous medium. Solutions are obtained by Laplace transform technique. Some of the important conclusions of the study are as follows:

- Temperature decreases with the increase of Pr and F.
- Concentration decreases as Sc and R increase.
- Velocity increases with increasing Gr, Gm and K and with decreasing M and F.
- Also increase in Sc, R and $\omega$t lead to decrease in velocity.

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