

Effect of isoelastic form of price dependent demand in cooperative advertising

Krishna Solanki^{*1}, Ravi Gor²

1. *L.D.R.P. Institute of Technology and Research, Gandhinagar, India*

2. *Dr. BabasahebAmbedkar Open University, Gujarat, India*

Abstract: *This is an extension of the co-operative advertising model developed earlier by the authors. We develop a manufacturer-retailer channel co ordination where the demand is modeled as a multiplicative effect of price and an additive sales response function. We change the form of demand here as an extension to earlier model developed by the authors .We use isoelastic form of the price dependent demand instead of the linear form to observe changes in the model if any. We develop both sequential and simultaneous moves non co-operative game structures where both retailer and manufacturer act simultaneously and independently and compare them through propositions. Finally we develop a cooperative model and discuss the optimality of pareto efficient scheme.*

Keywords: *Cooperative advertising, Game theory, Nash equilibrium, Stackelberg equilibrium, Supply chain coordination*

I. Introduction

Co operative advertising is a relation between manufacturer and retailer and here we develop the classic relationship between manufacturer and retailer named “leader-follower” two stage games. An equilibrium of this game is called the stackelberg equilibrium. In the stackelberg equilibrium, if the ratio of the manufacturer and retailer’s marginal profits is relatively high then manufacturer offers a positive amount for advertising to retailer but if marginal profit ratio is low then manufacturer offers zero amount of allowance to retailer. Kuehn, 1962; Thompson and Teng, 1984; Jorgensen and Zaccour, 1999, 2003 developed multiplicative effect of the price (using an appropriate function) and advertising (using a sale response function) earlier in their papers. Simon and Arndt (1980) concluded that diminishing returns characterize the shape of the advertising-sales response function. Similar approaches of relating demand and advertising expenditure were used in Kim and Staelin (1999) and Karray and Zaccour (2006). Assuming that both the types of advertising efforts; national and local, could influence sales and that their effects should be assessed separately (Jorgensen et al., 2000; Huang et al., 2002). Solanki and Gor (2013) developed two game theoretic models; a cooperative model and a non cooperative model in a stackelberg game framework. This paper follows some of the part with an isoelastic demand and different sales response function.

We develop a simultaneous move game in next section where the manufacturer and the retailer independently maximize their individual profits. This equilibrium is called Nash equilibrium. The manufacturer’s brand name investment is higher at Nash equilibrium than at stackelberg equilibrium. If the profits ratio of the retailer and the manufacturer is relatively low, then the local advertising expenditure is lower at Nash equilibrium than at stackelberg equilibrium; otherwise it is higher at Nash equilibrium than at stackelberg equilibrium.

We consider a Pareto efficient scheme in this paper. The system profit is higher with cooperation than with non-cooperation and maximized for every Pareto efficient co op advertising scheme, but not for any other schemes. The system profit is higher at any Pareto efficient scheme than at both noncooperative equilibriums. Also, if the marginal profits ratio of the retailer and manufacturer is relatively high, then the manufacturer’s brand name investment is higher at any Pareto efficient scheme than at both noncooperative equilibriums; otherwise the manufacturer’s brand name investment at any Pareto efficient scheme is higher than at Stackelberg equilibrium and is lower than at Nash equilibrium. The local advertising expenditure is higher at any Pareto efficient scheme than at both noncooperative equilibriums.

Proofs of the results are given in Appendix.

Assumptions

We consider a single-manufacturer-single retailer channel in which the retailer sells only the manufacturer’s brand within the product class. We can further extend it. Co operative advertising gives customers the reason such as low price and high quality to buy and also aware about when and where to obtain the product. Though manufacturer and retailer’s advertising performance are different but it creates positive effect on product sale.

Notations:

- p : retailer's selling (retail) price
- w : manufacturer's selling (wholesale) price
- a : retailer's local advertising expenditure, $a \geq 0$
- A : manufacturer's national advertising expenditure, $A \geq 0$
- t : the manufacturer participation rate, the percentage that the manufacturer agrees to pay the retailer to subsidize the local advertising cost

The consumer demand is as follow.

$$D(p, a, A) = g(p) \cdot h(a, A) \tag{1}$$

Where $g(p)$ reflects the impact of the retail price on the demand and $h(a, A)$, the sales response function reflects the impact of the advertising expenditures on the demand. The demand function $g(p)$ is a linearly decreasing function of p and is specifically taken as

$$g(p) = \gamma_0 p^{-\gamma_1} \tag{2}$$

where $\gamma_0 \gg \gamma_1$ and to ensure $g(p) > 0$ we need to restrict $p < (\gamma_0/\gamma_1)$

We model advertising effects on consumer demand as

$$h(a, A) = k_r \log a + k_m \log A \tag{3}$$

where k_r, k_m are positive constants reflecting the efficacy of each type of advertising in generating sales. The above equation captures both types of advertising effects which usually are not substitutes. The demand h is an increasing and concave function with respect to a and A , and has the property that is consistent with the commonly observed "advertising saturation effect", i.e., additional advertising spending generates continuously diminishing returns.

$$D(p, a, A) = (\gamma_0 p^{-\gamma_1}) \cdot (k_r \log a + k_m \log A) \tag{4}$$

The manufacturer's, retailer's and system's expected profit functions are as follows.

$$\pi_m = w (\gamma_0 p^{-\gamma_1}) \cdot (k_r \log a + k_m \log A) - ta - A \tag{5}$$

$$\pi_r = (p - w)(\gamma_0 p^{-\gamma_1}) \cdot (k_r \log a + k_m \log A) - (1 - t)a \tag{6}$$

$$\pi_{m+r} = p (\gamma_0 p^{-\gamma_1}) \cdot (k_r \log a + k_m \log A) - a - A \tag{7}$$

II. Stackelberg Equilibrium

We model the relationship between the manufacturer and retailer in which manufacturer is a leader and retailer a follower. It is a sequential noncooperative game.

Since the π_r is a concave function we can set its first derivative with respect to a to be zero.

$$\frac{\partial \pi_r}{\partial a} = (p - w) (\gamma_0 p^{-\gamma_1}) \frac{k_r}{a} - (1 - t) = 0 \tag{8}$$

Then, we have

$$a = \frac{(p - w)(\gamma_0 p^{-\gamma_1})k_r}{(1 - t)} \tag{9}$$

Equation (9) shows positive change in manufacturer's co-op advertising reimbursement policy and brand name investments. We can observe that

$$\frac{\partial a}{\partial t} = \frac{(p - w)(\gamma_0 p^{-\gamma_1})k_r}{(1 - t)^2} > 0 \tag{10}$$

$$\frac{\partial a}{\partial A} = 0 \tag{11}$$

Equation (10) shows that the more the manufacturer is willing to share the cost of local advertising, the more the retailer will spend on the local advertising.

Also, the optimal value of A and t are determined by maximizing the manufacturer's profit subject to the constraint imposed by eq. (9). The manufacturer's problem is as follow.

$$\max_{A,t} \pi_m = w (\gamma_0 p^{-\gamma_1}) (k_r \log a + k_m \log A) - ta - A \tag{12}$$

s.t. $0 \leq t \leq 1, A \geq 0$,

where, $a = \frac{(p - w)(\gamma_0 p^{-\gamma_1})k_r}{(1 - t)}$

Substituting the value of a in to (12), we get

$$\max_{A,t} \pi_m = w (\gamma_0 p^{-\gamma_1}) \left(k_r \log \left(\frac{(p-w)(\gamma_0 p^{-\gamma_1}) k_r}{(1-t)} \right) + k_m \log A \right) - t \left(\frac{(p-w)(\gamma_0 p^{-\gamma_1}) k_r}{(1-t)} \right) - A \quad (13)$$

s.t. $0 \leq t \leq 1, A \geq 0$.

Solving equation (13) for t and A and substituting then in eq. (9) we get unique equilibrium point, (a^*, t^*, A^*) of the two – stage game as follows.

$$a^* = (w(\gamma_0 p^{-\gamma_1}) k_r) \quad (14)$$

$$t^* = \begin{cases} \frac{2w-p}{w}, & 2w > p \\ 0, & \text{otherwise} \end{cases} \quad (15)$$

$$A^* = (w(\gamma_0 p^{-\gamma_1}) k_m) \quad (16)$$

Proposition 1. If $2w > p$, then (i) the manufacturer offers positive advertising allowance to the retailer, otherwise he will offer nothing, and (ii) the manufacturer’s advertising allowance for the retailer is positively and negatively correlated to changes in the manufacturer’s marginal profit and retailer’s marginal profit, respectively.

III. Nash Equilibrium

Let $\max_{A,t} \pi_m = w (\gamma_0 p^{-\gamma_1}) (k_r \log a + k_m \log A) - ta - A$ and

$$\max_{A,t} \pi_r = p (\gamma_0 p^{-\gamma_1}) (k_r \log a + k_m \log A) - (1-t)a$$

Solving by taking first derivative of π_m with respect to A and π_r with respect to a equal to zero and substituting $t = 0$ we get,

$$A^{**} = (w(\gamma_0 p^{-\gamma_1}) k_m)$$

$$a^{**} = (p-w)(\gamma_0 p^{-\gamma_1}) k_r$$

$$t^{**} = 0$$

In recent market studies we found that now retailers have increased their power relative to manufacturers. In this section we assume a symmetric relationship between the manufacturer and the retailer. It is assumed that the manufacturer and the retailer simultaneously and noncooperatively maximize their profits with respect to any possible strategies set by other member.

The following two propositions give the detailed comparisons among two different noncooperative game structures.

Proposition 2. i) The manufacturer always prefers the leader-follower structure rather than the simultaneous move structure. (ii) If $p \geq 2w$, the retailer prefers the simultaneous move game structure; otherwise he prefers the leader – follower game structure.

Proposition 3. (i) The manufacturer’s brand name investment is same at Nash and Stackelberg. (ii) If $p \geq 2w$, then the retailer’s local advertising expenditure is higher at Nash than at Stackelberg; otherwise, it is lower at Nash than at Stackelberg. (iii) The manufacturer’s advertising allowance for the retailer is zero.

IV. An Efficiency Co – Op Advertising Model

We discussed two noncooperative game structures above for a sequential move and a simultaneous move. The manufacturer who can promote its brand by national advertisement may not know about local market and retailer’s advertising behavior. So, the retailer knows how much if any, of the manufacturer’s money is spent on local advertising. But many retailers use the manufacturer’s money and co operative advertising programs for their own purposes and reduce their dependence on the manufacturers. Co – op advertising should not be used as a offer from a manufacturer to pay part of a retailer’s local advertising costs on the product, but should be used as a tool to enhance manufacturer’s brand name and retailer’s store reputation.

Here, we will consider the symmetric relationship between the manufacturer and retailer and discuss the efficiency of manufacturer and retailer transactions in a vertical co – op advertising agreements.

Pareto efficient scheme (a_0, t_0, A_0) is that if we cannot find any other scheme (a, t, A) such that neither the manufacturer’s nor the retailer’s profit is less at (a, t, A) but atleast one of them has profit higher at (a, t, A) than at (a_0, t_0, A_0) . So, (a_0, t_0, A_0) is Pareto efficient iff $\pi_m(a, t, A) \geq \pi_m(a_0, t_0, A_0)$ and $\pi_r(a, t, A) \geq \pi_r(a_0, t_0, A_0)$ for some (a, t, A) implies that $\pi_m(a, t, A) = \pi_m(a_0, t_0, A_0)$ and $\pi_r(a, t, A) = \pi_r(a_0, t_0, A_0)$.

Here, π_m and π_r are quasi – concave. So, the set of Pareto efficient schemes consists of those points where the manufacturer’s and retailer’s iso – profit surfaces are tangent to each other. i.e.

$$\nabla \pi_m(a, t, A) + \mu \nabla \pi_r(a, t, A) = 0 \quad (17)$$

For some $\mu \geq 0$, where $\nabla \pi_m = \left(\frac{\partial \pi_m}{\partial a}, \frac{\partial \pi_m}{\partial t}, \frac{\partial \pi_m}{\partial A} \right)$ stands for the gradient of π_m .

Proposition 4. The collection of Pareto efficient schemes is described by the set $Y = \{(\bar{a}^*, t, \bar{A}^*): 0 \leq t \leq 1\}$

$$\text{Where } \bar{a}^* = (p(\gamma_0 p^{-\gamma_1}) k_r) \text{ and } \bar{A}^* = (p(\gamma_0 p^{-\gamma_1}) k_m). \quad (18)$$

This proposition shows that Pareto efficient schemes are associated with a single local advertising expenditure \bar{a}^* and a single manufacturer's brand name investment \bar{A}^* and with manufacturer's share of local advertising expenditure between 0 and 1.

Proposition 5. An advertising scheme is Pareto efficient iff it is an optimal solution of the joint system profit maximization problem.

Appendix A. Proof of results

Proof of Proposition 2

(i) Since the retailer's response to every strategy of the manufacturer is unique in the two-stage game, the leader's payoff will not be less than that at Nash equilibrium. Therefore, $\pi_m^{**} \leq \pi_m^*$.

$$(i) \quad \pi_r^{**} - \pi_r^* = (p - w) (\gamma_0 p^{-\gamma_1}) k_r \log \left(\frac{p-w}{w} \right) \geq 0 \text{ if } p \geq 2w$$

$$< 0 \text{ otherwise}$$

Proof of Proposition 3

$$(i) \quad A^{**} - A^* = (w(\gamma_0 p^{-\gamma_1}) k_m) - (w(\gamma_0 p^{-\gamma_1}) k_m) = 0$$

$$(ii) \quad a^{**} - a^* = (-w) (\gamma_0 p^{-\gamma_1}) k_r - w (\gamma_0 p^{-\gamma_1}) k_r$$

$$= ((\gamma_0 p^{-\gamma_1}) k_r) \{p - 2w\} \geq 0 \text{ if } p \geq 2w$$

$$< 0 \text{ otherwise}$$

$$(ii) \quad t^{**} = 0$$

Proof of Proposition 4

$$\text{Since } \nabla \pi_m(a, t, A) = \left(\frac{w(\gamma_0 p^{-\gamma_1}) k_r}{a} - t, -a, \frac{w(\gamma_0 p^{-\gamma_1}) k_m}{A} - 1 \right)$$

$$\nabla \pi_r(a, t, A) = \left(\frac{(p-w)(\gamma_0 p^{-\gamma_1}) k_r}{a} - (1-t), a, \frac{(p-w)(\gamma_0 p^{-\gamma_1}) k_m}{A} \right)$$

Utilizing eq. (17) we can get $\mu = 1$, \bar{a}^* and \bar{A}^* in eq. (18) and with t between 0 and 1.

Proof of Proposition 5

Let the joint system maximization problem be as follows:

$$\bar{\pi}^* = \max_{a, A, t} \pi = \pi_{m+r} = p (\gamma_0 p^{-\gamma_1}) (k_r \log a + k_m \log A) - a - A \tag{19}$$

$$\text{s.t. } 0 \leq t \leq 1, A \geq 0, a \geq 0$$

It does not contain the variable t, any value of t between 0 and 1 can be component for any optimal solution of (19).

Taking the first derivatives of π with respect to a ad A, and setting them to zero, we have

$$\bar{a}^* = (p (\gamma_0 p^{-\gamma_1}) k_r)$$

and

$$\bar{A}^* = (p (\gamma_0 p^{-\gamma_1}) k_m)$$

Therefore, $(\bar{a}^*, t, \bar{A}^*)$ for any t in [0, 1] is an optimal solution of (19).

References

- [1]. Kuehn, A.A., (1962): How advertising performance depends upon other marketing Factors, *Journal of Advertising Research*, 2 (1), 2–10.
- [2]. Thompson, G.L., Teng, J., (1984): Optimal pricing and advertising policies for new product oligopoly models, *Marketing Science*, 3 (3), 148–168.
- [3]. Jorgensen, S., Zaccour, G., (1999): Equilibrium pricing and advertising strategies in a marketing channel, *Journal of Optimization Theory and Applications*, 102 (1), 111–125.
- [4]. Jorgensen, S., Zaccour, G., (2003a): Channel coordination over time: Incentive equilibria and credibility, *Journal of Economic Dynamics and Control*, 27 (5), 801–822.
- [5]. Jorgensen, S., Zaccour, G., (2003b): A differential game of retailer promotions, *Automatica*, 39 (7), 1145–1155.
- [6]. Simon, J.L., Arndt, J., (1980): The shape of the advertising function, *Journal of Advertising Research*, 20, 11–28.
- [7]. Kim, S.Y., Staelin, and R., (1999): Manufacturer allowances and retailer pass-through rates in a competitive environment, *Management Science*, 18 (1), 59–76.
- [8]. Karray, S., Zaccour, G., (2006): Could co-op advertising be a manufacturer's Counterstrategy to store brands? *Journal of Business Research*, 59, 1008–1015.
- [9]. Jorgensen, S., Sigue, S.P., Zaccour, G., (2000): Dynamic cooperative advertising in a Channel, *Journal of Retailing*, 76 (1), 71–92.
- [10]. Huang, Z., Li, S.X., Mahajan, V., (2002): An analysis of manufacturer–retailer supply chain coordination in cooperative advertising, *Decision Sciences*, 33 (3), 469–494.
- [11]. Solanki Krishna, Gor Ravi, (2013): A Game theoretic model on cooperative advertising where demand is dependent on price and advertising expenditures, *Journal of data and information processing*, volume I, iss 2, 28–33.
- [12]. Zhimin Huang, Susan X. Li (2000): Co-op advertising models in manufacturer-retailer supply chains: A game theory approach, *European journal of operational research* 135(2001)527-544.