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P-Pseudo Symmetric Ideals in Ternary Semiring

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Abstract: *In this paper we introduce and study about pseudo symmetric ideals and P-pseudo symmetric ideals in ternary semi rings. It is proved that (1) every completely P-Semiprime ideal A in a ternary semi ring T is a P-pseudo symmetric ideal, (2) If A is a P-pseudo symmetric ideal of a ternary semi ring T then (i) $A_2 = \{x : x^n \in A$ for some odd natural number $n \in N\}$ is a minimal completely P-Semiprime ideal of T, (ii) $A_4 = \{x : < x >^n \subseteq A$ for some odd natural number n} is the minimal P-Semiprime ideal of T containing A, (3) Every P-prime ideal Q minimal relative to containing a P-pseudo symmetric ideal A in a ternary semi ring T is completely P-prime, and (4) Let A be an ideal of a ternary semi ring T. Then A is completely P-prime iff A is P-prime and P-pseudo symmetric. Further we introduced the terms pseudo symmetric ternary semi ring and P-pseudo symmetric ternary semi ring. It is proved that (1) Every commutative ternary semi ring is a pseudo symmetric ternary semi ring, (2) Every commutative ternary semi ring is a P-pseudo symmetric ternary semi ring, (3) Every pseudo commutative ternary semi ring is a P-pseudo symmetric ternary semi ring and (4) If T is a ternary semiring in which every element is a midunit then T is a P-pseudo symmetric ternary semiring.*

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Key Words: *Pseudo Symmetric ideal, P-pseudo symmetric ideal, P-Prime, Completely P-Prime, P-Semiprime, Completely P-Semiprime, Pseudo Commutative, mid unit.*

I. Introduction:

Algebraic structures play a prominent role in mathematics with wide ranging applications in many disciplines such as theoretical physics, computer sciences, control engineering, information sciences, coding theory, topological spaces, and the like. The theory of ternary algebraic systems was introduced by D. H. Lehmer [5]. He investigated certain ternary algebraic systems called triplexes which turn out to be commutative ternary groups. After that W. G. Lister[6] studied about ternary semirings. About T. K. Dutta and S. Kar [1, 3] introduced and studied some properties of ternary semirings which is a generalization of ternary rings. Dheena. P, Manvisan. S [2] made a study on P-prime and small P-prime ideals in semirings. S. Kar [4] investigated on quasi ideals and bi-ideals in ternary semirings.

D. Madhusudhana Rao, A. Anjaneyulu and A. Gangadhara Rao [7] in 2011 introduced the notion of pseudo symmetric ideals in Γ-Semigroups. In 2012 Y. Sarala, A. Anjaneyulu and D. Madhusudhana Rao [13] introduced the same concept to the ternary semigroups. In 2014 D. MadhusudhanaRao and G. Srinivasa Rao [8, 9] investigated and studied about classification of ternary semirings and some special elements in a ternary semirings. D. Madhsusudhana Rao and G. Srinivasa Rao [10] introduced and investigated structure of certain ideals in ternary semirings. D. Madhusudhana Rao and G. Srinivasa Rao[11] also introduced the structure of completely P-prime, P-prime, Completely P-Semiprime and P-semiprime ideals in Ternary semirings. After that they [12] made a study and investigated prime radicals in ternary semitings. Our main purpose in this paper is to introduce the Structure of P-pseudo symmetric Ideals in ternary semirings.

II. Preliminaries:

Definition II.1[6] **:** A nonempty set T together with a binary operation called addition and a ternary multiplication denoted by [ ] is said to be a **ternary semiring** if T is an additive commutative semigroup satisfying the following conditions :
i) [[abc]de] = [a[bcd]e] = [ab[cde]],
ii) [(a + b)cd] = [acd] + [bcd],
iii) [a(b + c)d] = [abd] + [acd],
iv) [ab(c + d)] = [abc] + [abd] for all a; b; c; d; e ∈T.
Throughout Twill denote a ternary semiring unless otherwise stated.

Note II.2 : For the convenience we write $x_1x_2x_3$ instead of $\left[x_1x_2x_3\right]$

Note II.3 : Let T be a ternary semiring. If A,B and C are three subsets of T , we shall denote the set ABC = $\left\{\Sigma abc : a \in A, b \in B, c \in C\right\}$.

Note II.4 : Let T be a ternary semiring. If A,B are two subsets of T , we shall denote the set A + B = $\{a+b : a \in A, b \in B\}$.

Note II.5 : Any semiring can be reduced to a ternary semiring.

Example II.6 [6] : Let T be an semigroup of all m × n matrices over the set of all non negative rational numbers. Then T is a ternary semiring with matrix multiplication as the ternary operation.

Example II.7 [6]: Let S = {...,−2i, −i, 0, i, 2i, ...} be a ternary semiring withrespect to addition and complex triple multiplication.

Definition II.8 [7]: An element a of a ternary semiring T is said to be a **mid-unit** provided xayaz = xyz for allx, y, z∈ T.

Definition II.9 [6]: A ternary semiring T is said to be **commutative ternary semiring** providedabc = bca = cab = bac = cba = acbfor all a,b,c ∈ T.

Definition II.10 [6]: A ternary semiring T is said to be **left pseudo commutative** provided abcde = bcade = cabde = bacde = cbade = acbde ∀ a,b,c,d,e ∈ T.

Definition II.11 [6] : A ternary semiring T is said to be a **lateral pseudo commutative**ternary semiring provide abcde = acdbe = adbce = acbde = adcbe = abdcefor alla,b,c,d,e ∈ T.

Definition II.12 [6]: A ternary semiring T is said to be **right pseudo commutative** provided abcde = abdec = abecd = abdce = abedc = abced ∀ a,b,c,d,e ∈ T.

Definition II.13 [6]: A ternary semiring T is said to be **pseudo commutative**, provided T is a left pseudo commutative, right pseudo commutative and lateral pseudo commutative ternary semiring.

Definition II.14 [8]: A nonempty subset A of a ternary semiring T is said to be **ternary ideal** or simply an **ideal** of T if

(1) a, b ∈ A implies a + b∈ A

(2) b, c ∈ T , a ∈ A implies bca ∈ A, bac ∈ A, abc ∈ A.

Definition II.15 [9]:An ideal A of a ternary semiring T is said to be a **completely prime ideal** of T provided x, y, z ∈ T and xyz ∈ A implies either x ∈ A or y ∈ A or z ∈ A.

Definition II.16 [9]:An ideal A of a ternary semiring T is said to be a **completely P-prime ideal** of T provided x, y, z ∈ T and xyz+ P⊆A implies either x ∈ A or y ∈ A or z ∈ A for any ideal P.

Definition II.17 [9]: An ideal A of a ternary semiring T is said to be a P**-prime ideal** of T provided X,Y,Z are ideals of T and XYZ + P ⊆ A ⇒ X ⊆ A or Y ⊆ A or Z ⊆ A for any ideal P.

Theorem II.18[10]: Every completely P-prime ideal of a ternary semiring T is a P-prime ideal of T.

Theorem II.19[10] : Every completely P-semiprime ideal of a ternary semiring T is a P-semiprime ideal of T.

Definition II.20 [9]: An ideal A of a ternary semiring T is said to be a **completely P-semiprime ideal** provided x ∈ T, $x^n + p$ ∈ A for some odd natural number n>1 and p ∈ P implies x ∈ A.

Definition II.21 [9]: An ideal A of a ternary semiring T is said to be **semiprime ideal** provided X is an ideal of T and $X^n \subseteq$ A for some odd natural number nimplies X ⊆ A.

Definition 2.22 [9]: An ideal A of a ternary semiring T is said to be P**-semiprime ideal** provided X is an ideal of T and X^n+ P ⊆ A for some odd natural number nimplies X ⊆ A.

Notation II.23 [11]: If A is an ideal of a ternary semiring T, then we associate the following four types of sets.

A_1 = The intersection of all completely prime ideals of T containing A.

A_2 = {x ∈ T: x^n ∈ A for some odd natural numbers n}

A_3 = The intersection of all prime ideals of T containing A.

A_4 = {x ∈ T: $< x >^n \subseteq$ A for some odd natural number n}

Theorem II.24 [11]: If A is an ideal of a ternary semiring T, then $A \subseteq A_4 \subseteq A_3 \subseteq A_2 \subseteq A_1$**.**

III. P-Pseudo Symmetric Ideals In Ternary Semirings

We now introduce the notion of a pseudo symmetric ideal of a ternary semiring.

Definition III.1 : An ideal A of a ternary semiring T is said to be **pseudo symmetric** provided x, y, z ∈ T, xyz ∈ A implies xsytz ∈ A for all s, t ∈ T.

We now introduce the notion of a P-pseudo symmetric ideal of a ternary semiring.

Definition III.2 : A pseudo symmetric ideal A of a ternary semiring T is said to be P**-pseudo symmetric ideal** provided x, y, z ∈ T and P is an ideal of T, xyz + p ∈ A implies xsytz + p ∈ A for all s, t ∈ T and p ∈ P.

Note III.3 : A pseudo symmetric ideal A of a ternary semiring T is said to be P-**pseudo symmetric ideal** provided x, y, z $\in$ T and P is an ideal of T, xyz + P $\subseteq$ A implies xsytz + P $\subseteq$A for all s, t$\in$T.

Theorem III.4 : Let A be a pseudo symmetric ideal in a ternary semiring T and a_i, b_i, $c_i \in$T. Then $\sum_{i=1}^{n} a_i b_i c_i$ $\in$A if and only if < a > < b > < c > $\subseteq$ A.

Proof : Suppose < a > < b > < c >$\subseteq$A. $\sum_{i=1}^{n} a_i b_i c_i \in$ < a > < b > < c >$\subseteq$A$\Rightarrow \sum_{i=1}^{n} a_i b_i c_i \in$A.

Conversely suppose that $\sum_{i=1}^{n} a_i b_i c_i \in$ A. Let t $\in$< a > < b > < c>.

Then t = $s_1a_1s_2b_1s_3c_1s_4$where s_1 ,s_2, s_3, $s_4 \in T^1$.
$a_1b_1c_1 \in$A, s_2, s_3 $\in T^1$, A is pseudo symmetric ideal
$\Rightarrow a_1s_2b_1s_3c_1 \in A \Rightarrow s_1a_1s_2b_1s_3c_1s_4 \in$A $\Rightarrow$t $\in$A. Therefore < a > < b > < c >$\subseteq$A.

Corollary III.5 : Let A be any pseudo symmetric ideal in a ternary semiring T and a_1, a_2….,a_n $\in$ T where n is an odd natural number. Then $a_1a_2….a_n$ $\in$ A if and only if <a_1>< a_2>….<a_n>$\subseteq$ A.

Proof : Clearly if < a_1>< a_2>….<a_n>$\subseteq$A, then$a_1a_2….a_n \in$A where n is an odd natural number.
Conversely suppose that $a_1a_2….a_n \in$A where n is an odd natural number.
Let t$\in$<a_1>< a_2>….<a_n>. Then t =$s_1a_1 s_2a_2….a_ns_{n+1}$, where $s_i \in T^e$, i =1,2,…n+1.
$a_1a_2….a_n \in$A, A is pseudo symmetric ideal $\Rightarrow s_1a_1 s_2a_2….a_ns_{n+1} \in$A and hence t$\in$A.
Therefore <a_1>< a_2>….<a_n>$\subseteq$A.

Theorem III.6 : Let A be a P-pseudo symmetric ideal in a ternary semiring T and a_i, b_i, $c_i \in$T and p ∈ P. Then $\sum_{i=1}^{n} a_i b_i c_i + p$ ∈ A if and only if < a > < b > < c > + P $\subseteq$ A.

Proof : Suppose <a > < b > < c > + P$\subseteq$A.

Then $\sum_{i=1}^{n} a_i b_i c_i + p \in$ < a > < b > < c > + P$\subseteq$A$\Rightarrow \sum_{i=1}^{n} a_i b_i c_i + p \in$ A.

Conversely suppose that $\sum_{i=1}^{n} a_i b_i c_i + p \in A$. Let t $\in$< a > < b > < c > + P.

Then t = $s_1a_1s_2b_1s_3c_1s_4$ + p where s_1 ,s_2, s_3, $s_4 \in T^1$, p ∈ P.
$a_1b_1c_1$ + p $\in$A, s_2, s_3 $\in T^1$, A is P-pseudo symmetric ideal
$\Rightarrow a_1s_2b_1s_3c_1$ + p $\in$A. Then $a_1s_2b_1s_3c_1 \in$A and p ∈ A.
$\Rightarrow s_1a_1s_2b_1s_3c_1s_4 \in$A and p ∈ A $\Rightarrow$ $s_1a_1s_2b_1s_3c_1s_4$ + p ∈ A
⇒ t $\in$A. Therefore < a > < b > < c > + P $\subseteq$A.

Corollary III.7 : Let A be any P-pseudo symmetric ideal in a ternary semiring T and a_1, a_2….,a_n $\in$ T where n is an odd natural number and p ∈ P. Then $a_1a_2….a_n$ + p $\in$ A if and only if < a_1> < a_2>….<a_n> + P$\subseteq$ A.

Proof : Clearly if < a_1> < a_2>….< a_n > + P $\subseteq$A, then < a_1> < a_2>….< a_n > $\subseteq$A, P $\subseteq$A ⇒ $a_1a_2….a_n \in$A where n is an odd natural number and p ∈ A for all p ∈ A ⇒ $a_1a_2….a_n$ + p ∈ A .
Conversely suppose that $a_1a_2….a_n$ + p $\in$A where n is an odd natural number and p ∈ P.
Let t$\in$< a_1> < a_2>….< a_n> + P. Then t =s_1a_1 $s_2a_2….a_ns_{n+1}$ + p, where $s_i \in T^e$, i =1,2,…n+1, p ∈ P. $a_1a_2….a_n \in$A, p ∈ P and A is P-pseudo symmetric ideal $\Rightarrow s_1a_1 s_2a_2….a_ns_{n+1}$ + p$\in$A and hence t$\in$A.
Therefore <a_1>< a_2>….<a_n> + P$\subseteq$A.

Corollary III.8: Let A be a P-pseudo symmetric ideal in a ternary semiring T, then for any odd natural number n, a^n + p$\in$A implies < a >^{n} + P$\subseteq$A.

Proof : The proof follows from corollary 4.1.7, by taking a_1=a_2=….a_n=a.

Corollary III.9 : Let A be a P-pseudo symmetric ideal in a ternary semiring T. If $a^n \in$A, for some odd natural number n then < ast >^{n} + P, < sta >^{n} + P, < sat >^{n} + P $\subseteq$ A for all s, t∈ T and for some ideal P.

Theorem 4.1.10 : Every completely P-semiprime ideal A in a ternary semiring T is a P-pseudo symmetric ideal.

Proof : Let A be a completely P-semiprime ideal of the ternary semiring T.
Let x, y, z$\in$T, p ∈ P and xyz + p $\in$A ⇒ xyz$\in$A, p ∈ A. xyz$\in$A implies $(yzx)^3$ = (yzx)(yzx)(yzx) = yz(xyz)(xyz)x$\in$A and p ∈ A.

$(yzx)^3 + p \in A$, A is completely P-semiprime ideal $\Rightarrow$ $yzx \in A$.
Similarly $(zxy)^3 + p = (zxy)(zxy)(zxy) + p = z(xyz)(xyz)xy + p \in A$.
$(zxy)^3 + p \in A$, A is completely P-semiprime ideal $\Rightarrow zxy \in A$.
If s, $t \in T^1$, then $(xsytz)^3 + p = (xsytz)(xsytz)(xsytz) + p = xsyt[zx(syt)(zxs)y]tz + p \in A$.
$(xsytz)^3 + p \in A$, A is completely P-semiprime $\Rightarrow xsytz \in A$.
Therefore A is a P-pseudo symmetric ideal.

Note III.11 : The converse of theorem 4.1.9, is not true, i.e., a P-pseudo symmetric ideal of a ternary semiring need not be completely P-semiprime.

Example III.12 : Let T = {a, b, c} and P = {a}. Define a ternary operation [ ] on T as [abc] = a.b.c where . is binary operation and the binary operation defined as follows

+	a	b	c
a	a	b	c
b	b	b	c
c	c	c	c

.	a	b	c
a	a	a	a
b	a	a	a
c	a	b	c

Clearly (T, +, [ ]) is a ternary semiring and {a}, {a, b}, T are the ideals of T.
Now aaa + a $\in$ {a} $\Rightarrow$ aaaaa + a, ababa + a, acaca + a, aaaba + a, abaca + a, acaba + a $\in$ {a}
abb + a $\in$ {a} $\Rightarrow$ aabab + a, abbbb + a, acbcb + a, aabbb + a, abbcb + a, acbab + a $\in$ {a}
baa + a $\in$ {a} $\Rightarrow$ baaaa + a, bbbba + a, bcaca + a, baaba + a, bbaca + a, bcaaa + a $\in$ {a}
aba + a $\in$ {a} $\Rightarrow$ aabaa + a, abbba + a, acbca + a, aabba + a, abbca + a, acbaa + a $\in$ {a}
bab + a $\in$ {a} $\Rightarrow$ baaab + a, bbabb + a, bcbcb + a, baabb + a, bbacb + a, bcaab + a $\in$ {a}
bbb + a $\in$ {a} $\Rightarrow$ babab + a, bbbbb + a, bcbcb + a, babbb + a, bbbcb + a, bcbab + a $\in$ {a}
bba + a $\in$ {a} $\Rightarrow$ babaa + a, bbbba + a, bcbca + a, babba + a, bbbca + a, bcbaa + a $\in$ {a}
acc + a $\in$ {a} $\Rightarrow$ aacac + a, abcbc + a, acccc + a, aacbc + a, abccc + a, accac + a $\in$ {a}
caa + a $\in$ {a} $\Rightarrow$ caaaa + a, cbcba + a, ccaca + a, caaba + a, cbaca + a, ccaaa + a $\in$ {a}
aca + a $\in$ {a} $\Rightarrow$ aacaa + a, abcba + a, accca + a, aacba + a, abcca + a, accaa + a $\in$ {a}
cac + a $\in$ {a} $\Rightarrow$ caaac + a, cbabc + a, ccacc + a, caabc + a, cbacc + a, ccaac + a $\in$ {a}
cca + a $\in$ {a} $\Rightarrow$ cacaa + a, cbcba + a, cccca + a, cacba + a, cbcca + a, cccaa + a $\in$ {a}
abc + a $\in$ {a} $\Rightarrow$ aabac + a, abbbc + a, acbcc + a, aabbc + a, abbcc + a, acbac + a $\in$ {a}
bca + a $\in$ {a} $\Rightarrow$ bacaa + a, bbcba + a, bccca + a, bacba + a, bbcca + a, bccaa + a $\in$ {a}
cab + a $\in$ {a} $\Rightarrow$ caaab + a, cbabb + a, ccacb + a, caabb + a, abacb + a, ccaab + a $\in$ {a}.
Therefore {a} is a P-pseudo symmetric ideal in T. Here $b^3 + a = a \in$ {a}, but $b \notin$ {a}.
Therefore {a} is not a completely P-semiprime ideal.

Theorem III.13 : If A is a P-pseudo symmetric ideal of a ternary semiring T then $A_2 = A_4$.

Proof : By theorem II.24, $A_4 \subseteq A_2$. Let $x \in A_2$.
Then $x^n \in A$ for some odd natural number n.
Since A is P-pseudo symmetric, $x^n + p \in A \Rightarrow < x >^n + P \subseteq A \Rightarrow < x >^n \subseteq A$ and $P \subseteq A$
$\Rightarrow < x >^n \subseteq A \Rightarrow x \in A_4$. Therefore $A_2 \subseteq A_4$ and hence $A_2 = A_4$.

Theorem III.14: If A is a P-pseudo symmetric ideal of a ternary semiring T then $A_2 = \{x : x^n \in A$ for some odd natural number $n \in N\}$ is a minimal completely P-semiprme ideal of T.

Proof : Clearly $A \subseteq A_2$ and hence A_2 is a nonempty subset of T. Let $x \in A_2$ and s, $t \in T$.
Now $x \in A_2 \Rightarrow x^n \in A$ for some odd natural number n. $x^n \in A$, s, $t \in T$, A is a P-pseudo symmetric ideal of T
$\Rightarrow (xst)^n + p \in A, (sxt)^n + p \in A, (stx)^n + p \in A$ for $p \in P$
$\Rightarrow (xst)^n, (sxt)^n, (stx)^n \in A, p \in A \Rightarrow xst, sxt, stx \in A_2$.
Therefore A_2 is an ideal of T. Let $x \in T$ and $x^3 + p \in A_2$.
Now $x^3 + p \in A_2 \Rightarrow x^3 \in A_2$ and $p \in A_2 \Rightarrow x^3 \in A_2 \Rightarrow (x^3)^n \in A$ for some odd natural number n
$\Rightarrow x^{3n} \in A \Rightarrow x \in A_2$. So A_2 is a completely P-semiprime ideal of T.
Let Q be any completely P-semiprime ideal of T containing A. Let $x \in A_2$.
Then $x^n \in A$ for some odd natural number n. By corollary 4.1.7, $x^n + p \in A$
$\Rightarrow < x >^n + P \subseteq A \subseteq Q$. Since Q is completely P-semiprime, $< x >^n + P \subseteq Q \Rightarrow x \in Q$.
Therefore A_2 is the minimal completely P-semiprime ideal of T containing A.

Theorem III.15 : If A is a P-pseudo symmetric ideal of a ternary semiring T then $A_4 = \{x : < x >^n \subseteq A$ for some odd natural number n$\}$ is the minimal P-semiprime ideal of T containing A.

Proof : Clearly $A \subseteq A_4$ and hence A_4 is a nonempty subset of T. Let $x \in A_4$ and $s, t \in T$.
Since $x \in A_4$, $< x >^n \subseteq A$ for some odd natural number n.
Now $< xst >^n \subseteq <x>^n \subseteq A$, $< sxt >^n$ and $< stx >^n \subseteq < x >^n \subseteq A \implies xst, sxt, stx \in A_4$.
Then A_4 is an ideal of T containing A.
Let $x \in T$ such that $< x >^3 + P \subseteq A_4 \Rightarrow < x >^3 \subseteq A_4$, $P \subseteq A_4$.
Then $< x >^3 \subseteq A_4 \Rightarrow (<x>^3)^n \subseteq A \implies <x>^{3n} \subseteq A \implies x \in A_4$. Therefore A_4 is a P-semiprime ideal of T containing A. Suppose Q is a P-semiprime ideal of T containing A.
Let $x \in A_4$. Then $< x >^n \subseteq A \subseteq Q$ for some odd natural number n.
Since Q is a P-semiprime ideal of T, $< x >^n + P \subseteq Q$ for some odd natural number $n \implies x \in Q$. Therefore $A_4 \subseteq Q$ and hence A_4 is the minimal P-semiprime ideal of T containing A.

Theorem III.16 : Every P-prime ideal Q minimal relative to containing a P-pseudo symmetric ideal A in a ternary semiring T is completely P-prime.

Proof : Let S be a ternary sub semiring generated by T\Q. First we show that $A \cap S = \emptyset$. If $A \cap S \neq \emptyset$, then there exist $x_1, x_2, x_3, \ldots\ldots, x_n \in T\backslash Q$ such that $x_1x_2x_3\ldots\ldots x_n + p \in A$ where n is an odd natural number and $p \in P$. By corollary 4.1.7, $< x_1 >< x_2 > \ldots\ldots\ldots\ldots < x_n > + P \subseteq A \subseteq Q$. Since Q is a P-prime ideal, we have $< x_i > \subseteq Q$ for some i. It is a contradiction. Thus $A \cap S = \emptyset$. Consider the set $\Sigma = \{ B : B$ is an ideal in T containing A such that $B \cap S = \emptyset\}$. Since $A \in \Sigma$, Σ is nonempty. Now Σ is a poset under set inclusion and satisfies the hypothesis of Zorn's lemma. Thus by Zorn's lemma, Σ contains a maximal element, say M. Let X, Y and Z be three ideals in T such that $XYZ + P \subseteq M$. If $X \not\subseteq M$, $Y \not\subseteq M$, $Z \not\subseteq M$, then $M \cup X$, $M \cup Y$ and $M \cup Z$ are ideals in T containing M properly and hence by the maximality of M, we have $(M \cup X) \cap S \neq \emptyset$, $(M \cup Y) \cap S \neq \emptyset$ and $(M \cup Z) \cap S \neq \emptyset$. Since $M \cap S = \emptyset$, we have $X \cap S \neq \emptyset$, $Y \cap S \neq \emptyset$ and $Z \cap S \neq \emptyset$. So there exists $x \in X \cap S$, $y \in Y \cap S$ and $z \in Z \cap S$. Now, $xyz \in XYZ \cap T \subseteq M \cap T = \emptyset$. It is a contradiction. Therefore either $X \subseteq M$ or $Y \subseteq M$ or $Z \subseteq M$ and hence M is P-prime ideal containing A. Now, $A \subseteq M \subseteq T\backslash S \subseteq P$. Since Q is a minimal P-prime ideal relative to containing A, we have $M = T\backslash S = Q$ and $S = T\backslash Q$. Let $xyz + p \in Q$. Then $xyz \notin S$. Suppose if possible $x \notin Q$, $y \notin Q$, $z \notin Q$. Now $x \notin Q$, $y \notin Q$, $z \notin Q \Rightarrow x, y, z \in T\backslash Q \Rightarrow x, y, z \in S \Rightarrow xyz \in S$. It is a contradiction. Therefore either $x \in Q$ or $y \in Q$ or $z \in Q$. Therefore Q is a completely P-prime ideal of T.

Theorem III.17 : Let A be an ideal of a ternary semiring T. Then A is completely P-prime iff A is P-prime and P-pseudo symmetric.

Proof : Suppose A is a completely P-prime ideal of T. By theorem II.18, A is P-prime.
Let $x, y, z \in T$, $p \in P$ and $xyz + p \in A$.
$xyz + p \in A$, A is completely P-prime $\Rightarrow x \in A$ or $y \in A$ or $z \in A$
$\Rightarrow xsytz + p \in A$ for all $s, t \in T$ $p \in P$. Therefore A is a P-pseudo symmetric ideal.
Conversely Suppose that A is P-prime and P-pseudo symmetric.
Let $x, y, z \in T$, $p \in P$ and $xyz + p \in A$. $xyz + p \in A$, A is a P-pseudo symmetric ideal
$\Rightarrow < x > < y > < z > + P \subseteq A \Rightarrow < x > < y > < z > \subseteq A$ and $P \subseteq A$
$\Rightarrow < x > \subseteq A$ or $< y > \subseteq A$ or $< z > \subseteq A \Rightarrow x \in A$ or $y \in A$ or $z \in A$.
Therefore A is completely P-prime.

Corollary III.18: Let A be an ideal of a ternary semiring T. Then A is completely P-prime iff A is P-prime and completely P-semiprime.

Proof : The proof follows from theorem III.17,

Corollary III.19 : Let A be an ideal of a ternary semiring T. Then A is completely P-semiprime iff A is P-semiprime and P-pseudo symmetric.

Proof : Suppose that A is completely P-semiprime. By theorem II.19, A is P-semiprime and also by theorem III.10, A is P-pseudo symmetric.
Conversely suppose that A is P-semiprime and P-pseudo symmetric.
Let $x \in T$, $p \in P$ and $x^3 + p \in A$. $x^3 + p \in A$, A is P-pseudo symmetric
$\Rightarrow <x^3> + P \subseteq A \Rightarrow < x > \subseteq A \Rightarrow x \in A$.
Therefore A is a completely P-semiprime ideal of T.

Theorem III.20 : Let A be a P-pseudo symmetric ideal of a ternary semiring T. Then the following are equivalent.

1) A_1 = The intersection of all completely prime ideals of T containing A.

2) A_1^1 = The intersection of all minimal completely prime ideals of T containing A.

3) A_1^{11} = The minimal completely semiprime ideal of T relative to containing A.

4) $A_2 = \{ x \in T : x^n \in A$ for some odd natural number n$\}$

5) A_3 = The intersection of all prime ideals of T containing A.

6) A_3^1 = The intersection of all minimal prime ideals of T containing A.

7) A_3^{11} = The minimal semiprime ideal of T relative to containing A.

8) A_4 = {x ∈ T : < x >^{n} ⊆ A for some odd natural number n}

Proof: Since completely P-prime ideals containing A and minimal completely prime ideals containing A and minimal completely semiprime ideal relative to containing A are coincide, it follows that A_1= $A_1^1 = A_1^{11}$. Since prime ideals containing A and minimal prime ideals containing A and the minimal semiprime ideal relative to containing A are coincide, it follows that $A_3 = A_3^1 = A_3^{11}$. Since A is pseudo symmetric ideal, we have A_2=A_4. Now by corollary III.15, we have $A_1^{11} = A_3^{11}$. Therefore A_1= $A_1^1 = A_1^{11} = A_3 = A_3^1 = A_3^{11}$ and A_2=A_4. Hence the given conditions are equivalent.

Definition III.21 : A ternary semiring T is said to be a **pseudo symmetric ternary semiring** provided every ideal in T is a pseudo symmetric ideal.

Definition III.22 : A ternary semiring T is said to be a P-**pseudo symmetric ternary semiring** provided every ideal in T is a P-pseudo symmetric ideal.

Theorem III.23 : Every commutative ternary semiring is a pseudo symmetric ternary semiring.

Proof : Suppose T is commutative ternary semiring.
Then abc = bca = cab = bac = cba = acb for all a, b, c ∈ T. Let A be an ideal of T.
Let a, b, c ∈ T, abc ∈ A and s, t ∈ T. Then asbtc = abstc = absct = abcst ∈ A.
Therefore A is a pseudo symmetric ideal and hence T is a pseudo symmetric ternary semiring.

Theorem III.24 : Every commutative ternary semiring is a P-pseudo symmetric ternary semiring.

Proof : Suppose T is commutative ternary semiring.
Then abc = bca = cab = bac = cba = acb for all a, b, c∈ T. Let A be an ideal of T.
Let a, b, c ∈ T and p ∈ P where P is any ideal of A, abc + p ∈ A.
⇒ abc ∈ A and p ∈ A. abc ∈ A and s, t ∈ T. Then asbtc = abstc = absct = abcst ∈ A
⇒ asbtc + p = abstc + p = absct + p = abcst + p ∈ A
Therefore A is a P-pseudo symmetric ideal and hence T is a P-pseudo symmetric ternary semiring.

Theorem III.25 : Every pseudo commutative ternary semiring is a P-pseudo symmetric ternary semiring.

Proof : Let T be a pseudo commutative ternary semiring and A be any ideal of T.
Let x, y, z ∈ T, xyz + P ⊆ A where P is any ideal. Then xyz ∈ A and P ⊆ A.
If s, t ∈ T. Then xsytz = syxtz = syzxt = s(xyz)t∈ A.
Therefore xsytz + P ⊆ A for all s, t ∈ T. Therefore A is a P-pseudo symmetric ideal.
Therefore T is a P-pseudo symmetric semiring.

Theorem III.26 : If T is a ternary semiring in which every element is a midunit then T is a P-pseudo symmetric ternary semiring.

Proof : Let T be a ternary semiring in which every element is a midunit and A be any ideal of T. Let x, y, z ∈ T and xyz + P ⊆ A where P is any ideal A. Then xyz ∈ A and P ⊆ A.
If s ∈ T, then s is a midunit and hence, xsysz = xyz ∈ A ⇒ xsysz + P ⊆ A. Hence A is a P-pseudo symmetric ideal. Therefore T is a P-pseudo symmetric ternary semiring.

IV. Conclusion

In this paper mainly we studied about the P-pseudo symmetric ideals in ternary semirings.

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