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On Double Elzaki Transform and Double Laplace Transform

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Abstract: *In this paper, we applied the method double Elzaki transform to solve wave equation in one dimensional and the results are compared with the resultsof double Laplace transform.*
Keyword:*Double Elzaki troundaryansform, Double Laplace transform, Inverse Double Elzaki transform, Convolution.*

I. Introduction

The wave equation is an important second-order linear partial differential equation which generally describes all kinds of waves, such as sound waves, light waves and water waves. It arises in many different fields, such as acoustics, electromagnetics, and fluid dynamics. Variations of the wave equation are also found in quantum mechanics and general relativity. Historically, the problem of a vibrating string such as that of a musical instrument was studied by Jean le Rondd'Alembert, Leonhard Euler, Daniel Bernoulli, and Joseph-Louis Lagrange. In 1746, d'Alambert discovered the one-dimensional wave equation, and within ten years Euler discovered the three-dimensional wave equation.

In recent years, many researches have paid attention to find the solution of partial differential equations by using various methods. Among theseare the double Laplace transform, the double Sumudu transform [3-7], differential transform method [15], various ways have been proposed recently to deal with these partial differential equations, one of these combination is Elzaki transform method [8-14]. The Elzaki transform a kind of modified Laplace's / Sumudu, was introduce by Elzaki in 2011 and it is defined by

$$E\left[f(t)\right]=\int_0^{\infty} v\, f(t)\exp(-t/v)dt = T(v). \quad (1)$$

For $E\left[f(t)\right]=T(v)$, Where $f(t)$ isa functionfor all real numbers $t>0$.

Where Elzaki transform defined over the set of function.

$$A=\left\{ f(t):\exists\ \mathrm{M}, \mathrm{k}_1, \mathrm{k}_2>0, \left|f(t)\right|<Me^{\frac{|t|}{k_j}},\ \text{if } \mathrm{t}\in(-1)^j\times[0,\infty) \right\}$$

the constant M must be finite number, $\mathrm{k}_1,\mathrm{k}_2$ may be finite or infinite.

II. Double Elzaki Transform And Double Laplace Transform

Let $f(x,t)$ be a function that can be express as convergent infinite series andlet $(x,t)\in \boldsymbol{R}_+^2$, thenthe double Elzaki transform is denoted by $E_2\left[f(x,t)\right]$ and defined by

$$E_2\left[f(x,t):(u,v)\right]=uv\int_0^{\infty}\int_0^{\infty} f(x,t)e^{-\left(\frac{x}{u}+\frac{t}{v}\right)}dx\ dt = T(u,v), \quad (2)$$

where $x,\ t>0$ and $u,\ v$ are transform variables for x and t respectively, whenever the improper integral is convergent.

Double Elzaki transform of the second partial derivative with respect to x is of form

$$E_2\left[\frac{\partial^2 f(x,t)}{\partial x^2}:(u,v)\right]=uv\int_0^{\infty}\int_0^{\infty}\frac{\partial^2 f(x,t)}{\partial x^2}e^{-\left(\frac{x}{u}+\frac{t}{v}\right)}dxdt = v\int_0^{\infty} e^{-\frac{t}{v}}\left(u\int_0^{\infty}\frac{\partial^2 f(x,t)}{\partial x^2}e^{-\frac{x}{u}}dx\right)dt,$$

the integral inside the bracket is

$$u\int_0^\infty \frac{\partial^2 f(x,t)}{\partial x^2} e^{-\frac{x}{u}} dx = \frac{T(u,t)}{u^2} - f(0,t) - u\frac{\partial f(0,t)}{\partial x}, \quad (3)$$

By taking Elzaki transform with respect to t for equation (3), we get adouble Elzaki transform in the form

$$E_2\left[\frac{\partial^2 f(x,t)}{\partial x^2} : (u,v)\right] = \frac{T(u,v)}{u^2} - T(0,v) - u\frac{\partial T(0,v)}{\partial x}. \quad (4)$$

Similarly, we get double Elzaki transformof $\frac{\partial^2 f(x,t)}{\partial t^2}$ as

$$E_2\left[\frac{\partial^2 f(x,t)}{\partial t^2} : (u,v)\right] = \frac{T(u,v)}{v^2} - T(u,0) - v\frac{\partial T(u,0)}{\partial t}. \quad (5)$$

The double Laplace transform of a function of two variables defined in the positive quadrant of the xt -plane is given by:

$$L_x L_t\left[f(x,t):(p,s)\right] = \int_0^\infty e^{-px}\int_0^\infty f(x,t)e^{-st} dx\ dt = F(p,s), \quad (6)$$

where $x,\ t > 0$ and $p,\ s$ are transform variables for x and t respectively, whenever the improper integral is convergent.

Double Laplace transform of first order partial derivative definedas follow

$$L_x L_t\left[\frac{\partial f(x,t)}{\partial x}\right] = pF(p,s) - F(0,s), \quad (7)$$

Double Laplace transform for second partial derivative with respect to x is defined as

$$L_x L_x\left[\frac{\partial^2 f(x,t)}{\partial x^2}\right] = p^2F(p,s) - pF(0,s) - \frac{\partial F(0,s)}{\partial x}. \quad (8)$$

Similarly, double Laplace transform for second partial derivative with respect to t is

$$L_t L_t\left[\frac{\partial^2 f(x,t)}{\partial t^2}\right] = s^2F(p,s) - sF(p,0) - \frac{\partial F(p,0)}{\partial t}. \quad (9)$$

Double Laplace transformsof a mixed partial derivative with respect to x and t can be defined as

$$L_x L_t\left[\frac{\partial^2 f(x,t)}{\partial x\,\partial t}\right] = psF(p,s) - pF(p,0) - sF(0,s) - F(0,0). \quad (10)$$

Theorem (1): Considera function f in the set A defined by

$$f(x,t) = \left\{f(x,t)\in A \,\middle|\, \exists M, k_1, k_2 > 0 \text{ such that } |f(x,t)| \le M\ e^{\frac{x+t}{k_i^2}},\ i = 1,2 \text{ and } (x,t)\in \boldsymbol{R}_+^2\right\}$$

with double Laplace transform $F(p,s)$, and double Elzaki transform $T(u,v)$,

then $T(u,v) = uvF\left(\frac{1}{u},\frac{1}{v}\right)$, where $M,\ k_1,\ k_2 \in R^+$

Proof: Let $f(x,t)\in A$ and $k_1 < u,v < k_2$, $T(u,v) = u^2v^2\int_0^\infty\int_0^\infty f(ux,vt)e^{-(x+t)}dxdt$,

Let $\eta = ux$ and $\lambda = vt$, we have

$$T(u,v) = u^2v^2\int_0^\infty\int_0^\infty f(ux,vt)e^{-(x+t)}dxdt = uv\int_0^\infty\int_0^\infty f(\eta,\lambda)e^{-\left(\frac{\eta}{u}+\frac{\lambda}{v}\right)}d\eta d\lambda = uvF\left(\frac{1}{u},\frac{1}{v}\right)$$

Note:

The double Laplace transform and double Elzaki transform having strong relation.

$T(u,v)=L_x L_t\left(f(x,t);\left(\frac{1}{u},\frac{1}{v}\right)\right)=uvF\left(\frac{1}{u},\frac{1}{v}\right)$, or

$$T(p,s)=L_x L_t\left(f(x,t);\left(\frac{1}{p},\frac{1}{s}\right)\right)=ps\ F\left(\frac{1}{p},\frac{1}{s}\right) \qquad (11)$$

Definition (1): Let $f(x,t)$ and $g(x,t)$ be piecewise continuous functions on $[0,\infty)$ and having double Laplace transform $F(p,s)$ and $G(p,s)$ respectively, then the double convolution of the functions $f(x,t)$ and $g(x,t)$ exist and defined by

$$\left(f**g\right)(x,t)=\int_0^t\int_0^x f(\alpha,\beta)g(x-\alpha,t-\beta)d\alpha d\beta, \qquad (12)$$

$$L_x L_t\left[\left(f**g\right)(x,t);(p,s)\right]=F(p,s)G(p,s), \qquad (13)$$

Theorem(2): Let $f(x,t)$ and $g(x,t)$ be defined in A and having the double Laplace transform $F(p,s)$ and $G(p,s)$ respectively, and also having double Elzaki transform $M(u,v)$ and $N(u,v)$ respectively, then the double Elzaki transform of the convolution of $f(x,t)$ and $g(x,t)$ is given by

$$E_2\left[\left(f**g\right)(x,t);(u,v)\right]=\frac{1}{uv}M(u,v)N(u,v)$$

Proof: The Laplace transform of $\left(f**g\right)(x,t)$ is given by

$$L_x L_t\left[\left(f**g\right)(x,t);(p,s)\right]=F(p,s)G(p,s).$$

From theorem(1) we have

$$E_2\left[\left(f**g\right)(x,t);(u,v)\right]=uvL_x L_t\left[\left(f**g\right)(x,t);(p,s)\right],$$

Since $M(u,v)=uvF\left(\frac{1}{u},\frac{1}{v}\right)$, $N(u,v)=uv\ G\left(\frac{1}{u},\frac{1}{v}\right)$, then

$$E_2\left[\left(f**g\right)(x,t);(u,v)\right]=uv\left(F\left(\frac{1}{u},\frac{1}{v}\right)G\left(\frac{1}{u},\frac{1}{v}\right)\right)=uv\left[\frac{M(u,v)}{uv}.\frac{N(u,v)}{uv}\right]=\frac{1}{uv}\left[M(u,v).N(u,v)\right].$$

III. Applications

In this section, we assume that the inverse double Elzaki transform is exists. We apply the inverse double Elzaki transform to find the solution of the wave equation in one dimension with initial and boundary conditions.

Example (1): Consider the homogeneous wave equation in the form:

$$U_{tt}=c^2\ U_{xx}, \qquad (14)$$

with initial conditions

$$U(x,0)=\sin x, \qquad U_t(x,0)=2, \qquad (15)$$

and boundary conditions

$$U(0,t)=2t, \qquad U_x(0,t)=\cos(ct), \qquad (16)$$

where $c=\left(T/\rho\right)^{\frac{1}{2}}$, is T the tension, and ρ is its linear density. The quantity c has the dimensions of velocity.

By taking the double Elzaki transform to Eq.(14) we get,

$$\left[\frac{T(u,v)}{v^2}-T(u,0)-v\ \frac{\partial T(u,0)}{\partial t}\right]=c^2\left[\frac{T(u,v)}{u^2}-T(0,v)-u\ \frac{\partial T(0,v)}{\partial x}\right], \qquad (17)$$

The single Elzaki transform of initial conditions gives

$$T(u,0)=\frac{u^3}{u^2+1},\qquad \frac{\partial T(u,0)}{\partial t}=2u^2, \quad (18)$$

The single Elzaki transform of boundary conditions gives

$$T(0,v)=2v^3,\qquad \frac{\partial T(0,v)}{\partial x}=\frac{v^2}{c^2v^2+1}, \quad (19)$$

By substituting (18) & (19) into equation (17), we get

$$\frac{T(u,v)}{v^2}-c^2\frac{T(u,v)}{u^2}=T(u,0)+v\,\frac{\partial T(u,0)}{\partial t}-c^2T(0,v)-c^2u\,\frac{\partial T(0,v)}{\partial x}, \text{ then}$$

$$T(u,v)=\left(\frac{u^3}{u^2+1}\right)\left(\frac{v^2}{c^2v^2+1}\right)+2u^2v^3, \quad (20)$$

ApplyinginversedoubleElzaki transformof equation (20) gives the solution of wave equation (14) in the form

$$U(x,t)=2t+\sin x\ \cos(ct). \quad (21)$$

By taking the double Laplace transform to Eq (14) we get,

$$s^2F(p,s)-sF(p,0)-\frac{\partial F(p,0)}{\partial t}=c^2\left(p^2F(p,s)-pF(0,s)-\frac{\partial F(0,s)}{\partial x}\right) \quad (22)$$

Thesingle Laplace transform of initial conditions gives

$$F(p,0)=\frac{1}{p^2+1},\qquad \frac{\partial F(p,0)}{\partial t}=\frac{2}{p}, \quad (23)$$

The singleLaplace transformof boundary conditions

$$F(0,s)=\frac{2}{s^2}\ ,\qquad \frac{\partial F(0,s)}{\partial x}=\frac{s}{s^2+c^2}, \quad (24)$$

By substituting (23)&(24) into equation(22), we get

$$\left(s^2-c^2p^2\right)F(p,s)=sF(p,0)+\frac{\partial F(p,0)}{\partial t}-c^2pF(0,s)-c^2\frac{\partial F(0,s)}{\partial x}, \text{ then}$$

$$F(p,s)=\left(\frac{1}{p^2+1}\right)\left(\frac{s}{s^2+c^2}\right)+\frac{2}{ps^2}. \quad (25)$$

Applyinginversedouble Laplace transform of equation (25) gives the solution of wave equation (14) in the form

$$U(x,t)=2t+\sin x\ \cos(ct). \quad (26)$$

Example (2): Consider the inhomogeneous wave equation in the form:

$$U_{tt}-U_{xx}=6t+2x,\ t>0. \quad (27)$$

With initial conditions

$$U(x,0)=0,\qquad U_t(x,0)=\sin x, \quad (28)$$

and boundary conditions

$$U(0,t)=t^3,\qquad U_x(0,t)=t^2+\sin t, \quad (29)$$

By taking the double Elzaki transform to Eq.(27) gives,

$$\left[\frac{T(u,v)}{v^2}-T(u,0)-v\,\frac{\partial T(u,0)}{\partial t}\right]-\left[\frac{T(u,v)}{u^2}-T(0,v)-u\,\frac{\partial T(0,v)}{\partial x}\right]=6v^3u^2+2u^3v^2, \quad (30)$$

The singleElzaki transform of initial conditions gives

$$T(u,0)=0,\qquad \frac{\partial T(u,0)}{\partial t}=\frac{u^3}{u^2+1}, \quad (31)$$

Thesingle Elzaki transform of boundary conditions gives

$$T(0,v)=6v^5, \qquad \frac{\partial \mathrm{T}(0,v)}{\partial x}=2v^4+\frac{v^3}{v^2+1}, \quad (32)$$

By substituting(31) &(32) intoequation (30), we get

$$\left[\frac{u^2-v^2}{u^2v^2}\right]T(u,v)=6v^3u^2+2u^3v^2-6v^5-2v^4u+\frac{v\,u^3}{u^2+1}-\frac{v^3u}{v^2+1}, \text{ then}$$

$$T(u,v)=2u^3v^4+6v^5u^2+\left(\frac{u^3}{u^2+1}\right)\left(\frac{v^3}{v^2+1}\right). \quad (33)$$

By applyinginverse double Elzaki transform of equation (33) gives the solution of wave equation (27) in the form

$$U(x,t)=xt^2+t^3+\sin x\ \ \sin t. \quad (34)$$

By taking double Laplace transform to Eq (27),we get

$$s^2F(p,s)-sF(p,0)-\frac{\partial F(p,0)}{\partial t}-\left(p^2F(p,s)-pF(0,s)-\frac{\partial F(0,s)}{\partial x}\right)=\frac{6}{p\,s^2}+\frac{2}{p^2\,s} \quad (35)$$

Thesingle Laplacetransform of initial conditions gives

$$F(p,0)=0, \qquad \frac{\partial F(p,0)}{\partial t}=\frac{1}{p^2+1}, \quad (36)$$

The single Laplace transformof boundary conditions gives

$$F(0,s)=\frac{3!}{s^4}\ , \qquad \frac{\partial F(0,s)}{\partial x}=\frac{2!}{s^3}+\frac{1}{s^2+1}, \quad (37)$$

By substituting (36)&(37) into equation(35), weget

$$\left(s^2-p^2\right)F(p,s)=\frac{6}{s^2p}+\frac{2}{p^2s}-\frac{6p}{s^4}-\frac{2}{s^3}+\frac{s^2-p^2}{\left(s^2+1\right)\left(p^2+1\right)} \text{ then}$$

$$F(p,s)=\frac{6}{s^4p}+\frac{2}{p^2s^3}+\frac{1}{\left(s^2+1\right)\left(p^2+1\right)}, \quad (38)$$

By applyinginverse double Laplace transform of equation (38) gives the solution of wave equation (27) in the form

$$U(x,t)=xt^2+t^3+\sin x\ \ \sin t. \qquad (39)$$

Example (3): Consider the inhomogeneous wave equation in the form:

$$U_{tt}-U_{xx}=-3e^{2x+t},\ (\mathrm{x,y})\in \boldsymbol{R}_+^2. \quad (40)$$

With initial conditions

$$U(x,0)=e^{2x}+e^x, \qquad U_t(x,0)=e^{2x}+e^x, \quad (41)$$

and boundary conditions

$$U(0,t)=2e^t, \qquad U_x(0,t)=3e^t, \quad (42)$$

By taking the double Elzaki transform to Eq. (40),we get

$$\left[\frac{T(u,v)}{v^2}-T(u,0)-v\,\frac{\partial T(u,0)}{\partial t}\right]-\left[\frac{T(u,v)}{u^2}-T(0,v)-u\,\frac{\partial T(0,v)}{\partial x}\right]=-3\left(\frac{u^2}{1-2u}\right)\left(\frac{v^2}{1-v}\right), \quad (43)$$

Thesingle Elzaki transform of initial conditions gives

$$T(u,0)=\frac{u^2}{1-2u}+\frac{u^2}{1-u}, \qquad \frac{\partial \mathrm{T}(u,0)}{\partial t}=\frac{u^2}{1-2u}+\frac{u^2}{1-u}, \quad (44)$$

Thesingle Elzaki transform of boundary conditions gives

$$T(0,v)=\frac{2v^2}{1-v}, \qquad \frac{\partial T(0,v)}{\partial x}=\frac{3v^2}{1-v}, \text{(45)}$$

By substituting (44) & (45) intoequation (43), we get

$$T(u,v)=\left(\frac{u^2}{1-2u}\right)\left(\frac{v^2}{1-v}\right)+\left(\frac{u^2}{1-u}\right)\left(\frac{v^2}{1-v}\right), \text{(46)}$$

Applying inverse double Elzaki transform of equation (46)gives the solution of wave equation(40) in the form

$$U(x,t)=e^{2x+t}+e^{x+t}. \text{(47)}$$

By takingdouble Laplace transform to Eq.(40), we get ,

$$s^2F(p,s)-sF(p,0)-\frac{\partial F(p,0)}{\partial t}-\left(p^2F(p,s)-pF(0,s)-\frac{\partial F(0,s)}{\partial x}\right)=\frac{-3}{(s-1)(p-2)}, \text{(48)}$$

The single Laplace transform of initial conditions gives

$$F(p,0)=\frac{1}{s(p-2)}+\frac{1}{s(p-1)}, \qquad \frac{\partial F(p,0)}{\partial t}=\frac{1}{s(p-2)}+\frac{1}{s(p-1)}, \quad \text{(49)}$$

The single Laplace transformof boundary conditions gives

$$F(0,s)=\frac{2}{p(s-1)} \quad , \qquad \frac{\partial F(0,s)}{\partial x}=\frac{3}{p(s-1)}, \qquad \text{(50)}$$

By substituting (49) & (50) in equation(48), weget

$$F(p,s)=\frac{1}{(s-1)(p-2)}+\frac{1}{(p-1)(s-1)}, \quad \text{(51)}$$

Applying doubleinverse Laplace transform of equation(51)gives the solution of waveequation (40) in the form

$$U(x,t)=e^{2x+t}+e^{x+t}. \text{(52)}$$

Example (4): Consider the inhomogeneous wave equation in the form:

$$U_{tt}=U_{xx}-3U+3, \text{(53)}$$

With initial conditions

$$U(x,0)=1, \qquad U_t(x,0)=2\sin x, \qquad \text{(54)}$$

and boundary conditions

$$U(0,t)=1, \qquad U_x(0,t)=\sin 2t, \quad \text{(55)}$$

By taking the double Elzaki transform to Eq.(53), we get,

$$\left[\frac{T(u,v)}{v^2}-T(u,0)-v\,\frac{\partial T(u,0)}{\partial t}\right]=\left[\frac{T(u,v)}{u^2}-T(0,v)-u\,\frac{\partial T(0,v)}{\partial x}\right]-3T(u,v)+3u^2v^2 \qquad \text{(56)}$$

The single Elzaki transform of initial conditions gives

$$T(u,0)=u^2, \qquad \frac{\partial T(u,0)}{\partial t}=\frac{2u^3}{u^2+1}, \qquad \text{(57)}$$

The single Elzaki transform of boundary conditions gives

$$T(0,v)=v^2, \qquad \frac{\partial T(0,v)}{\partial x}=\frac{2v^3}{4v^2+1}, \text{(58)}$$

By substituting (57) & (58) into equation (56), we get

$$\left[\frac{u^2-v^2+3u^2v^2}{u^2v^2}\right]T(u,v)=u^2+\frac{2u^3v}{u^2+1}-v^2-\frac{2v^3u}{4v^2+1}+3u^2v^2, \text{ then}$$

$$T(u,v)=u^2v^2+\left(\frac{u^3}{u^2+1}\right)\left(\frac{2v^3}{4v^2+1}\right) \qquad \text{(59)}$$

Applying doubleinverse Elzaki transform of equation (59) gives the solution of wave equation (53) in the form
$U(x,t)=1+\sin x \ \sin 2t.$ (60)

By taking double Laplace transform to Eq. (53), we get

$$s^2F(p,s)-sF(p,0)-\frac{\partial F(p,0)}{\partial t}=\left(p^2F(p,s)-pF(0,s)-\frac{\partial F(0,s)}{\partial x}\right)-3F(p,s)+\frac{3}{s} \qquad (61)$$

The single Laplace transform of initial conditions gives

$$F(p,0)=\frac{1}{p}, \qquad \frac{\partial F(p,0)}{\partial t}=\frac{2}{p^2+1}, \qquad (62)$$

The singleLaplace transformof boundary conditions

$$F(0,s)=\frac{1}{s}\ , \qquad \frac{\partial F(0,s)}{\partial x}=\frac{2}{s^2+4}, \qquad (63)$$

By substituting (62)&(63) into equation(61) , we have

$$\left(s^2-p^2+3\right)F(p,s)=sF(p,0)+\frac{\partial F(p,0)}{\partial t}-pF(0,s)-\frac{\partial F(0,s)}{\partial x}+\frac{3}{s}, \text{ then}$$

$$F(p,s)=\frac{1}{ps}+\left(\frac{1}{p^2+1}\right)\left(\frac{2}{s^2+4}\right). \ (64)$$

Applying doubleinverse Laplace transform of equation (64) gives the solution of wave equation (53) in the form
$U(x,t)=1+\sin x \ \sin 2t.$(69)

IV. Conclusions

Double Elzaki transform is applied to obtain the solution of wave equation of one dimensional, the result are compared with result of double Laplace transform. The wave equation in one dimensional under the initial and boundary conditions, give similar results when we use the double Elzaki transform and double Laplace transform.

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