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Slightly S_g^*-continuous functions and totally S_g^*-continuous functions in Topological spaces

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Abstract: *The aim of this paper is to introduce two new classes of functions, namely slightly S_g^*-continuous functions and totally S_g^*-continuous functions and study its properties.*
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1. Introduction

Continuous functions in topology found a valuable place in the applications of mathematics as it has applications to engineering especially to digital signal processing and neural networks. Topologists studied weaker and stronger forms of continuous functions in topology using the sets stronger and weaker than open and closed sets.

In 1997, Slightly continuity was introduced by Jain[3] and has been applied for semi-open and pre open sets by Nour[5] and Baker[1] respectively. Recently, S.Pious Missier and J.Arul Jesti have introduced the concept of S_g^*-open sets[6], and introduced some more functions in S_g^*-open sets. Continuing this work we shall introduce a new functions called slightly S_g^*-continuous functions and totally S_g^*-continuous functions and investigated their properties in terms of composition and restriction. Also we establish the relationship between slightly S_g^*-continuous functions and totally S_g^*-continuous functions with other functions.

II. Preliminaries

Throughout this paper (X, τ), (Y, σ) and (Z, η) (or X,Y and Z) represent non-empty topological spaces on which no separation axioms are assumed unless explicitly stated. For a subset A of a space (X, τ), S_g^*Cl(A) and S_g^*Int(A) denote the S_g^*-closure and the S_g^*-interior of A respectively.

Definition 2.1: A subset A of a topological space (X,τ) is called a $\boldsymbol{S_g^*}$**-open set** [6] if there is an open set U in X such that U⊆A⊆ sCl*(U). The collection of all S_g^*-open sets in (X, τ) is denoted by S_g^*O(X,τ).

Definition 2.2: A subset A of a topological space (X,τ) is called a $\boldsymbol{S_g^*}$**-closed set**[6] if X\A is S_g^*-open. The collection of all S_g^*-closed sets in (X, τ) is denoted by S_g^*C(X,τ).

Theorem 2.3 [6]**:**Every open set is S_g^*-open and every closed set is S_g^*-closed set

Definition 2.4: A topological space (X, τ) is said to be $\boldsymbol{S_g^*}$-$\boldsymbol{T_{1/2}}$**space** [7] if every S_g^*-open set of X is open in X.

Definition 2.5: A topological space (X, τ) is said to be $\boldsymbol{S_g^*}$**-locally indiscrete space** [8] if every S_g^*-open set of X is closed in X.

Definition 2.6: A function f: X → Y is said to be **contra-** $\boldsymbol{S_g^*}$**-continuous** [8] if the inverse image of every open set in Y is S_g^*-closed in X.

Definition 2.7: A function f: (X, τ) ⟶ (Y, σ) is called a **contra-continuous** [2] if f^{-1} (O) is closed in (X, τ) for every open subset O of (Y, σ).

Definition 2.8: A mapping f: X → Y is said to be $\boldsymbol{S_g^*}$**-continuous** [7] if the inverse image of every open set in Y is S_g^*-open in X.

Defintion 2.9: A map f: X → Y is said to be $\boldsymbol{S_g^*}$**-irresolute**[7] if the inverse image of every S_g^*-open set in Y is S_g^*-open in X.

Definition 2.10: A mapping f: X → Y is said to be **strongly** $\boldsymbol{S_g^*}$**-continuous** [7]if the inverse image of every S_g^*-open set in Y is open in X.

Definition 2.11: A mapping f: X → Y is said to be **perfectly** $\boldsymbol{S_g^*}$**-continuous** [7]if the inverse image of every S_g^*-open set in Y is open and closed in X.

Definition 2.12: A function f: X ⟶ Y is called **slightly continuous** [3] if the inverse image of every clopen set in Y is open in X.

III. Slightly S_g^*- continuous function

Definition 3.1: A function $f:(X,\tau) \to (Y,\sigma)$ is said to be **slightly S_g^*- continuous** at a point $x \in X$ if for each subset V of Y containing f(x), there exists a S_g^* -open subset U in X containing x such that $f(U) \subseteq V$. The function f is said to be slightly S_g^*- continuous if f is slightly S_g^*-continuous at each of its points.

Definition 3.2: A function $f:(X,\tau) \to (Y,\sigma)$ is said to be **slightly S_g^*-continuous** if the inverse image of every clopen set in Y is S_g^*-open in X.

Example 3.3: Let $X = \{a,b,c\} = Y$, $\tau = \{X,\emptyset,\{a\},\{b\},\{a,c\},\{a,b\}\}$, $\sigma = \{\emptyset,Y,\{a\},\{b,c\}\}$.
$S_g^*O(X) = \{\emptyset,X,\{a\},\{b\},\{a,c\},\{a,b\}\}$. The function f is defined as $f(a) = c, f(b) = a, f(c) = b$. The function f is slightly S_g^*-continuous.

Proposition 3.4: The definition 3.1 and 3.2 are equivalent.

Proof: Suppose the definition 3.1 holds. Let V be a clopen set in Y and let $x \in f^{-1}(V)$. Then $f(x) \in V$ and thus there exists a S_g^*-open set U_x such that $x \in U_x$ and $f(U_x) \subseteq V$. Now $x \in U_x \subseteq f^{-1}(V)$. And $f^{-1}(V) = \bigcup_{x \in f^{-1}(V)} U_x$. Since arbitrary union of S_g^*-open sets is S_g^*-open, $f^{-1}(V)$ is S_g^*-open in X and therefore f is slightly S_g^*-continuous.

Suppose the definition 3.2 holds. Let $f(x) \in V$ where V is a clopen set in Y. Since f is slightly S_g^*-continuous, $x \in f^{-1}(V)$ where $f^{-1}(V)$ is S_g^*-open in X. Let $U = f^{-1}(V)$. Then U is S_g^*-open in X, $x \in U$ and $f(U) \subseteq V$.

Theorem 3.5: Let $f:(X,\tau) \to (Y,\sigma)$ be a function then the following are equivalent.

(1) f is slightly S_g^*- continuous .

(2) The inverse image of every clopen set V of Y is S_g^*-open in X.

(3) The inverse image of every clopen set V of Y is S_g^*-closed in X.

(4) The inverse image of every clopen set V of Y is S_g^*-clopen in X.

Proof:

(1)⟹(2): Follows from the Theorem 3.4.

(2) ⟹(3): Let V be a clopen set in Y which implies V^c is clopen in Y. By (2), $f^{-1}(V^c) = (f^{-1}(V))^c$ is S_g^*-open in X. Therefore $f^{-1}(V)$ is S_g^*-closed in X.

(3) ⟹ (4): By (2) and (3) $f^{-1}(V)$ is S_g^*-clopen in X.

(4) ⟹ (1): Let V be a clopen subset of Y containing f(x). By (4) $f^{-1}(V)$ is S_g^*-clopen in X. Put $U = f^{-1}(V)$ then $f(U) \subseteq V$. Hence f is slightly S_g^*-continuous.

Theorem 3.6: Every slightly continuous function is slightly S_g^*-continuous.

Proof: Let $f: X \to Y$ be slightly continuous. Let U be a clopen set in Y . Then $f^{-1}(U)$ is open in X. Since every open set is S_g^*-open, $f^{-1}(U)$ is S_g^*-open. Hence f is slightly S_g^*-continuous.

Remark 3.7: The converse of the above theorem need not be true as can be seen from the following example

Example 3.8: Let $X = \{a,b,c,d\}$ with $\tau = \{X,\emptyset,\{a\},\{b,c\},\{a,b,c,\}\}$. $S_g^*O(X,\tau) = \{X,\emptyset,\{a\},\{b,c\},\{a,b,c\},\{b,c,d\}\}$. Let $Y = \{p,q,r\}$ with $\sigma=\{Y,\emptyset,\{p\},\{q,r\}\}$. Define $f:(X,\tau)\to(Y,\sigma)$ by $f(a) = p, f(b) = q, f(c) = f(d) = r$. Hence $f^{-1}\{q,r\} = \{b,c,d\}$ is S_g^*-open but not open in X. Thus f is slightly S_g^*-continuous but not slightly continuous.

Theorem 3.9: Every S_g^*-continuous function is slightly S_g^*-continuous.

Proof: Let $f: X \to Y$ be a S_g^*-continuous function. Let U be a clopen set in Y .Then $f^{-1}(U)$ is S_g^*-open in X and S_g^*-closed in X. Hence f is slightly S_g^*-continuous.

Remark 3.10: The converse of the above theorem need not be true as can be seen from the following example.

Example 3.11: Let $X=\{a,b,c\}, Y = \{p,q\}$. $\tau = \{X,\emptyset,\{a\},\{b\},\{a,b\}\} = S_g^*O(X,\tau)$. $\sigma = \{Y,\emptyset,\{p\}\}$. Define $f:(X,\tau) \to (Y,\sigma)$ by $f(a) = q$, $f(b) = f(c) = p$. The function f is slightly S_g^*-continuous but not S_g^*-continuous since $f^{-1}\{p\} = \{b,c\}$ is not S_g^*-open in X.

Theorem 3.12: If the function $f:(X,\tau) \to (Y,\sigma)$ is slightly S_g^*-continuous and (Y,σ) is a locally indiscrete space then f is S_g^*-continuous.

Proof: Let U be an open subset of Y. Since Y is locally indiscrete, U is closed in Y. Since f is slightly S_g^*-continuous, $f^{-1}(U)$ is S_g^*-open in X. Hence f is S_g^*-continuous.

Theorem 3.13: If the function $f:(X,\tau) \to (Y,\sigma)$ is slightly S_g^*-continuous and (X,τ) is a $S_g^* - T_{1/2}$ space then f is slightly continuous.

Proof: Let U be a clopen subset of Y. Since f is slightly S_g^*-continuous, $f^{-1}(U)$ is S_g^*-open in X. Since X is a $S_g^* - T_{1/2}$ space, $f^{-1}(U)$ is open in X. Hence f is slightly continuous.

Theorem 3.14: Let $f:(X,\tau) \to (Y,\sigma)$ and $g:(Y,\sigma) \to (Z,\eta)$ be function

(i) If f is S_g^*-irresolute and g is slightly S_g^*-continuous then $gof:(X,\tau) \to (Z,\eta)$ is slightly S_g^*-continuous.

(ii) If f is S_g^*-irresolute and g is S_g^*-continuous then gof is slightly S_g^*-continuous.

(iii) If f is S_g^*-irresolute and g is slightly continuous then gof is slightly S_g^*-continuous.

(iv) If f is S_g^*-continuous and g is slightly continuous then gof is slightly S_g^*-continuous.

(v) If f is strongly S_g^*-continuous and g is slightly S_g^*-continuous then gof is slightly continuous.

(vi) If f is slightly S_g^*-continuous and g is perfectly S_g^*-continuous then gof is S_g^*-irresolute.

(vii) If f is slightly S_g^*-continuous and g is contra-continuous then gof is slightly S_g^*-continuous.

(viii) If f is S_g^*-irresolute and g is contra-S_g^*-continuous then gof is slightly S_g^*-continuous.

Proof:

(i) Let U be a clopen set in Z. Since g is slightly S_g^*-continuous, g^{-1} (U) is S_g^*-open in Y. Since f is S_g^*-irresolute, $f^{-1}(g^{-1}(U))$ is S_g^*-open in X. Since $(gof)^{-1}(U) = f^{-1}(g^{-1}(U)$, gof is slightly S_g^*-continuous.

(ii) Let U be a clopen set in Z. Since g is S_g^*-continuous, g^{-1} (U) is S_g^*-open in Y. Also since f is S_g^*-irresolute, $f^{-1}\left(g^{-1}(U)\right)$ is S_g^*-open in X. Hence gof is slightly S_g^*-continuous.

(iii) Let O be a clopen set in Z. Then g^{-1} (U) is S_g^*-open in Y. Therefore $f^{-1}\left(g^{-1}(U)\right)$ is S_g^*-open in X, since f is S_g^*-irresolute. Hence gof is slightly S_g^*-continuous.

(iv) Let U be a clopen set in Z. Then g^{-1} (U) is open in Y, since g is slightly continuous. Also since f is S_g^*-continuous. Also since f is S_g^*-continuous, $f^{-1}\left(g^{-1}(U)\right)$ is S_g^*-open in X. Hence gof is slightly S_g^*-continuous.

(v) Let U be a clopen set in Z. Then g^{-1} (U) is S_g^*-open in Y, since g is slightly S_g^*-continuous. Also $f^{-1}\left(g^{-1}(U)\right)$ is open in X, since f is strongly S_g^*-continuous. Therefore gof is slightly continuous.

(vi) Let U be a S_g^*-open in Z. Since g is perfectly S_g^*-continuous, g^{-1} (U) is open and closed in Y. Then $f^{-1}\left(g^{-1}(U)\right)$ is S_g^*-open in X. Since f is slightly S_g^*-continuous. Hence gof is S_g^*-irresolute.

(vii) Let U be a closed and open set in Z. Since g is contra-continuous, g^{-1} (U) is open and closed in Y. Since f is slightly S_g^*-continuous, $f^{-1}(g^{-1}(U))$ is S_g^*-open in X. Since $(gof)^{-1}(U) = f^{-1}(g^{-1}(U)$, gof is slightly S_g^*-continuous.

(viii) Let O be a clopen set in Z. Since g is contra-S_g^*-continuous, g^{-1} (U) is S_g^*-open and S_g^*-closed in Y. Therefore $f^{-1}\left(g^{-1}(U)\right)$ is S_g^*-open and S_g^*-closed in X, since f is S_g^*-irresolute. Hence gof is slightly S_g^*-continuous.

Theorem 3.15: If $f:(X,\tau) \to (Y,\sigma)$ is slightly S_g^*-continuous and A is an open subset of X then the restriction $f_{/A}:(A,\tau_A) \to (Y,\sigma)$ is slightly S_g^*-continuous.

Proof: Let V be a clopen subset of Y. Then $(f_{/A})^{-1}(V) = f^{-1}(V) \cap A$. Since $f^{-1}(V)$ is S_g^*-open and A is open, $(f_{/A})^{-1}(V)$ is S_g^*-open in the relative topology of A. Hence $f_{/A}$ is slightly S_g^*-continuous.

IV. Totally S_g^*-continuous function

Definition 4.1: A map $f:(X,\tau) \to (Y,\sigma)$ is said to be **totally S_g^*-continuous** if the inverse image of every open set in (Y,σ) is S_g^*-clopen in (X,τ).

Example 4.2: Let X=Y={a, b, c} with $\tau = \{X, \emptyset, \{a\}, \{b, c\}\} = S_g^*O(X,\tau)$ and $\sigma = \{Y, \emptyset, \{a\}\}$. Define $f:(X,\tau) \to (Y,\sigma)$ by $f(a) = a, f(b) = b, f(c) = c$. Then f is totally S_g^*-continuous.

Theorem 4.3: Every perfectly S_g^*-continuous function is totally S_g^*-continuous.

Proof: Let $f:(X,\tau) \to (X,\sigma)$ be a perfectly S_g^*-continuous function. Let U be an open set in (Y,σ). Then U is S_g^*-open in (Y,σ). Since f is perfectly S_g^*-continuous, $f^{-1}(U)$ is both open and closed in (X,τ)which implies $f^{-1}(U)$ is both open and closed in (X,τ) which implies $f^{-1}(U)$ is both S_g^*-open and S_g^*-closed in (X,τ). Hence f is totally S_g^*-continuous.

Remark 4.4: The converse of the above theorem need not be true as can be seen from the following example,

Example 4.5: Let $X = \{a, b, c\}$ with $\tau = \{\emptyset, X, \{a\}, \{b, c\}\} = \tau^c$ and $Y = \{d, e, f\}$ with $\sigma=\{\emptyset, Y, \{d\}\}$. Define $f:(X,\tau) \to (Y,\sigma)$ by $f(a) = d$, $f(b) = f$ and $f(c) = e$. Here $S_g^*O(X,\tau) = \tau$ and $S_g^*O(Y,\sigma)=\{\emptyset, Y, \{d\}, \{d, e\}, \{d, f\}\}$. Here $\{d, e\}$ and $\{d, f\}$ are S_g^*-open in Y, but $f^{-1}\{d, e\} = \{a, c\}$ and $f^{-1}\{d, f\} = \{a, b\}$ are not open as well as not closed in X. So f is not perfectly-S_g^*-continuous but totally S_g^*-continuous.

Theorem 4.6: Every totally S_g^*-continuous function is S_g^*-continuous

Proof: Suppose $f:(X,\tau)\to(Y,\sigma)$ is totally S_g^*-continuous and A is any open set in (Y,σ). Since f is totally S_g^*-continuous, $f^{-1}(A)$ is S_g^*-clopen in (X,τ). Hence f is S_g^*-continuous function.

Remark 4.7: The converse of the above theorem need not be true as can be seen from the following example

Example 4.8: Let $X=\{a,b,c,d\}$ with $\tau=\{\emptyset,X,\{a\}\}$ and $\sigma=\{\emptyset,Y,\{a\}\}$. Here $S_g^*O(X,\tau)=\{\emptyset,X,\{a\},\{a,b\},\{a,c\},\{a,d\},\{a,b,c\},\{a,c,d\},\{a,b,d\}\}$. The identity map $f:(X,\tau)\to(Y,\sigma)$is S_g^*-continuous but not totally S_g^*-continuous since $f^{-1}\{a\}=\{a\}$ is S_g^*-open in (X,τ) but not S_g^*-closed in (X,τ).

Remark 4.9: The following two examples shows that totally S_g^*-continuous and strongly S_g^*-continuous are independent.

Example 4.10: Let $X=\{a,b,c\}$ with $\tau=\{\emptyset,X,\{a\},\{b,c\}\}=\tau^c$ and $Y=\{d,e,f\}$ with $\sigma=\{\emptyset,Y,\{d\}\}$. Define $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=d$, $f(b)=f$ and $f(c)=e$. Here $S_g^*O(X,\tau)=\tau$ and $S_g^*O(Y,\sigma)=\{\emptyset,Y,\{d\},\{d,e\},\{d,f\}\}$. Here $\{d,e\}$ and $\{d,f\}$ are S_g^*-open in Y, but $f^{-1}\{d,e\}=\{a,c\}$ and $f^{-1}\{d,f\}=\{a,b\}$ are not open in X. Hence f is not strongly S_g^*-continuous but totally S_g^*-continuous.

Example 4.11: Let $X=Y=\{a,b,c\}$ with $\tau=\{\emptyset,X,\{a\},\{b\},\{a,b\}\}=S_g^*O(X,\tau)$ and $\sigma=\{\emptyset,Y,\{a,b\}\}=S_g^*O(Y,\sigma)$. The identity map $f:(X,\tau)\to(Y,\sigma)$ is strongly S_g^*-continuous but not totally S_g^*-continuous. For, the subset $\{a,b\}$ of (Y,σ), $f^{-1}\{a,b\}=\{a,b\}$ is S_g^*-open in (X,τ) but not S_g^*-closed in (X,τ).

Theorem 4.12: Let $f:(X,\tau)\to(Y,\sigma)$and $g:(Y,\sigma)\to(Z,\eta)$ be function. Then $gof:(X,\tau)\to(Z,\eta)$

(i) If f is S_g^*-irresolute and g is totally S_g^*-continuous then gof is totally S_g^*-continuous.

(ii) If f is totally S_g^*-continuous and g is continuous then gof is totally S_g^*-continuous.

Proof:

(i) Let U be a open set in Z. Since g is totally S_g^*-continuous, $g^{-1}(U)$ is S_g^*-clopen in Y. Since f is S_g^*-irresolute, $f^{-1}(g^{-1}(U))$ is S_g^*-open and S_g^*-closed in X. Since $(gof)^{-1}(U)=f^{-1}(g^{-1}(U)$, gof is totally S_g^*-continuous.

(ii) Let U be a open set in Z. Since g is continuous, $g^{-1}(U)$ is open in Y. Also since f is totally S_g^*-continuous, $f^{-1}(g^{-1}(U))$ is S_g^*-clopen in X. Hence gof is totally S_g^*-continuous.

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