

General Production and Sales System with SCBZ Machine Time and Manpower

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Abstract: A production and sale system is considered. During the operation time a machine produces random number of products. After operation time, sale time starts, and it has one among two distinct distributions depending on the magnitude of production time is within or exceeding a random threshold magnitude. Two models are treated. In model I, the machine operation time has exponential distribution and the life time of the machine is governed by SCBZ property; repair, recruitment, production and sale times have general distributions. In model II, when the operation time is more than a threshold, the sales are done altogether and when it is less than the threshold, the sales are done one by one. Joint transforms of the variables, their means and Co-variances with numerical results are presented.

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I. Introduction

In manufacturing models to get the return on investment and to pay minimum interest, it is natural that when the production time is more, the sale time is made short so as to cut cost. It has been noticed that when the units produced are more, financial supports for the customers are provided to clear products early. These are widely felt in perishable commodity sectors where many banking institutions provide required finance for the purchase. Storage systems of (s, S) type was studied by Arrow, Karlin and Scarf [1]. Such systems with random lead times and unit demand were treated by Danial and Ramanarayanan [2]. Models with bulk demands were analyzed by Ramanarayanan and Jacob [9]. Murthy and Ramanarayanan [5, 6, 7, 8] considered several (s, S) inventory systems. Kun-Shan Wu, Ouyang and Liang-Yuh [3] studied (Q, r, L) inventory model with defective items. Usha, Nithyapriya and Ramanarayanan [10] considered storage systems with random sales time depending on production. General Manpower and Machine system with Markovian production were analyzed by Hari kumar.k [4]. In this paper, two models are treated. In model I, the machine operation time is exponential and the life time of the machine is governed by SCBZ property; repair, recruitment, production and sale times have general distributions. Sales are done one by one. In model II, when the operation time is more than a threshold, the sales are done altogether and when it is less than the threshold, the sales are done one by one. The joint transforms, the means of production time, repair-recruitment times, sale time and Covariance of production and sale time with the numerical examples are presented.

II. Model -I

2.1 Assumptions:

1. Inter-production times of products are independent and identically distributed (i.i.d.) random variables with Cdf $F(X)$ and pdf $f(x)$.
2. The sales time of products are independent with Cdf $G(Z)$ and pdf $g(z)$ and the products are sold one by one.
3. Sales of the products begin and the production is stopped when the machine fails or with probability p when an employee leaves. When the machine is in operation the production is continued with probability q when an employee leaves, $p+q=1$.
4. The repair time of the machine is general with Cdf $R(y)$ and pdf $r(y)$. The recruitment time of each employee is general with Cdf $R_1(y)$ and pdf $r_1(y)$. All recruitments and repair of the machine are done one by one when the machine is stopped.
5. The machine operation time has exponential distribution with parameter 'a'. If the machine does not fail within an exponential time with parameter 'c', then the parameter 'a' changes to 'b'.
6. The inter-departure time of employees have iid exponential distribution with parameter μ .

2.2 Analysis:

The life time distribution of the machine is governed by SCBZ property in which the parameter changes after a random exponential time. Its pdf $h(x)$ satisfies the following equation.

$$h(x) = ae^{-ax}e^{-cx} + \int_0^x ce^{-cz}e^{-az}be^{-b(x-z)}dz \tag{1}$$

The first term of equation (1) is written considering the case that the machine fails before the parameter changes. The second term is written considering the parameter changes and then the failure occurs. On simplification the pdf becomes

$$h(x) = \alpha(c+a)e^{-(c+a)x} + \beta be^{-bx} \tag{2}$$

Here $\alpha = \frac{a-b}{c+a-b}$ and $\beta = \frac{c}{c+a-b}$, $\alpha + \beta = 1$ (3)

The Cdf of the machine failure time is

$$H(x) = 1 - \alpha e^{-(c+a)x} - \beta e^{-bx} \tag{4}$$

To study the model I, the joint probability density function of the three variables namely (X, \hat{R}, \hat{S}) where (i)

X is the operation time which is the minimum of the machine life time and manpower service time (ii) \hat{R} is the sum of all repairs and recruitment times and (iii) \hat{S} is the total sales time may be written as follows.

Using (2) and (4)

$$f(x, y, z) = \left[h(x) \left(\sum_{i=0}^{\infty} e^{-\mu x} \frac{(\mu x)^i}{i!} q^i r(y) \odot r_{1i}(y) \right) + (1 - H(x)) \left(\sum_{i=1}^{\infty} \mu e^{-\mu x} \frac{(\mu x)^{i-1}}{(i-1)!} q^{i-1} pr_{1i}(y) \right) \right] \left[\sum_{k=0}^{\infty} (F_k(x) - F_{k+1}(x)) g_k(z) \right] \tag{5}$$

The first term of equation (5) inside the square bracket is the part of the pdf that the machine fails, ‘i’ employees have left not causing stoppage and the repair and the recruitments are done one by one. The second term of the pdf inside the square bracket is that the machine does not fail, the production stoppage occurs when the i-th employee leaves and recruitments are done one by one. The second square bracket indicates the number of products produced and is sold one by one. The suffix letter indicates the convolution of pdf or Stieltjes convolution of Cdf and \odot indicates the convolution of the functions.

Now, the triple Laplace transform of the joint pdf is seen as follows.

$$f^*(\xi, \eta, \varepsilon) = \int_0^{\infty} \int_0^{\infty} \int_0^{\infty} e^{-\xi x - \eta y - \varepsilon z} f(x, y, z) dx dy dz \tag{6}$$

Here * indicates the Laplace Transform, this gives upon integration of y and z.

$$f^*(\xi, \eta, \varepsilon) = \int_0^{\infty} e^{-\xi x} \left(\alpha(c+a)r^*(\eta)e^{-(c+a)x} + \beta be^{-bx}r^*(\eta) + \alpha e^{-(c+a)x}r_1^*(\eta)\mu p + \beta e^{-bx}\mu pr_1^*(\eta) \right) e^{-\mu x(1-qr_1^*(\eta))} \left[\sum_{k=0}^{\infty} (F_k(x) - F_{k+1}(x)) \right] g^{*k}(\varepsilon) dx \tag{7}$$

On simplification we obtain

$$f^*(\xi, \eta, \varepsilon) = \frac{\alpha[(c+a)r^*(\eta) + \mu pr_1^*(\eta)][1 - f^*(\xi + c + a + \mu(1 - qr_1^*(\eta)))]}{[\xi + c + a + \mu(1 - qr_1^*(\eta))][1 - g^*(\varepsilon)f^*(\xi + c + a + \mu(1 - qr_1^*(\eta)))]} + \frac{\beta[br^*(\eta) + \mu pr_1^*(\eta)][1 - f^*(\xi + b + \mu(1 - qr_1^*(\eta)))]}{[\xi + b + \mu(1 - qr_1^*(\eta))][1 - g^*(\varepsilon)f^*(\xi + b + \mu(1 - qr_1^*(\eta)))]} \tag{8}$$

This gives,

$$E(X) = -\frac{\partial}{\partial \xi} f^*(\xi, \eta, \varepsilon) | \xi = \eta = \varepsilon = 0$$

$$E(\hat{R}) = -\frac{\partial}{\partial \eta} f^*(\xi, \eta, \varepsilon) | \xi = \eta = \varepsilon = 0$$

$$E(\hat{S}) = -\frac{\partial}{\partial \varepsilon} f^*(\xi, \eta, \varepsilon) | \xi = \eta = \varepsilon = 0$$

as follows.

$$E(X) = \frac{\alpha}{c+a+\mu p} + \frac{\beta}{b+\mu p} \tag{9}$$

$$E(\hat{R}) = \frac{\alpha[(c+a)E(R) + \mu E(R_1)]}{(c+a+\mu p)} + \frac{\beta[bE(R) + \mu E(R_1)]}{(b+\mu p)} \tag{10}$$

$$E(\hat{S}) = E(G) \left[\frac{\alpha f^*(c+a+\mu p)}{1-f^*(c+a+\mu p)} + \frac{\beta f^*(b+\mu p)}{1-f^*(b+\mu p)} \right] \tag{11}$$

Equation (8) gives the joint Laplace transform of $E(X, \hat{S})$ as follows.

$$f^*(\xi, 0, \varepsilon) = \frac{\alpha(c+a+\mu p)}{(\xi+c+a+\mu p)} \frac{(1-f^*(\xi+c+a+\mu p))}{(1-g^*(\varepsilon)f^*(\xi+c+a+\mu p))} + \frac{\beta(b+\mu p)}{(\xi+b+\mu p)} \frac{(1-f^*(\xi+b+\mu p))}{(1-g^*(\varepsilon)f^*(\xi+b+\mu p))} \tag{12}$$

The product moment $E(X \hat{S})$ is given by

$$E(X \hat{S}) = \frac{\partial^2}{\partial \xi \partial \varepsilon} f^*(\xi, 0, \varepsilon) | \xi = 0 = \varepsilon$$

$$E(X \hat{S}) = E(G) \left\{ \frac{\alpha}{(1-f^*(c+a+\mu p))} \left[\frac{f^*(c+a+\mu p)}{(c+a+\mu p)} - \frac{f'^*(c+a+\mu p)}{(1-f^*(c+a+\mu p))} \right] + \frac{\beta}{(1-f^*(b+\mu p))} \left[\frac{f^*(b+\mu p)}{(b+\mu p)} - \frac{f'^*(b+\mu p)}{(1-f^*(b+\mu p))} \right] \right\} \tag{13}$$

The Covariance of X and \hat{S} , $Cov(X, \hat{S}) = E(X \hat{S}) - E(X)E(\hat{S})$ may be seen using (13), (11) and (9).

III. Model II:

In this model we treat the previous model I with all assumptions except the assumption (2) given for sales.

3.1 Assumption for Sales:

(2.1) When the operation time X is more than a threshold time U , the sales are done all together. It is assigned to an agent whose sales time distribution function is $G_1(z)$ with pdf $g_1(z)$.

(2.2) When the operation time X is less than a threshold time U , the sales are done one by one with Cdf $G_2(z)$ and pdf $g_2(z)$.

(2.3) The threshold U has exponential distribution with parameter δ .

3.2 Analysis:

Using the arguments given for model I, we note the joint pdf (X, \hat{R}, \hat{S}) (Operation time, repair-recruitment time, sales time) is as follows.

$$f(x, y, z) = \left[h(x) \left(\sum_{i=0}^{\infty} e^{-\mu x} \frac{(\mu x)^i}{i!} q^i r(y) \odot r_{i_i}(y) \right) + (1 - H(x)) \left(\sum_{i=1}^{\infty} \mu e^{-\mu x} \frac{(\mu x)^{i-1}}{i-1} q^{i-1} pr_{i_i}(y) \right) \right] \sum_{k=0}^{\infty} (F_k(x) - F_{k+1}(x)) [(1 - e^{-\delta x}) g_1(z) + e^{-\delta x} g_{2,k}(z)] \quad (14)$$

The same arguments given for model I may be used for all terms except the last square bracket where sales time pdf is $g_1(z)$ when $(X > U)$, the operation time is greater than the threshold and when the operation time is less than the threshold $(X < U)$, the k products are sold one by one with sales time pdf $g_2(z)$ where the suffix k indicates k -fold convolution.

The triple Laplace transform can be seen as

$$f^*(\xi, \eta, \varepsilon) = \alpha[(c+a)r^*(\eta) + \mu pr_1^*(\eta)] \left\{ g_1^*(\varepsilon) \left(\frac{1}{\theta_1} - \frac{1}{\theta_1 + \delta} \right) + \frac{(1 - f^*(\theta_1 + \delta))}{(\theta_1 + \delta)(1 - g_2^*(\varepsilon)f^*(\theta_1 + \delta))} \right\} + \beta[br^*(\eta) + \mu pr_1^*(\eta)] \left\{ g_1^*(\varepsilon) \left(\frac{1}{\theta_2} - \frac{1}{\theta_2 + \delta} \right) + \frac{(1 - f^*(\theta_2 + \delta))}{(\theta_2 + \delta)(1 - g_2^*(\varepsilon)f^*(\theta_2 + \delta))} \right\} \quad (15)$$

where $\theta_1 = \xi + c + a + \mu(1 - r_1^*(\eta))$ and $\theta_2 = \xi + b + \mu(1 - qr_1^*(\eta))$

$E(X)$ and $E(\hat{R})$ are same for the model II as that of model I. Expected sales time $E(\hat{S})$ becomes

$$E(\hat{S}) = E(G_1) \left[1 - \frac{\alpha(c+a+\mu p)}{c+a+\delta+\mu p} - \frac{\beta(b+\mu p)}{b+\mu p+\delta} \right] + E(G_2) \left[\frac{\alpha(c+a+\mu p)}{(c+a+\delta+\mu p)} \frac{f^*(\delta+c+a+\mu p)}{(1-f^*(\delta+c+a+\mu p))} + \frac{\beta(b+\mu p)}{(b+\mu p+\delta)} \frac{f^*(\delta+b+\mu p)}{(1-f^*(\delta+b+\mu p))} \right] \quad (16)$$

Equation (15) gives the joint Laplace transform of the pdf of (X, \hat{S}) as follows.

$$f^*(\xi, 0, \varepsilon) = g_1^*(\varepsilon) \left[\alpha(c+a+\mu p) \left(\frac{1}{\xi+c+a+\mu p} - \frac{1}{\xi+\delta+c+a+\mu p} \right) + \beta(b+\mu p) \left(\frac{1}{\xi+b+\mu p} - \frac{1}{\xi+\delta+b+\mu p} \right) \right] + \frac{\alpha(c+a+\mu p)}{(\xi+c+a+\mu p+\delta)} \frac{(1-f^*(\xi+\delta+c+a+\mu p))}{(1-g_2^*(\varepsilon)f^*(\xi+\delta+c+a+\mu p))} + \frac{\beta(b+\mu p)}{(\xi+b+\mu p+\delta)} \frac{(1-f^*(\xi+\delta+b+\mu p))}{(1-g_2^*(\varepsilon)f^*(\xi+\delta+b+\mu p))} \quad (17)$$

The product moment $E(X \hat{S})$ is given by

$$E(X \hat{S}) = \frac{\partial^2}{\partial \xi \partial \varepsilon} f^*(\xi, 0, \varepsilon) |_{\xi=0} = \varepsilon$$

$$E(X \hat{S}) = E(G_1) \left[\frac{\alpha}{c+a+\mu p} - \frac{\alpha(c+a+\mu p)}{(\delta+c+a+\mu p)^2} + \frac{\beta}{(b+\mu p)} - \frac{\beta(b+\mu p)}{(\delta+b+\mu p)^2} \right] + E(G_2) \left[\frac{\alpha(c+a+\mu p)}{(\delta+c+a+\mu p)^2} \frac{f^*(\delta+c+a+\mu p)}{(1-f^*(\delta+c+a+\mu p))} - \frac{\alpha(c+a+\mu p)}{(\delta+c+a+\mu p)} \frac{f^*(\delta+c+a+\mu p)}{(1-f^*(\delta+c+a+\mu p))^2} + \frac{\beta(b+\mu p)}{(\delta+b+\mu p)^2} \frac{f^*(\delta+b+\mu p)}{(1-f^*(\delta+b+\mu p))} - \frac{\beta(b+\mu p)}{(\delta+b+\mu p)} \frac{f^*(\delta+b+\mu p)}{(1-f^*(\delta+b+\mu p))^2} \right] \quad (18)$$

$Cov(X, \hat{S}) = E(X \hat{S}) - E(X)E(\hat{S})$ can be written using equations (18), (16) and (9).

IV. Numerical Examples For Model I & II

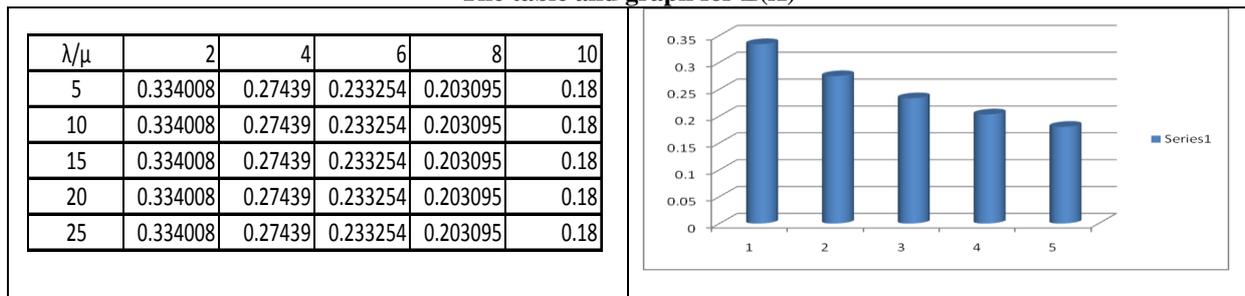
The usefulness of the results obtained is presented by numerical examples. The two models I and II are considered together, since there is only change in sales pattern $E(X)$ and $E(\hat{R})$ are same for models I and II.

4.1 Model I

Let $p = 0.3$, the probability the machine fails employee leaves.
Let $a = 3$, the machine operation time has exponential distribution.

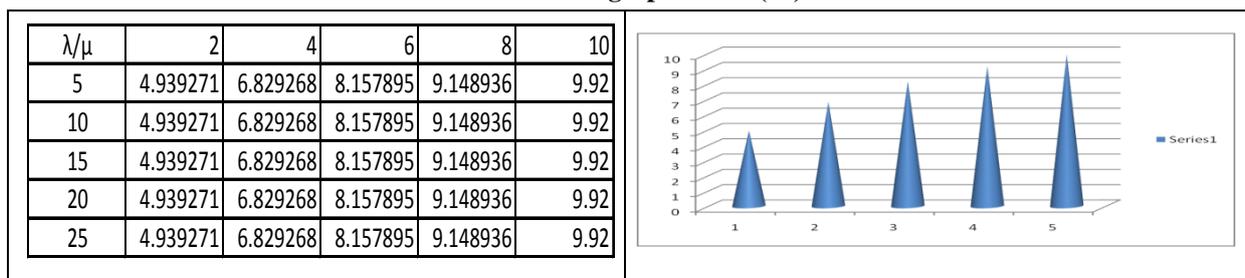
If the machine does not fail within an exponential time with parameter 'c', the parameter 'a' changes to 'b'. Let $b = 2$ and $c = 4$. $\mu = 2, 4, 6, 8, 10$, the inter departure time of employees.
Here $f(\cdot)$ is an exponential density function with parameter ' λ ' and varies from 5 to 25 with an increment of 5.
Let $E(R) = 2$, $E(R_1) = 5$.

The table and graph for $E(X)$



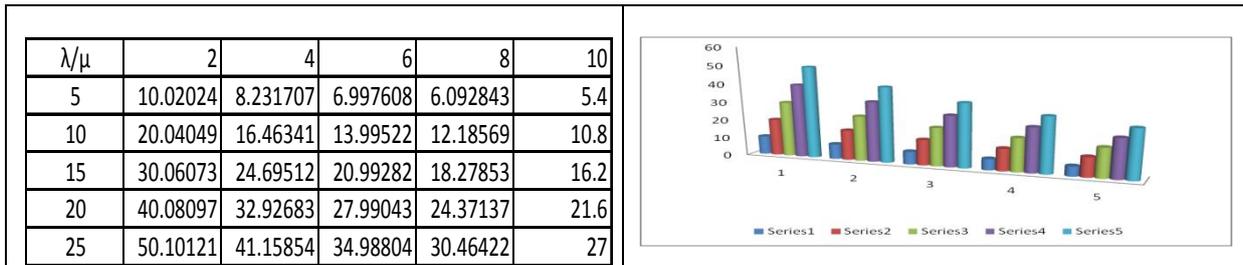
It is observed that when μ increases, the expected operation time $E(X)$ decreases.

The table and graph for $E(\hat{R})$



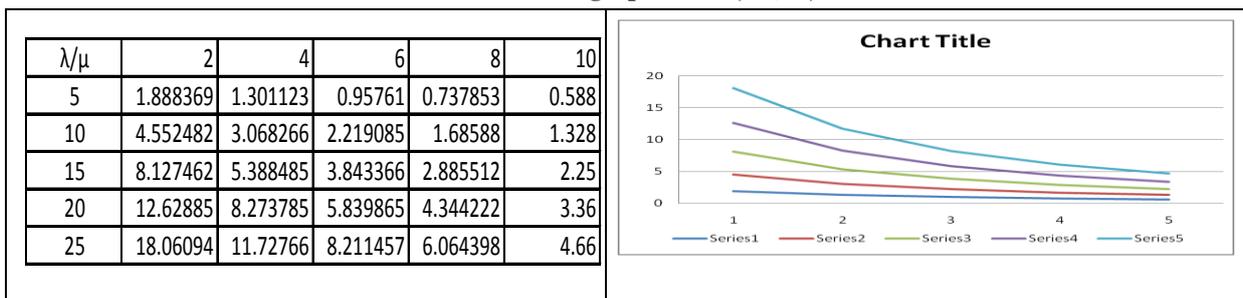
It is observed that when μ increases, the expected repair and recruitment time $E(\hat{R})$ increases.

The table and graph for $E(\hat{S})$



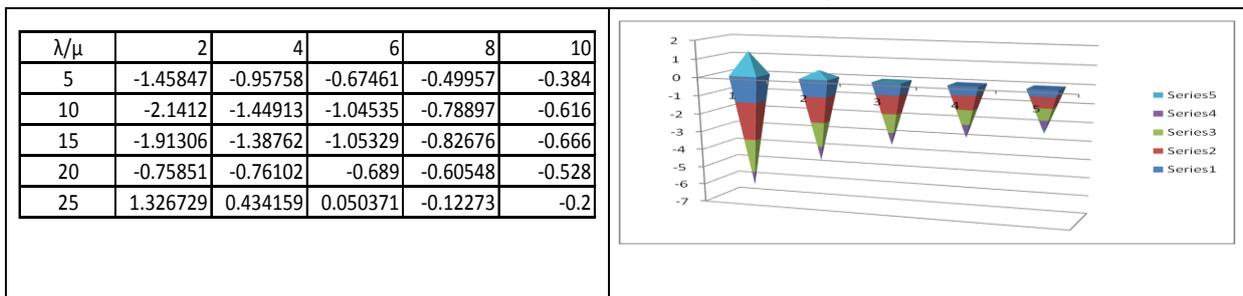
It is observed that when μ increases, the expected sales time $E(\hat{S})$ decreases and when λ increases the expected sales time increases.

The table and graph for $E(X, \hat{S})$



It is observed that when μ increases, the joint moment $E(X, \hat{S})$ decreases and when λ increases $E(X, \hat{S})$ increases.

The table and graph for $Cov(X, \hat{S})$

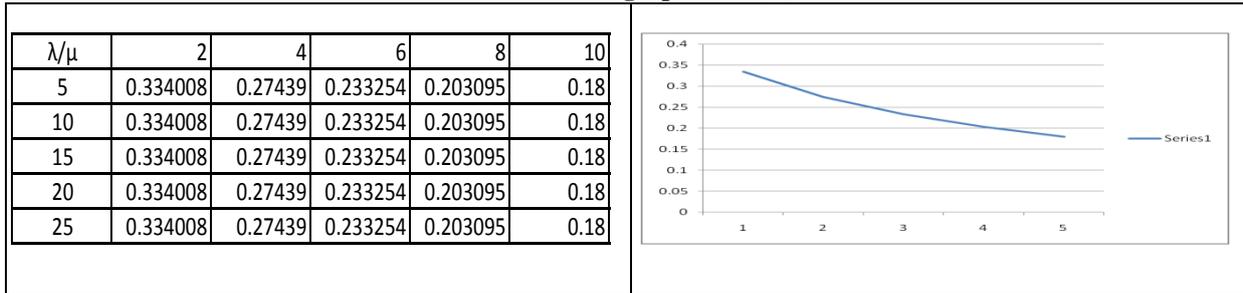


It is observed that when λ increases, the production decreases and the $Cov(X, \hat{S})$ oscillates and becomes positive and therefore sale decreases. When μ increases, the operation time decreases and the Covariance tend to zero.

4.2 Model II

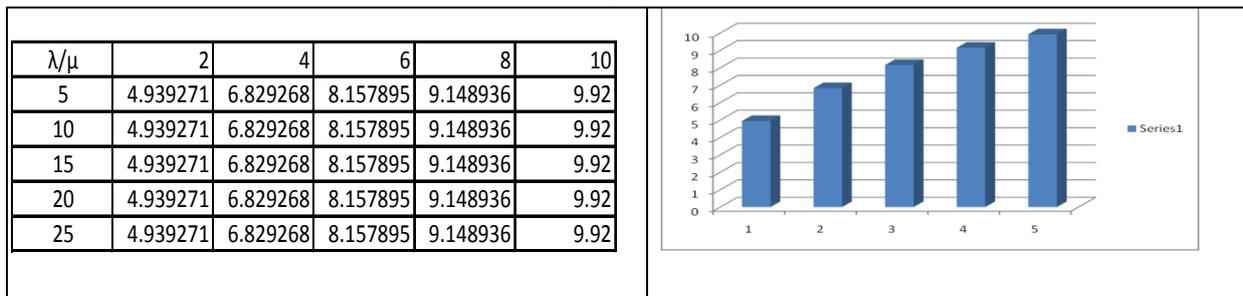
Consider the values as given in model I with additional assumptions for $E(G_1) = 5$, $E(G_2) = 10$ and the threshold has exponential distribution δ and let $\delta = 10$.

The table and graph for $E(X)$



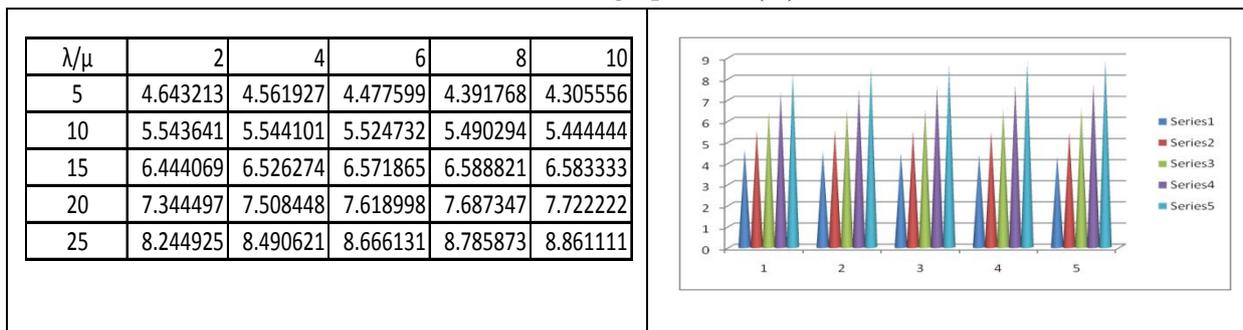
It is observed that when μ increases, the expected operation time $E(X)$ decreases.

The table and graph for $E(\hat{R})$



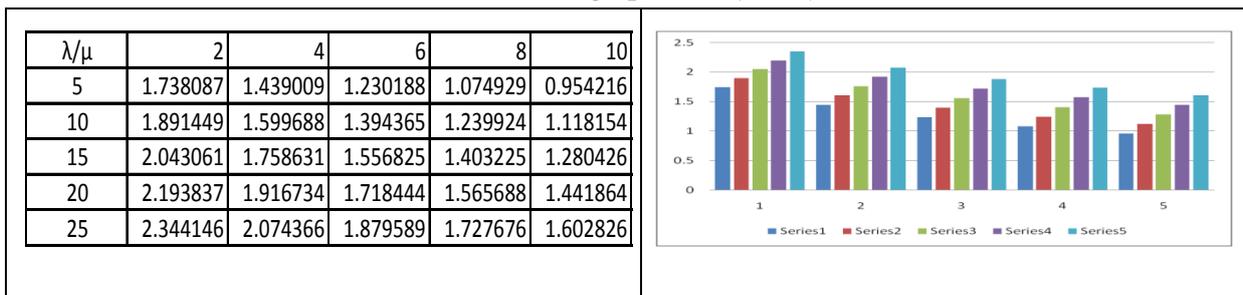
It is observed that when μ increases, the expected repair and recruitment time $E(\hat{R})$ increases.

The table and graph for $E(\hat{S})$



It is observed that when μ increases, the expected sales time $E(\hat{S})$ decreases and when λ increases the expected sales time increases.

The table and graph for $E(X, \hat{S})$



It is observed that when μ increases, the joint moment $E(X, \hat{S})$ decreases and when λ increases the $E(X, \hat{S})$ increases.

The table and graph for $Cov(X, \hat{S})$

λ/μ	2	4	6	8	10
5	0.187217	0.187261	0.185772	0.182983	0.179216
10	0.039828	0.078441	0.105701	0.124873	0.138154
15	-0.10931	-0.03211	0.023914	0.06507	0.095426
20	-0.25929	-0.14351	-0.05871	0.004428	0.051864
25	-0.40973	-0.25538	-0.14182	-0.05669	0.007826

It is observed that when μ increases the operation time may decrease and so the Covariance tends to decrease.

When λ increases, $Cov(X, \hat{S})$ tends to zero and tends to change the sign.

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