IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 11, Issue 2 Ver. I (Mar - Apr. 2015), PP 53-59
www.iosrjournals.org

Transaction exposure risk modelled in a newsvendor framework under the multiplicative demand error

Sanjay Patel[1], Ravi Gor[2]
[1](Mathematics Department, St. Xavier's College, Ahmedabad, India)
[2](Dr. Babasaheb Ambedkar Open University, Ahmedabad, India)

Abstract: *We consider a global supply chain consisting of one retailer and one manufacturer, both from different countries. As there is a time lag between the payments made while placing the order and the time when the order is realized, risk in the form of the exchange rate fluctuation affects the optimal pricing and order quantity decisions. We explore the derivations of analytic expressions involving the transaction exposure when the retailer or manufacturer undertakes to share the exchange rate risk and the demand error is modelled in the multiplicative form in the news vendor framework.*
Keywords: *transaction exposure, exchange rate, global supply chain, newsvendor problem, optimal pricing and quantity*

I. Introduction

Suppose two different countries having different currencies are into a business. When the exchange rate between the two currencies gets an exposure to unexpected changes, there exists a financial risk and this risk is known as foreign exchange risk (or exchange rate risk). A transaction exposure arises only when there exists a time lag between the time of the financial obligation has been incurred and the time its due to be settled. This is because of the purchase price to buyer/ retailer on the settlement day may differ from that when it was incurred, if the debt is denominated in the manufacturer/ supplier currency. Arcelus, Gor and Srinivasan [1] have developed a mathematical model in news vendor framework to find optimum ordering and pricing policies for retailer/manufacturer, when the foreign exchange rate between the two countries doing the business, faces transaction exposure. Our main contribution in this paper is to derive analytic expressions for such a global supply chain within the newsvendor framework and involving the transaction exposure under the general form of the demand with multiplicative error.

II. Literature Review and transaction exposure model

Cases of transaction exposure when a firm has an accounts receivable or payable denominated in a foreign currency has been reported in Goel [2]. The nature of global trade is that either the buyer or the seller has to bear what is commonly known in international finance as transaction exposure, Eitemann et al.[3], Shubita et al.[4]. The very important newsvendor framework introduced by Petruzzi and Dada [5] and the price dependent demand forms in the additive and multiplicative error structures by Mills[6] and Karlin and Carr[7] have been used.

Suppose a retailer/buyer(say in India) wants to order q units from a foreign manufacturer/seller (say in U.S) of a certain product. The retailer does not know the demand (D) of the product, which is uncertain. But it partly depends upon the price(p) and partly random. The fluctuation or error in the demand (i.e. the randomness) can be of various types. In this paper we take the price dependent demand with multiplicative error which can be described as $D(p,\in) = g(p)\cdot \in$,where $\in$ is multiplicative error in the demand and it follows some distribution (say $f(\in)$) with mean μ in some interval [A,B] and g(p) is the deterministic demand. [Generally g(p) is taken as decreasing iso-elastic function of p say, $g(p) = ap^{-b}$ in multiplicative demand error case with the restrictions $a>0$, $b>1$.]

Let the exchange rate be 'r ' in the retailer currency when the order is placed by him [e.g. 1$ = r Rs.]. Let w be the cost of one unit of the product in the manufacturer currency. If the buyer pays immediately then he has to pay wr (Rs) per unit of the product.

But suppose there is a time lag (some fixed period) between the order is placed and the amount is paid for the product when it is acquired by the retailer. Thus there exists transaction exposure exchange rate risk, since the exchange rate (r) may get fluctuate or change. So the buyer has to pay more or less according to the existing rate at the time of the arrival of the product. Generally the fluctuation in the exchange rate r is very small and uncertain. Also the fluctuation in r is always some percentage of r, hence we can take the future exchange rate as, $r + r\in_r = r(1+\in_r)$.[e.g. 1 $ = $r(1+\in_r)$ Rs.]. Note that $\in_r$ is also a random variable together with the random variable D. One can also take the future exchange rate as $r+\in_r$ but we consider the former in the model. The fluctuation $\in_r$ is unknown but its distribution is known (say $\psi(\in_r)$).

If the fluctuation $\in_r$ is positive buyer has to pay more and if it is negative seller will get less. So the question arises here is that who will bare the exchange rate risk? Buyer/retailer OR seller/manufacturer? In this paper we discuss the two situations under the multiplicative demand error. In each case the retailer's optimal policy is to determine the optimum order(q) and selling price(p) of the product so that his expected profit is maximum. At the same time we obtain the manufacturer's optimal policies as well.

III. Assumptions and Notations

The following assumptions are made in the foreign exchange transaction exposure model:

(i) The standard newsvendor problem assumptions apply.
(ii) The global supply chain consists of single retailer- single manufacturer.
(iii) The error in demand is multiplicative.
(iv) Only one of the two-retailer or manufacturer- bears the exchange rate risk.

The following notations are used in the paper:

q= order quantity
p= selling price per unit
D= demand of the product= no. of units required
$\in$= demand error= randomness in the demand.
v= salvage value per unit
s=penalty cost per unit for shortage
c= cost of manufacturing per unit for manufacturer
w_r = purchase cost for retailer
$\in_r$= the exchange rate fluctuation= exchange rate error= randomness in exchange rate
Π= profit function.

Case-1: Retailer bears the exchange Rate Risk

In the case-1 we assume that the retailer bears the exchange rate risk and manufacturer does not. Thus the manufacturer will get w per unit at any point of time and the buyer will have to pay according to the existing exchange rate. So the buyer will be paying $wr(1+\in_r)$ per unit, on the settlement day or when the product is acquired by him. This amount in terms of manufacturer currency is $wr(1+\in_r)/r = w(1+\in_r) = w_r$ (say). Thus w_r is the purchase cost to buyer in seller's currency.

Now the retailer/ buyer will choose the selling price *p* and the order quantity *q* so as to maximize his expected profit. The profit function for the retailer is given by,

$\Pi(p,q)$ = [revenue from *q* items] – [expenses for the *q* items]

$$\Pi(p,q)=\begin{cases}[pD+v(q-D)]-[qw_r] & if\ \ D\le q\ \ (overstocking)\\ [pq]-[s(D-q)+qw_r] & if\ \ D>q\ \ \ \ (shortage)\end{cases} \qquad (1)$$

Note that all the parameters *p, v, s*, w_r are taken in manufacturer's currency and the salvage value *v* is taken as an income from the disposal of each of the *q*-D leftover.

Since the demand, $D(p,\in)=g(p)\cdot\in$ the retailer's profit function (1) for ordering *q* units and keeping selling price *p* is given by,

$$\Pi(p,q)=\begin{cases}[p(g(p)\in)+v\{q-(g(p)\in)\}]-[qw_r] & if\ \ D\le q\\ [pq]-[s(\{g(p)\in\}-q)+qw_r] & if\ \ D>q\end{cases}$$

$$\Rightarrow\Pi(p,q)=\begin{cases}p(g(p)\in)+vq-vg(p)\in-qw_r & if\ \ D\le q\\ pq-s(g(p)\in)+sq-qw_r & if\ \ D>q\end{cases}$$

Put $g(p)=g$ and define $z = q / g(p) = q / g$ i.e. $q=g\,z$, for the multiplicative demand error.

Now $D\le q \Leftrightarrow g\in\le q \Leftrightarrow \in\le q/g \Leftrightarrow \in\le z$ and similarly $D>q \Leftrightarrow \in>z$.

$$\Rightarrow \Pi(z,p)=\begin{cases} pg\in+vgz-vg\in-gzw_r & if\ \in\le z \\ pgz-sg\in+sgz-gzw_r & if\ \in> z \end{cases}$$

$$\Rightarrow \Pi(\mathrm{z},p)=\begin{cases} pg\in+vg(z-\in)-gzw_r & if\ \in\le z \\ pgz-sg(\in-\mathrm{z})-gzw_r & if\ \in> z \end{cases} \tag{2}$$

The equation (2) describes the profit function for the retailer in the manufacturer currency. Note that the parameter *q* is replaced by *z*. Now the retailer wants to find the optimal order quantity q say q* and optimal price p= p* to maximize his expected profit. In order to do this he must find optimal values of the price p and the parameter z, say p* and z* respectively which maximizes his expected profit so that he can determine the optimal order q*=z*·g(p*). The profit Π is a function of the random variable ∈ with support [A, B]. Thus the retailer's expected profit is given by,

$$E\left[\Pi(z,p)\right]=\int_A^B \Pi(z,p)f(\in)d\in$$

$$\Rightarrow E\left[\Pi(z,p)\right]=\int_A^Z \Pi(z,p)f(\in)d\in+\int_Z^B \Pi(z,p)f(\in)d\in \text{ , as } A\le z\le B$$

Take ∈=*u* for simplicity in (2) and then writing the expected profit we get,

$$E[\Pi(z,p)]=\int_A^z[pgu+vg(z-u)-gzw_r]\cdot f(u)\,du+\int_z^B[pgz-sg(u-z)-gzw_r]\cdot f(u)\,du$$

$$=\int_A^z[pgu+vg(z-u)]\cdot f(u)\,du+\int_z^B[pgz-sg(u-z)]\cdot f(u)\,du-gzw_r\left[\int_A^z f(u)du+\int_z^B f(u)du\right]$$

$$\Rightarrow E[\Pi(z,p)]=\int_A^z[pgu+vg(z-u)]\cdot f(u)\,du+\int_z^B[pgz-sg(u-z)]\cdot f(u)\,du-gzw_r \tag{3}$$

Define $\Lambda(z)=\int_A^z (z-u)f(u)du$ [expected leftovers] and

$\Phi(z)=\int_Z^B (\mathrm{u}-z)f(u)du$ [expected shortages].

Also put $X=(g\mu)(p-w_r)$ [riskless profit as it does not contain ∈] and

$$Y=g[(w_r-v)\Lambda+(p+s-w_r)\Phi] \text{ [loss function]} \tag{4}$$

Here $\mu=\int_A^B uf(u)du$ in the equation (4) and it gives the expected value of the randomness *u* in the demand D.

Now we shall show that $E\left[\Pi(z,p)\right]=X-Y$.

Consider X – Y

$$=[(g\mu)(p-w_r)]-g[(w_r-v)\Lambda+(p+s-w_r)\Phi]$$

$$=p\,\mathrm{g}\,\mu-w_r\,\mathrm{g}\,\mu-g(w_r-v)\Lambda-g(p+s-w_r)\Phi$$

$$=p\,\mathrm{g}\int_A^B uf(u)du-w_r\,\mathrm{g}\int_A^B uf(u)du-(\mathrm{g}\,w_r-gv)\int_A^z(\mathrm{z}-\mathrm{u})f(u)du-(\mathrm{g}\,p+gs-gw_r)\int_z^B(\mathrm{u}-\mathrm{z})f(u)du$$

$$= p\,\mathrm{g}\int_A^B uf(u)du - w_r\,\mathrm{g}\int_A^B uf(u)du - \mathrm{g}\,w_r\int_A^z (\mathrm{z}-\mathrm{u})\,f(u)du + gv\int_A^z (\mathrm{z}-\mathrm{u})\,f(u)du$$

$$-\mathrm{gp}\int_z^B (\mathrm{u}-\mathrm{z})\,f(u)du - gs\int_z^B (\mathrm{u}-\mathrm{z})\,f(u)du + gw_r\int_z^B (\mathrm{u}-\mathrm{z})\,f(u)du$$

$$= p\,\mathrm{g}\left\{\int_A^z uf(u)du + \int_z^B uf(u)du\right\} - w_r\,\mathrm{g}\int_A^B uf(u)du - \mathrm{g}\,w_r\int_A^z (\mathrm{z}-\mathrm{u})\,f(u)du + gv\int_A^z (\mathrm{z}-\mathrm{u})\,f(u)du$$

$$-\mathrm{gp}\int_z^B (\mathrm{u}-\mathrm{z})\,f(u)du - gs\int_z^B (\mathrm{u}-\mathrm{z})\,f(u)du + gw_r\int_z^B (\mathrm{u}-\mathrm{z})\,f(u)du$$

$$= p\,\mathrm{g}\int_A^z uf(u)du + p\,\mathrm{g}\int_z^B uf(u)du - w_r\,\mathrm{g}\int_A^B uf(u)du - \mathrm{g}\,w_r\int_A^z (\mathrm{z}-\mathrm{u})\,f(u)du + gv\int_A^z (\mathrm{z}-\mathrm{u})\,f(u)du$$

$$-\mathrm{gp}\int_z^B (\mathrm{u}-\mathrm{z})\,f(u)du - gs\int_z^B (\mathrm{u}-\mathrm{z})\,f(u)du + gw_r\int_z^B (\mathrm{u}-\mathrm{z})\,f(u)du$$

$$= \int_A^z [pgu + gv(z-u)]\,f(u)du + \int_z^B [pgu - pg(u-z) - sg(u-z)]\,f(u)du$$

$$-w_r g\int_A^B uf(u)du - gw_r\left\{\int_A^z (z-u)\,f(u)du + \int_z^B (z-u)\,f(u)du\right\}$$

$$= \int_A^z [pgu + gv(z-u)]\,f(u)du + \int_z^B [pgu - pg(u-z) - sg(u-z)]\,f(u)du - w_r g\int_A^B uf(u)du - gw_r\int_A^B (z-u)\,f(u)du$$

$$= \int_A^z [pgu + gv(z-u)]\,f(u)du + \int_z^B [pgu - pg(u-z) - sg(u-z)]\,f(u)du - gw_r\int_A^B [u + (z-u)]\,f(u)du$$

$$= \int_A^z [pgu + gv(z-u)]\,f(u)du + \int_z^B [pgu - pg(u-z) - sg(u-z)]\,f(u)du - gzw_r\int_A^B f(u)du$$

$$= \int_A^z [pgu + gv(z-u)]\,f(u)du + \int_z^B [pgz - sg(u-z)]\,f(u)du - gzw_r$$

$$= E\left[\Pi(z,p)\right] \qquad \text{(from (3))}$$

Hence we have proved that $X - Y = E\left[\Pi(z,p)\right]$.

$$\Rightarrow E\left[\Pi(z,p)\right] = [(g\mu)(p - w_r)] - g[(w_r - v)\Lambda + (p + s - w_r)\Phi] \tag{5}$$

The equation (5) represents the expected profit of the retailer as a function of *z* and *p*. We use Whitin's method [8] to maximize the expected profit function. In this method first we keep *p* fixed in (5) and use the second order optimality conditions $\frac{\partial E}{\partial z} = 0$ and $\frac{\partial^2 E}{\partial z^2} < 0$ to find the optimum value of z* as a function of *p*. Then we substitute the value of z* in the expected profit (5) so that it becomes a function of single variable *p* and hence the optimal p* can also be obtained.

Now $E\left[\Pi(z,p)\right] = (g\mu)(p - w_r) - g[(w_r - v)\Lambda + (p + s - w_r)\Phi]$. Differentiate partially w.r.t. *z*.

$$\Rightarrow \frac{\partial E}{\partial z} = 0 - g(w_r - v)\frac{\partial \Lambda}{\partial z} - \mathrm{g}(p + s - w_r)\frac{\partial \Phi}{\partial z}$$

But $\Lambda(z) = \int_A^z (\mathrm{z}-\mathrm{u})\, f(u)du \Rightarrow \frac{\partial \Lambda}{\partial z} = \int_A^z (1)\, f(u)\, du + 0 - 0 = \int_A^z f(u)\, du$ and

$$\Phi(z) = \int_z^B (\mathrm{u}-\mathrm{z})\, f(u)du \Rightarrow \frac{\partial \Phi}{\partial z} = \int_z^B (-1)\, f(u)\, du + 0 - 0 = -\int_z^B f(u)\, du .$$

$$\Rightarrow \frac{\partial E}{\partial z} = -g(w_r - v)\int_A^z f(u)du + \mathrm{g}(p + s - w_r)\int_z^B f(u)du$$

$$\Rightarrow \frac{\partial E}{\partial z} = -g(w_r - v)\left\{\int_A^z f(u)\, du + \int_z^B f(u)\, du - \int_z^B f(u)\, du\right\} + \mathrm{g}(p + s - w_r)\int_z^B f(u)du$$

$$\Rightarrow \frac{\partial E}{\partial z} = -g(w_r - v)\int_A^B f(u)\, du + g(w_r - v)\int_z^B f(u)\, du + \mathrm{g}(p + s - w_r)\int_z^B f(u)du$$

$$\Rightarrow \frac{\partial E}{\partial z} = -g(w_r - v) + g\int_z^B [(w_r - v) + (p + s - w_r)]\, f(u)\, du$$

$$\Rightarrow \frac{\partial E}{\partial z} = -g(w_r - v) + g\int_z^B (p + s - v)\, f(u)\, du$$

$$\Rightarrow \frac{\partial E}{\partial z} = -g(w_r - v) + g(p + s - v)\int_z^B f(u)\, du$$

$\rightarrow$If we use the CDF $F(z) = \int_A^z f(u)\, du$ then $1 - F(z) = \int_z^B f(u)\, du$.

$$\Rightarrow \frac{\partial E}{\partial z} = -g(w_r - v) + g(p + s - v)[1 - F(z)] \tag{6}$$

Again differentiating w.r.t z we get,

$$\frac{\partial^2 E}{\partial z^2} = (p + s - v)\left[-\frac{\partial F}{\partial z}\right]$$

$$\Rightarrow \frac{\partial^2 E}{\partial z^2} = -(p + s - v)\, f(z) \tag{7}$$

For optimal value of the expected profit we must have $\frac{\partial E}{\partial z} = 0$.

$\Rightarrow -g(w_r - v) + g(p + s - v)[1 - F(z)] = 0$ [from(6)]

$$\Rightarrow g(p + s - v)[1 - F(z)] = g(w_r - v)$$

$$\Rightarrow F(z) = 1 - \frac{(w_r - v)}{(p + s - v)}$$

$$\Rightarrow F(z) = \frac{p + s - w_r}{p + s - v}$$

$$\Rightarrow z = F^{-1}\left(\frac{p + s - w_r}{p + s - v}\right) = z^* \text{ (say)} \tag{8}$$

This z* gives the optimum solution for maximum profit as a function of *p*, since $\frac{\partial^2 E}{\partial z^2} < 0$ from equation (7). Now substitute this z* in $E[\Pi(z, p)]$ and obtain optimum p* using the second order optimality criteria.

Hence the retailer's optimal order q =q* is given by,

$$q^* = g(p^*)z^* = g(p^*)F^{-1}\left(\frac{p^* + s - w_r}{p^* + s - v}\right) \text{,where } F^{-1} \text{ is the inverse CFD.} \quad (9)$$

Also the manufacturer's profit when the buyer bears the risk is [(selling price of seller)-(cost of purchase to seller)]× no. of units sold. $\Pi_m = (w-c)q^*$.

Case-2: Manufacturer bears the exchange Rate Risk

In the case-2 we assume that the manufacturer bears the exchange rate risk and retailer does not. Thus the retailer pays w per unit in manufacturer's currency at any point of time and the manufacturer will get according to the existing exchange rate. So the manufacturer will be getting $wr/(r(1+\in_r)) = w_m$ per unit on the settlement day in his currency. Now the retailer's profit function, his expected profit and optimal policies to get maximum expected profit can be obtained by replacing wr by w in case-1. So we get the retailer's profit as,

$$\Pi(p,q) = \begin{cases} [pD + v(q-D)] - [qw] & if \ \ D \le q \ (overstocking) \\ [pq] - [s(D-q) + qw] & if \ \ D > q \quad (shortage) \end{cases}$$

And his expected profit as,

$$E[\Pi(z,p)] = [(g\mu)(p-w)] - g[(w-v)\Lambda + (p+s-w)\Phi]$$

The optimal policy z* is given by $z^* = F^{-1}\left(\frac{p^* + s - w}{p^* + s - v}\right)$ and hence the optimum order quantity is,

$$q^* = g(p^*) + z^* = g(p^*)F^{-1}\left(\frac{p^* + s - w}{p^* + s - v}\right) \text{, where } F^{-1} \text{ is the inverse CFD.} \quad (9)$$

Also the manufacturer's profit when the buyer bears the risk is [(selling price of seller)-(cost of purchase to seller)]× no. of units sold. $\Pi_m = (w_m - c)q^*$.

We have the summary for both cases under multiplicative demand error as follows:

	Case-1: Buyer bears the risk	**Case-2: Seller bears the risk**
Demand function	D= g(p)∈	
Selling price to seller (in$)	w	$w_m = wr/(r(1+\in_r))$
Purchase cost to buyer(in $)	w_r = wr(1+$\in_r$)/r =w(1+$\in_r$)	wr
Expected profit of buyer $E[\Pi(z,p)]$	$(g\mu)(p-w_r) - g[(w_r - v)\Lambda + (p+s-w_r)$	$(g\mu)(p-w) - g[(w-v)\Lambda + (p+s-w)\Phi]$
Optimum order quantity q*	$q^* = g(p^*)F^{-1}\left(\frac{p^* + s - w_r}{p^* + s - v}\right)$	$q^* = g(p^*)F^{-1}\left(\frac{p^* + s - w}{p^* + s - v}\right)$
Expected profit of seller Π_m	(w-c)q*	(w_m-c)q*

IV. Conclusion

We consider a global supply chain consisting of one retailer and one manufacturer from different countries in a newsvendor framework and derive analytic expressions when the retailer or supplier bears the transaction exposure risk. The main contribution of the research is the derivation of analytical expressions using a general demand function and the multiplicative inclusion of the demand errors. This work will be useful to carry forward applications of the model to more complex situations using the optimal decisions arrived at. Attempts to formulate the model using more realistic forms of demand could be a possible future work. More sophisticated work using various forms of the error distributions and using simulations could also give useful contribution to the body of literature of exchange rate risks.

References

[1] F.J. Arcelus, Ravi Gor, G. Srinivasan, Foreign exchange transaction exposure in a newsvendor setting, European Journal of Operational Research, 227(2013) 552-557

[2] Goel, M., 2012. Management of transaction exposure: a comparative analysis of MNCs in India. International Journal of Service Science,
Management, Engineering and Technology 3, 37–54.

[3] Eitemann, D.K., Stonehill, A.I., Moffett, M.H., 2010. Multinational Business Finance,12th ed. Prentice Hall, Boston.

[4] Shubita, M.F., Harris, P., Malindretos, J., Bobb, L.M., 2011. Foreign exchange exposure: an overview. International Research Journal of Finance and Economics 78, 171–177.

[5] Petruzzi, N.C., Dada, M., 1999. Pricing and the newsboy problem: a review with extensions. Operations Research 47, 183–194.

[6] Mills, E.S., 1958. Uncertainty and price theory. Quarterly Journal of Economics 73, 116–130.

[7] Karlin, S., Carr, C.R., 1962. Prices and optimal inventory policy. In: Arrow, K.J., Scarf, H. (Eds.), Studies in Applied Probability and Management Science. Stanford University Press, Palo Alto, CA, pp. 159–172.

[8] Whitin, T. M., 1955. Inventory control and price theory, Management Sci. 2 61-68.