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# Analysis of A Novel Chaotic Dynamic System withTen quadratic nonlinearities

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***Abstract:****In this paper introduces a novel non-linear ten-dimensional autonomous system which has ten quadratic nonlinearities and twelve positive real constant parameters , complex chaotic dynamics behaviors and gives analysis of novel system. More importantly. We analyze the novel system by means of phase portraits , equilibrium points , calculated Lyapunov exponents, fractional dimension and attractors of the system. The qualitative properties and the phase portraits of the novel chaotic system have been described in detail. Numerical simulations using MATEMATICA are provided to illustrate phase portraits and the qualitative properties of the novel chaotic system.*

***Keywords:****numerical simulation, ten-dimensional chaotic system, chaotic attractor andLyapunov exponents.*

## I. Introduction

Chaos analysis and applications in dynamical systems are observed in many practical applications in engineering, computer cryptography [1-2].Since Lorenz found the first chaotic attractor in a three dimensional autonomous system in 1963, the three-dimensional chaotic system has been a focal point of study for many researchers in the past few decades. The chaotic characteristics of the hyperchaotic system are more complex Thus, to use the hyperchaotic system signal as the encrypted signal has more extensive application prospect [3-6]. In [7-10], the four-dimensional system, five-dimensional system, six-dimensional, seven-dimensional system and their realizing circuits are given, which lays a foundation of the construction of the higher dimensional hyperchaotic system[11] . In this paper, we introduce a novel 10-D chaotic system with ten quadratic nonlinearities and discuss its qualitative properties. The new 10-D chaotic system has three unstable equilibrium points. It is organized as follows: Section 2 Establishment of the Novel ten-dimensional Dynamic System; Section 3Dynamics Analysis of the System .Section 4 Observation of Chaotic and Complex Dynamics discusses basic properties of the system and gives numerical results. Simulation results show that the system can generate complex chaotic attractors when the system parameters are chosen appropriable. Section 4 Waveform analysis of the novel chaotic system. Sensitivity to initial conditions are presented in Section 6.

## II. Establishment of the Novel ten-dimensional Dynamic System

The novel ten-dimensional autonomous system is obtained as follows:

$$
\begin{aligned}
\frac{dx}{dt} &= \sigma(y-x)-\rho(zs+w)\\
\frac{dy}{dt} &= \delta x - xz - y\\
\frac{dz}{dt} &= xy - \eta z\\
\frac{du}{dt} &= -v + \eta sq - u\\
\frac{dv}{dt} &= -\lambda(v+wq)-u \qquad\qquad (1)\\
\frac{dw}{dt} &= \gamma x + zx\\
\frac{dp}{dt} &= \mu r\\
\frac{dq}{dt} &= -\mu(q+pr)\\
\frac{dr}{dt} &= -\varphi p + qx + \xi qs\\
\frac{ds}{dt} &= -\beta pr - \omega(s-p)
\end{aligned}
$$

Where x, y, z, u, v, u, w, p, q, r and s are the states of system and $\sigma,\rho,\delta,\gamma,\eta,\lambda,\gamma,\mu,\varphi,\xi,\beta$ and $\omega$ are real positive parameters of the system.

The 10-D system (1) exhibits a chaotic attractor, when the system parameter values are chosen as:

$\sigma = 20, \rho = 2.1, \delta = 15, \eta = 2, \lambda = 8, \gamma = 10, \ \mu = 5, \ \varphi = 25, \xi = 5.1, \beta = 1, \omega = 1.9$ *(2)*

We take the initial conditions as:

$x(0) = -1, y(0) = 4, z(0) = 1, u(0) = 0, v(0) = 0, w(0) = 1, p(0) = 0,$

$q(0) = -1, r(0) = 8, s(0) = 5.$

## III. Dynamics Analysis of the System

In this section, basic properties and complex dynamics of the new system (1) are investigated, such as Equilibrium Point, Lyapunov exponents, fractal dimensions, and attractors. The novel dynamic system has the following basic properties.

### 3.1. Equilibrium Point

We can obtain that the system (1) has three equilibrium points:

$E_0$ (x=0 ,y=0 , z=0 ,u=0 , v=0 , w=0 , p=0 , q=0 , r=0 , s=0 )

$E_1$(x= $\frac{-\delta+\sqrt{\delta^2-4\gamma^2\eta\sigma}}{2\gamma}$ ,y= $-\gamma\eta$ ,z= $\frac{\delta-\sqrt{\delta^2-4\gamma^2\eta\sigma}}{2}$ ,u=0 ,v=0,w= $\frac{\frac{\delta\sigma}{\gamma}-2\gamma\eta\sigma-\frac{\sqrt{\delta^2-4\gamma^2\eta\sigma}}{\gamma}}{2\rho}$ , p=0 , q=0 , r=0 , s=0 )

$E_2$(x= $\frac{-\delta-\sqrt{\delta^2-4\gamma^2\eta\sigma}}{2\gamma}$ ,y= $-\gamma\eta$,z= $\frac{\delta+\sqrt{\delta^2-4\gamma^2\eta\sigma}}{2}$ ,u=0,v=0 , w= $\frac{\frac{\delta\sigma}{\gamma}-2\gamma\eta\sigma+\frac{\sqrt{\delta^2-4\gamma^2\eta\sigma}}{\gamma}}{2\rho}$ , p=0 , q=0 , r=0 , s=0 )

When $\sigma = 20, \rho = 2.1, \delta = 15, \eta = 2, \lambda = 8, \gamma = 10, \ \mu = 5, \ \varphi = 25, \xi = 5.1, \beta = 1, \omega = 1.9$

The three equilibrium points becomes *:*

$E_0${u=0 ,w=0 ,y=0 ,v=0 ,q=0 ,z=0 ,x=0 ,r=0 ,s=0 ,p=0},

$E_1${u=0 , w=-183.333-11.4186i , y=-20 , v=0 , q=0 , z=7.5-11.9896i , x=-0.75+1.19896i ,r=0 ,s=0 ,p=0} and

$E_2${u=0 ,w=-183.333+11.4186i ,y=-20 ,v=0 ,q=0 ,z=7.5+11.9896i , x=-0.75-1.19896i ,r=0 ,s=0 ,p=0}

The Jacobian matrix of system (1) , let

$$f = \begin{cases} f1 = \frac{dx}{dt} = \sigma(y-x) - \rho(zs+w) \\ f2 = \frac{dy}{dt} = \delta x - xz - y \\ f3 = \frac{dz}{dt} = xy - \eta z \\ f4 = \frac{du}{dt} = -v + \eta sq - u \\ f5 = \frac{dv}{dt} = -\lambda(v + wq) - u \\ f6 = \frac{dw}{dt} = \gamma x + zx \\ f7 = \frac{dp}{dt} = \mu r \\ f8 = \frac{dq}{dt} = -\mu(q + pr) \\ f9 = \frac{dr}{dt} = -\varphi p + qx + \xi qs \\ f10 = \frac{ds}{dt} = -\beta pr - \omega(s-p) \end{cases} \tag{3}$$

$$J=\begin{bmatrix}
\frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} & \frac{\partial f_1}{\partial z} & \frac{\partial f_1}{\partial u} & \frac{\partial f_1}{\partial v} & \frac{\partial f_1}{\partial w} & \frac{\partial f_1}{\partial p} & \frac{\partial f_1}{\partial q} & \frac{\partial f_1}{\partial r} & \frac{\partial f_1}{\partial s}\\
\frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} & \frac{\partial f_2}{\partial z} & \frac{\partial f_2}{\partial u} & \frac{\partial f_2}{\partial v} & \frac{\partial f_2}{\partial w} & \frac{\partial f_2}{\partial p} & \frac{\partial f_2}{\partial q} & \frac{\partial f_2}{\partial r} & \frac{\partial f_2}{\partial s}\\
\frac{\partial f_3}{\partial x} & \frac{\partial f_3}{\partial y} & \frac{\partial f_3}{\partial z} & \frac{\partial f_3}{\partial u} & \frac{\partial f_3}{\partial v} & \frac{\partial f_3}{\partial w} & \frac{\partial f_3}{\partial p} & \frac{\partial f_3}{\partial q} & \frac{\partial f_3}{\partial r} & \frac{\partial f_3}{\partial s}\\
\frac{\partial f_4}{\partial x} & \frac{\partial f_4}{\partial y} & \frac{\partial f_4}{\partial z} & \frac{\partial f_4}{\partial u} & \frac{\partial f_4}{\partial v} & \frac{\partial f_4}{\partial w} & \frac{\partial f_4}{\partial p} & \frac{\partial f_4}{\partial q} & \frac{\partial f_4}{\partial r} & \frac{\partial f_4}{\partial s}\\
\frac{\partial f_5}{\partial x} & \frac{\partial f_5}{\partial y} & \frac{\partial f_5}{\partial z} & \frac{\partial f_5}{\partial u} & \frac{\partial f_5}{\partial v} & \frac{\partial f_5}{\partial w} & \frac{\partial f_5}{\partial p} & \frac{\partial f_5}{\partial q} & \frac{\partial f_5}{\partial r} & \frac{\partial f_5}{\partial s}\\
\frac{\partial f_6}{\partial x} & \frac{\partial f_6}{\partial y} & \frac{\partial f_6}{\partial z} & \frac{\partial f_6}{\partial u} & \frac{\partial f_6}{\partial v} & \frac{\partial f_6}{\partial w} & \frac{\partial f_6}{\partial p} & \frac{\partial f_6}{\partial q} & \frac{\partial f_6}{\partial r} & \frac{\partial f_6}{\partial s}\\
\frac{\partial f_7}{\partial x} & \frac{\partial f_7}{\partial y} & \frac{\partial f_7}{\partial z} & \frac{\partial f_7}{\partial u} & \frac{\partial f_7}{\partial v} & \frac{\partial f_7}{\partial w} & \frac{\partial f_7}{\partial p} & \frac{\partial f_7}{\partial q} & \frac{\partial f_7}{\partial r} & \frac{\partial f_7}{\partial s}\\
\frac{\partial f_8}{\partial x} & \frac{\partial f_8}{\partial y} & \frac{\partial f_8}{\partial z} & \frac{\partial f_8}{\partial u} & \frac{\partial f_8}{\partial v} & \frac{\partial f_8}{\partial w} & \frac{\partial f_8}{\partial p} & \frac{\partial f_8}{\partial q} & \frac{\partial f_8}{\partial r} & \frac{\partial f_8}{\partial s}\\
\frac{\partial f_9}{\partial x} & \frac{\partial f_9}{\partial y} & \frac{\partial f_9}{\partial z} & \frac{\partial f_9}{\partial u} & \frac{\partial f_9}{\partial v} & \frac{\partial f_9}{\partial w} & \frac{\partial f_9}{\partial p} & \frac{\partial f_9}{\partial q} & \frac{\partial f_9}{\partial r} & \frac{\partial f_9}{\partial s}\\
\frac{\partial f_{10}}{\partial x} & \frac{\partial f_{10}}{\partial y} & \frac{\partial f_{10}}{\partial z} & \frac{\partial f_{10}}{\partial u} & \frac{\partial f_{10}}{\partial v} & \frac{\partial f_{10}}{\partial w} & \frac{\partial f_{10}}{\partial p} & \frac{\partial f_{10}}{\partial q} & \frac{\partial f_{10}}{\partial r} & \frac{\partial f_{10}}{\partial s}
\end{bmatrix} \tag{4}$$

$$J=\begin{bmatrix}
-\sigma & \sigma & -\rho s & 0 & 0 & -\rho & 0 & 0 & 0 & -\rho z\\
\delta-z & -1 & -x & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
y & x & -\eta & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & -1 & -1+\eta s & 0 & 0 & 0 & 0 & \eta v\\
0 & 0 & 0 & -1 & -\lambda & -\lambda r & 0 & 0 & -\lambda w & 0\\
\gamma & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu & 0\\
0 & 0 & 0 & 0 & 0 & 0 & -\mu r & -\mu & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & -\varphi & 1 & \xi s & \xi r\\
0 & 0 & 0 & 0 & 0 & 0 & -\beta r+\omega & 0 & -\beta p & -\omega
\end{bmatrix} \tag{5}$$

For equilibrium point $E_0${u=0 ,w=0 ,y=0 ,v=0 ,q=0 ,z=0 ,x=0 ,r=0 ,s=0 ,p=0},
and $\sigma=20, \rho=2.1, \delta=15, \eta=2, \lambda=8, \gamma=10,\ \mu=5,\ \varphi=25, \xi=5.1, \beta=1, \omega=1.9$
, the Jacobian matrix has the following result:

$$J_0=\begin{bmatrix}
-20 & 20 & 0 & 0 & 2.1 & 0 & 0 & 0 & 0 & 0\\
15 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & -1 & -8 & 0 & 0 & 0 & 0 & 0\\
10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -5 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & -25 & 1 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 1.9 & 0 & 0 & -1.9
\end{bmatrix} \tag{6}$$

To gain its eigenvalues, let **[$\lambda I$-$J_0$]**=0, Then the eigenvalues that corresponding to equilibrium $E_0$(0, 0, 0,0,0,0,0,0,0,0) are respectively obtained as follows:
$\lambda_1$ =-30.4981, $\lambda_2$ =7.77156×$10^{-16}$+11.1803 i, $\lambda_3$ =7.77156×$10^{-16}$-11.1803 i, $\lambda_4$ =9.53245, $\lambda_5$ =-8.14005, $\lambda_6$ =-5., $\lambda_7$ =-2., $\lambda_8$ =-1.9, $\lambda_9$ =-0.859945 and $\lambda_{10}$ =-0.0343972.
Therefore, the equilibrium $E_0$(0, 0, 0,0,0,0,0,0,0,0) is a saddle point.
So, and the hyperchaotic system is unstable at the point $E_0$.

At the same time, it is easy to prove that both the equilibrium points $E1$ and $E2$ are also unstable saddle points.
In the same way, the eigenvalues that corresponding to equilibrium point $E_2$ are obtained as:
$\lambda_1$ = -25.1424+7.20292 i , $\lambda_2$ =4.95248 -10.0364 i , $\lambda_3$ =3.10862×$10^{-15}$+11.1803i ,

$\lambda_4$ =1.38778×10$^{-15}$-11.1803i, $\lambda_5$ =-8.14005+8.12227×10$^{-16}$i, $\lambda_6$ =-5., $\lambda_7$ =-2.82956+2.86132i, $\lambda_8$ =-1.9, $\lambda_9$ = -0.859945+1.86974×10$^{-16}$ i and $\lambda_{10}$ = 0.0194993 -0.0278046 i.

The eigenvalues that corresponding to equilibrium point $E2$ are obtained as :
$\lambda_1$ = -25.1424-7.20292 i, $\lambda_2$ =4.95248 +10.0364 i, $\lambda_3$ =3.10862×10$^{-15}$+11.1803 i, $\lambda_4$ =3.10862×10$^{-15}$-11.1803 i, $\lambda_5$ =-8.14005-8.12227×10$^{-16}$ i, $\lambda_6$ =-5, $\lambda_7$ =-2.82956-2.86132 i, $\lambda_8$ =-1.9, $\lambda_9$ = -0.859945-1.86974×10$^{-16}$ i and $\lambda_{10}$ = 0.0194993 +0.0278046 i.
For each of the two equilibrium points $E1$ and $E\ 2$ , the results show that λ6 and λ8are a negative real numbers, λ3 and λ4 become a pair of complex conjugate eigenvalues with positive real parts. Therefore, equilibrium points *E1* and $E2$ are all saddle-focus points; so, these equilibrium points are all unstable.

**3.2 Dissipativity**
The system (1) can be expressed in vector notation as ( 3) :

The divergence of the vector field *f* on R$^{10}$ is given by

$$\nabla \cdot f = \frac{\partial f1}{\partial x} + \frac{\partial f2}{\partial y} + \frac{\partial f3}{\partial z} + \frac{\partial f4}{\partial u} + \frac{\partial f5}{\partial v} + \frac{\partial f6}{\partial w} + \frac{\partial f7}{\partial p} + \frac{\partial f8}{\partial q} + \frac{\partial f9}{\partial r} + \frac{\partial f10}{\partial s} \qquad (7)$$

We note that $\nabla \cdot f$ measures the rate at which volumes change under the flow $\Phi_t$ of ***f*** .
Let ***D*** be a region in **R$^{10}$**with a smooth boundary and let ***D(t)=***$\Phi_t(t)$ ,the image of ***D*** under $\Phi_t$ ,the time t of the flow of f . let ***V(t)*** be the volume of ***D(t)***. By Liouville's theorem, we get :

$$\frac{dV}{dt} = \int_{D(t)} (\nabla \cdot f)\, dxdydzdudvdwdpdqdrds \qquad (8)$$

For the system (1), we find that

$$\nabla \cdot f = \frac{\partial f1}{\partial x} + \frac{\partial f2}{\partial y} + \frac{\partial f3}{\partial z} + \frac{\partial f4}{\partial u} + \frac{\partial f5}{\partial v} + \frac{\partial f6}{\partial w} + \frac{\partial f7}{\partial p} + \frac{\partial f8}{\partial q} + \frac{\partial f9}{\partial r} + \frac{\partial f10}{\partial s}$$

$$= -(\sigma + \eta + \lambda + \mu + \omega + 2) < 0 \qquad (9)$$

because $\sigma, \eta, \lambda, \mu$ and $\omega$ are positive constants.
Substituting (9) into (8) and simplifying, we get

$$\frac{dV}{dt} = -(\sigma + \eta + \lambda + \mu + \omega + 2) \int_{D(t)} dxdydzdudvdwdpdqdrds = -(\sigma + \eta + \lambda + \mu + \omega + 2)\,V(t) \qquad (10)$$

Solving the first order linear differential equation(10),we obtain the unique solution

$$V(t) = V(0)e^{-(\sigma+\eta+\lambda+\mu+\omega+2)\,t} = V(0)\, e^{-38.9t} \qquad (11)$$

Eq(11) shows that any volume V(t) must shrink exponentially fast to zero with time .Thus, the dynamical system described by (1) is a dissipative system.
As (1) is a dissipative system , all obits of the system (1) are eventually confined to a specific of ***R$^{10}$*** that has zero volume.Hence , the asymptotic motion of system (1) settles onto an attractor of the system(1).

**3.3 Lyapunov Exponents and Lyapunov Dimensions.**
According to the nonlinear dynamical theory, a quantitative measure approach of the sensitive dependence on the initial conditions is calculating the Lyapunov exponent. It is the average rate of divergence (or convergence) of two neighboring trajectories. Moreover, the ten Lyapunov exponents of the nonlinear dynamical system (1) with parameters

$$\sigma = 20, \rho = 2.1, \delta = 15, \eta = 2, \lambda = 8, \gamma = 10, \mu = 5, \varphi = 25, \xi = 5.1, \beta = 1, \omega = 1.9$$

Are obtained as follows:
$L_1$= 18.94059 , $L_2$= 9.96383 , $L_3$= 1.00877 , $L_4$= 0.828434 , $L_5$= 0.0490522 , $L_6$= -0.0132193 , $L_7$= -0.0646262 , $L_8$= -1.00973 , $L_9$= -28.8771 , $L_{10}$= -39.729.
It can be seen that the largest Lyapunov exponent is $L_1$= 18.94059 , indicating that the system has chaotic characteristics. Since the $L_1$,$L_2$,$L_3$,$L_4$ and $L_5$ are a positive Lyapunov exponent, and the rest five Lyapunov exponents are negative. Thus, the system is hyperchaotic. The fractal dimension is also a typical characteristic of chaos calculated Kaplan-Yorke dimension by Lyapunov exponents, and $D_{KY}$ can be expressed as:

$$D_{KY} = j + \frac{1}{|L_{j+1}|}\sum_{i=1}^{j} L_i \qquad (12)$$

where j says the first j Lyapunov exponent is nonnegative , namely, j is the maximum value of $i$ value which meets both $\sum_{i=1}^{j} L_i >> 0$ and $\sum_{i=1}^{j} L_i < 0$ at the same time. $L_i$ is in descending order of the sequence according to the sequence of Lyapunov exponents. $D_{KY}$ is the upper bound of the dimension of the system information. For the system in this paper, by observing the values of ten Lyapunov exponents in the above, we determine that the value of j is nine , and then the Kaplan-Yorke dimension can be expressed from the above due to $L_1+L_2+L_3+L_4+L_5+L_6+L_7+L_8+L_9>0$

and

$L_1+L_2+L_3+L_4+L_5+L_6+L_7+L_8++L_{10}<0$, the Lyapunov dimension of the novel chaotic system is

$$D_{KY} = j + \frac{1}{|L_{j+1}|}\sum_{i=1}^{j} L_i$$

$$D_{KY} = 9 + \frac{1}{|L_{j+1}|}\sum_{i=1}^{9} L_i = 9 + \frac{L_1+L_2+L_3+L_4+L_5+L_6+L_7+L_8+L_9}{L_{10}}$$

$$= 9 + \frac{18.96383 + 9.94059 + 1.00877 + 0.828434 + 0.0490522 + -0.0132193 + -0.0646262 + -1.00973 + -28.8771}{39.729}$$

$= 9.04596$

which means that the Lyapunov dimension of system (1) is fractional. Because of the fractal nature, the new system has non-periodic orbits; what is more, its nearby trajectories diverge. Therefore, there is really chaos [11] in this nonlinear system.

## IV. Observation of Chaotic and Complex Dynamics

The initial values of the system are selected as:

$$x(0) = -1, y(0) = 4, z(0) = 1, u(0) = 0, v(0) = 0, w(0) = 1, p(0) = 0,$$
$$q(0) = -1, r(0) = 8, s(0) = 5.$$

Using MATHEMATICAprogram, the numerical simulation have been completed. This nonlinear system exhibits the complex and abundant chaotic dynamics behaviors, the strange attractors are shown in Figs.1,2&3.

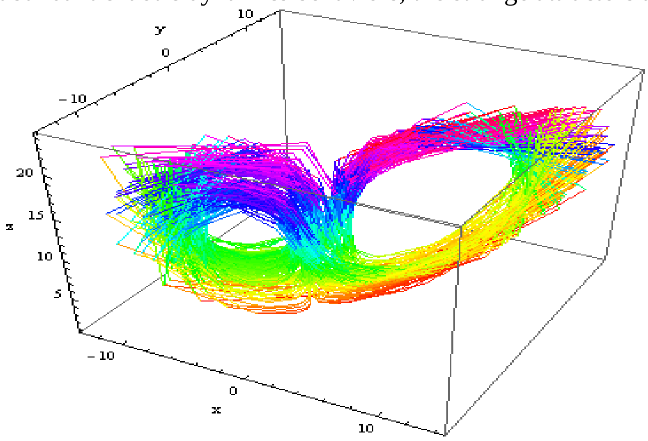


**Fig.1.** Chaotic attractors ,three- dimensional view (x-y-z)

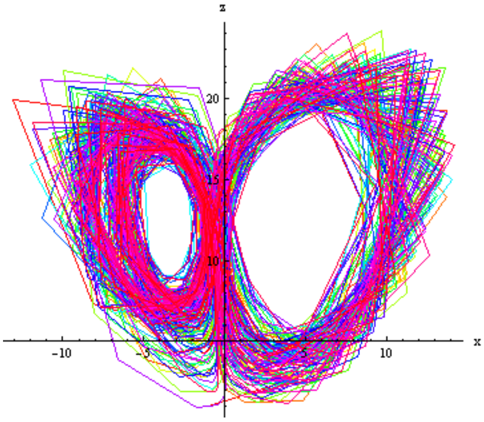


**Fig.2***Chaotic attractors ,z–x* phase plane

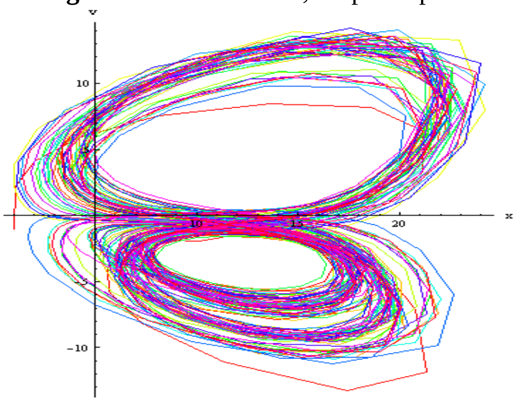


**Fig.3** *Chaotic attractors ,v–x* phase plane

## V. Waveform analysis of the novel chaotic system

As is well known, the waveform of a chaotic system should be aperiodic. In order to demonstrate that the proposed system is a chaotic system. Figs 4 &5 shows the time versus the states plot obtained from the **Mathematica** simulation.

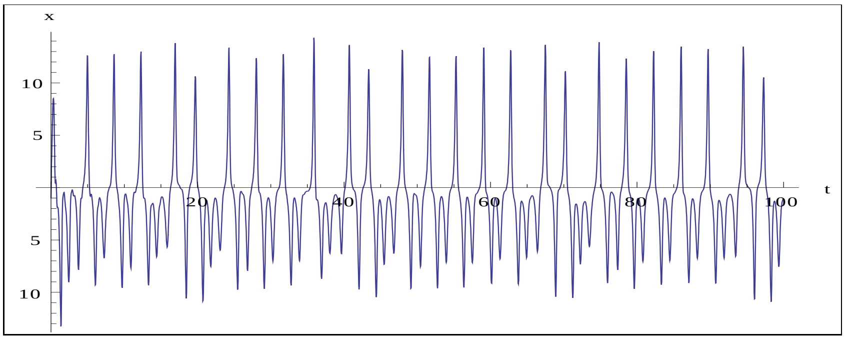


**Fig. 4** Time versus *x* of the novel chaotic system

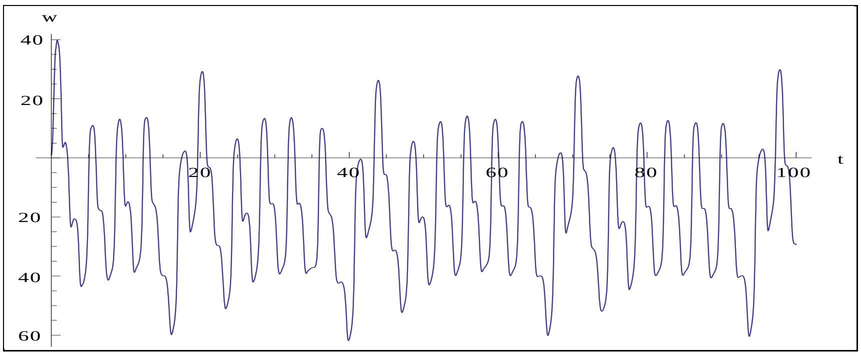


**Fig.5** Time versus *w* of the novel chaotic system

## VI. Sensitivity to initial conditions

Perhaps the most characteristic feature of a chaotic system is its long-term unpredictability. This comes about because of sensitive dependence of solutions on initial conditions. Two different initial conditions, no matter how close, will eventually become widely separated. Thus for any finite number of digits of accuracy in an initial condition, there will be a future time at which no accurate predictions can be made about the state of the system.

Fig.6 shows that the evolution of the chaos trajectories is very sensitive to initial conditions. The initial values of the system(1) are set to

$$x(0) = -1, y(0) = 4, z(0) = 1, u(0) = 0, v(0) = 0, w(0) = 1, p(0) = 0,$$
$$q(0) = -1, r(0) = 8, s(0) = 5.$$

for the solid line and

$$x(0) = -1, y(0) = 4, z(0) = 1, u(0) = 0, v(0) = 0, w(0) = 1, p(0) = 0,$$
$$q(0) = -1, r(0) = 8, s(0) = 5.0001.$$

for the dashed line.

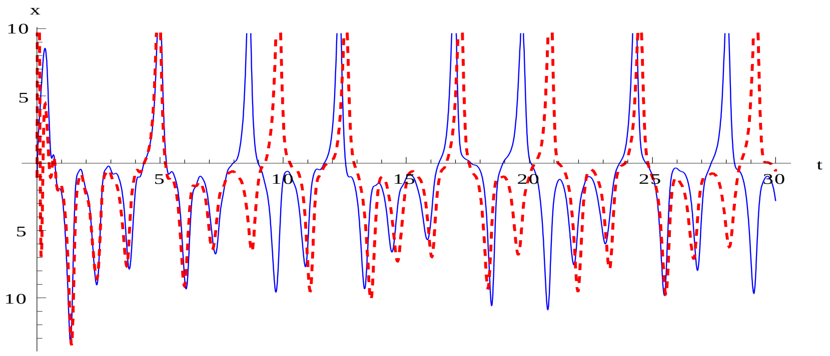


**Fig.6 .**Sensitivity tests of the novel system x(t)

Obviously, the waveform of system (1) is non-periodic and has better sensitivity to initial conditions and we call it sensitive dependence on initial conditions.

## VII. Conclusions

In this paper presented a novel ten-dimensional nonlinear system. Some basic properties of the system have been investigated.The new 10-D chaotic system has three unstable equilibrium points and calculated Lyapunov exponents, the Lyapunov exponents of the system are :

$L_1$= 18.94059 , $L_2$= 9.96383 , $L_3$= 1.00877,$L_4$= 0.828434 ,$L_5$= 0.0490522 ,$L_6$= -0.0132193 ,$L_7$= -0.0646262 , $L_8$= -1.00973 , $L_9$= -28.8771 , $L_{10}$= -39.729,the maximal Lyapunov exponent (MLE) of the novel system is $L_1$= 18.94059. In addition the Lyapunov dimension of the novel chaotic system is obtained as $D_{KY}$ =9.04596.

We firmly believe that more detailed theory analysis about the new system will be obtained in the near future.

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