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Vertex Odd Mean and Even Mean Labeling Of Some Graphs

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Abstract: *A graph G with p vertices and q edges is a mean graph if there is an injective function f from the vertices of G to {0,1,2,….q} such that when each edge* uv *is labeled with* $\frac{f(u)+f(v)}{2}$ *if* $f(u)+f(v)$ *is even and* $\frac{f(u)+f(v)+1}{2}$ *if* $f(u)+f(v)$ *is odd then the resulting edges are distinct. In this paper we investigate vertex odd and even mean labeling of Umbrella graph , Mongolian tent and* $K_1 + C_n$.
Keywords*: Mean labeling, Mongolian tent, Umbrella graph, vertex odd mean labeling, vertex even mean labeling.*

I. Introduction

By a graph $G = (V(G), E(G))$ with p vertices and q edges we mean a simple, connected and undirected graph. In this paper a brief summary of definitions and other information is given in order to maintain compactness. The term not defined here are used in the sense of Harary [3]. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. A useful survey on graph labeling by J. A. Gallian (2014) can be found in [2]. Somasundaram and Ponraj [5] have introduced the notion of mean labeling of graphs. In this paper we investigate vertex odd and even mean labeling of Umbrella graph, Mongolian tent and $K_1 + C_n$.

Definition 1.1A graph G with q edges to be an vertex odd mean graph if there is an injective function f from the vertices of G to $\{1,3,5,\ldots\ldots,2q-1\}$ such that when each edge uv is labeled with $\frac{f(u)+f(v)}{2}$ if $f(u)+f(v)$ is even and $\frac{f(u)+f(v)+1}{2}$ if $f(u)+f(v)$is odd then the resulting edges are distinct. Such a function is called a vertex odd mean labeling.

Definition 1.2A graph G with q edges to be an vertex even mean graph if there is an injective function f from the vertices of G to $\{2,4,6\ldots\ldots,2q\}$ such that the edge labels are given by $\frac{f(u)+f(v)}{2}$ are distinct. Such a function is called a vertex even mean labeling.

Definition 1.3

For any integers $m > 2$ and $n > 1$, the Umbrella graph $U(m,n)$ whose vertex and edge set is defined as

$$V\big(U(m,n)\big) = \{x_1, x_2, \ldots\ldots x_m, y_1, y_2, \ldots\ldots . y_n\}$$

$$E\big(U(m,n)\big) = \begin{cases} (x_i, x_{i+1}) & i = 1,2,\ldots\ldots m-1 \\ (y_i, y_{i+1}) & i = 1,2,\ldots\ldots n-1 \\ (x_i, y_1) & i = 1,2,\ldots\ldots m \end{cases}$$

Definition 1.4:

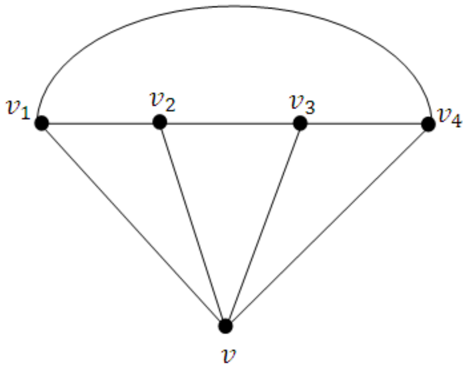
A Mongolian tent as a graph obtained from $P_m \times P_n$ by adding one extra vertex above the grid and joining every other of the top row of $P_m \times P_n$ to the new vertex.

Definition 1.5:

The join of graphs K_1 and C_n , $K_1 + C_n$, is obtained by joining a vertex of K_1 with every vertex of C_n with an edge.

II. Results On Vertex Odd Mean And Even Mean Labeling

Theorem 2.1: The graph $K_1 + C_n$ has vertex odd mean labeling.
Proof: The graph $K_1 + C_n$ has n+1 vertices and 2n edges.
Let v be vertex of K_1 and $v_1, v_2, \ldots, v_n$ be the vertices of the cycle.
The ordinary labeling of $K_1 + C_4$ is is given in figure.

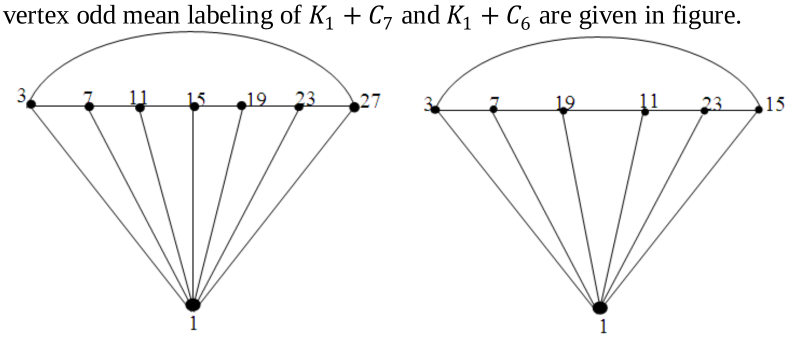


Define a vertex labeling $f: V(K_1 + C_n) \longrightarrow \{1,3,5,\dots,2q-1\}$ by

$$f(u) = 1$$
$$f(v_i) = 4i - 1, 1 \le i \le n \qquad \text{if } n \text{ is odd}$$
$$f(v_i) = \begin{cases} 3 & i = 1 \\ 2i + 3 & i \text{ is even} \\ q + 2i + 1 & i \text{ is odd} \end{cases} \qquad \text{if } n \text{ is even}$$

Clearly labels of the edges received by the mean of the labels on end vertices are all distinct.
Hence the graph $K_1 + C_{n,}$ has vertex odd mean labeling.

Example 2.2: The vertex odd mean labeling of $K_1 + C_7$ and $K_1 + C_6$ are given in figure.



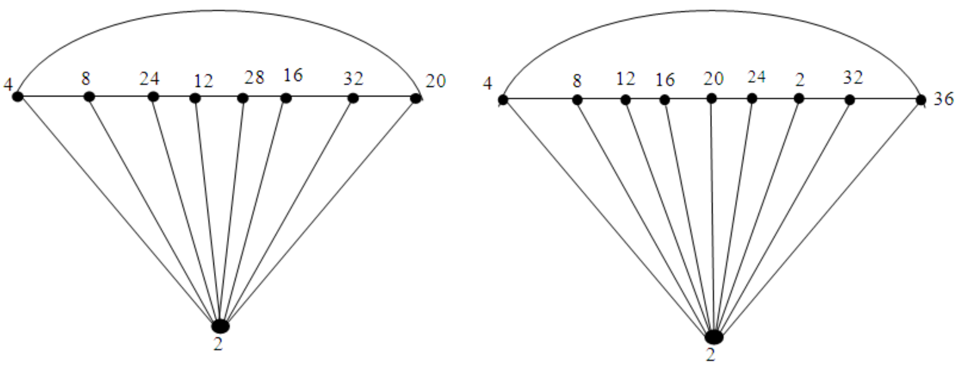
Theorem 2.3: The graph $K_1 + C_n$, has vertex even mean labeling.

Proof: Define a vertex labeling $f: V(K_1 + C_n) \longrightarrow \{2,4,6,\dots,2q\}$ by

$$f(u) = 1$$
$$f(v_i) = 4i \qquad 1 \le i \le n \qquad \text{if } n \text{ is odd}$$
$$f(v_i) = \begin{cases} 4 & i = 1 \\ 2i + 4 & i \text{ is even} \\ q + 2i + 2 & i \text{ is odd} \end{cases} \qquad \text{if } n \text{ is even}$$

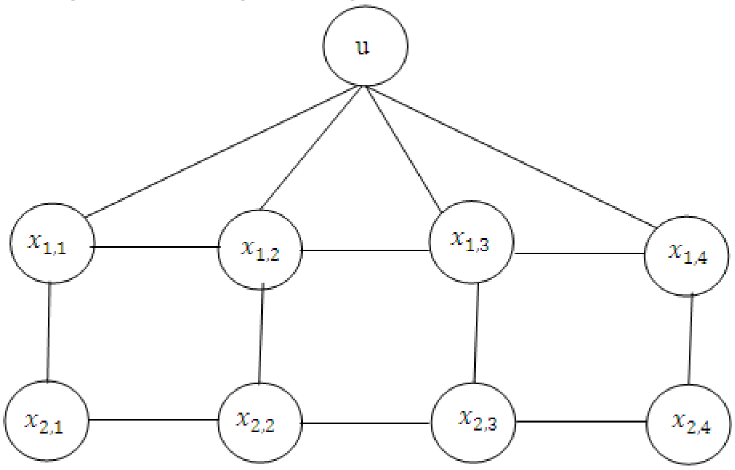
Clearly labels of the edges received by the mean of the labels on end vertices are all distinct.
Hence the graph $K_1 + C_{n,}$ has vertex even mean labeling.

Example 2.4: The vertex even mean labeling for $K_1 + C_8$ and $K_1 + C_9$ are given in figure.



Theorem 2.5: For any integer $m > 2$ and $n = 2$, the Mongolian tent has vertex odd mean labeling.

Proof: Consider $M(m,n)$ with the vertex set $\{u, x_{1,1}, x_{1,2}, \dots, x_{1,m}, x_{2,1}, x_{2,2}, \dots x_{2,m}\}$ where u is the pendent vertex. The ordinary labeling for $M(4,2)$ is given in figure.

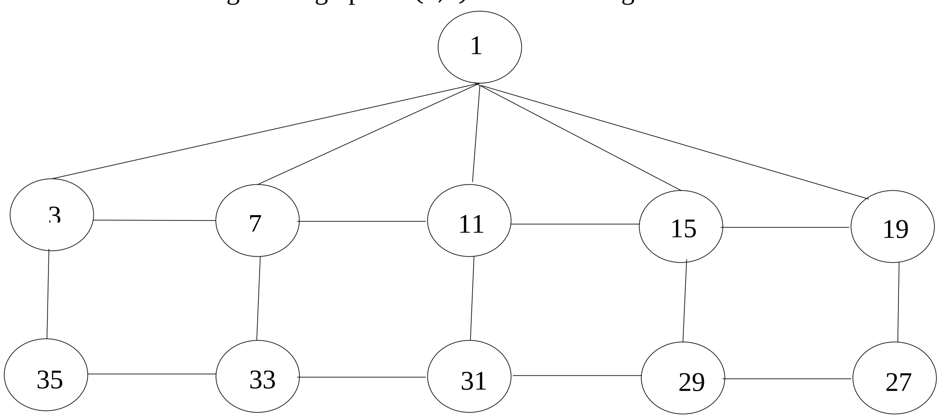


Define a vertex labeling $f : V(M(m,n)) \longrightarrow \{1,3,5, \dots, 2q-1\}$ by

$$f(u) = 1$$
$$f(x_{1,i}) = 4i - 1 \qquad i = 1,2, \dots, m$$
$$f(x_{2,i}) = 2q - (2i - 1) \qquad i = 1,2, \dots, m$$

Clearly labels of the edges received by the mean of the labels on end vertices are all distinct.
Hence the Mongolian tent hasvertex odd mean labeling.

Example 2.6
The vertex odd mean labeling of the graph $M(5,2)$ is shown in figure



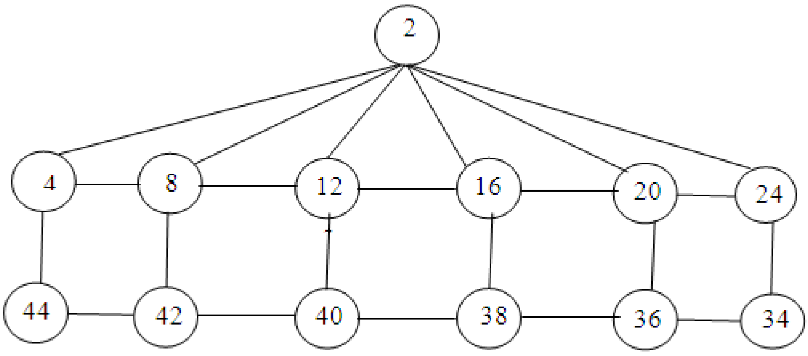
Theorem 2.7:For any integer $m > 2$ and $n = 2$, the Mongolian tent has an even mean labeling.

Proof: Define a vertex labeling$f : V(M(m,n)) \longrightarrow \{2,4,6, \dots 2q\}$ by

$$f(u) = 2$$
$$f(x_{1,i}) = 4i \qquad i = 1,2, \dots, m$$
$$f(x_{2,i}) = 2q - (2i - 2) \qquad i = 1,2, \dots, m$$

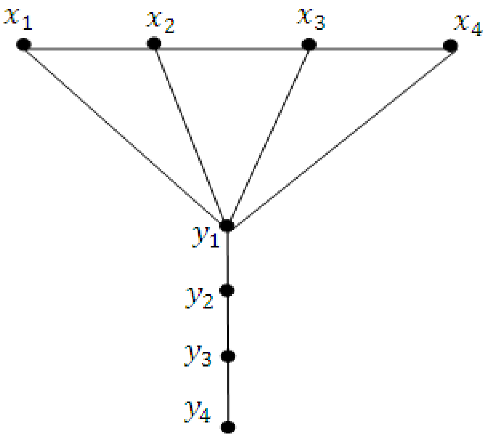
Clearly labels of the edges received by the mean of the labels on end vertices are all distinct.
Hence the Mongolian graph has vertex even mean labeling.

Example 2.8
The vertex even mean labeling of the graph $M(6,2)$ is shown in figure



Theorem 2.9: For any integer $m > 2$ and $n > 1$,the Umbrella graph $U(m,n)$ has vertex odd mean labeling.

Proof: The graph $U(m,n)$ has $m+n$ vertices and $2m+n-2$ edges.
The ordinary labeling for $U(4,3)$ is given in the figure

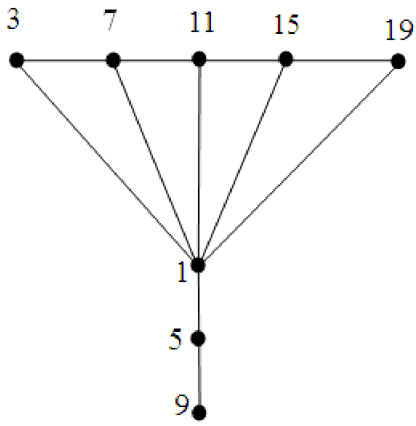


Define a vertex labeling $f : V(U(m,n)) \longrightarrow \{1,3,5,\ldots\ldots,2q-1\}$ by

$$f(x_i) = 4i - 1 \qquad i = 1,2,\ldots\ldots,m$$
$$f(y_i) = 4i - 3 \qquad i = 1,2,\ldots\ldots,n$$

Clearly labels of the edges received by the mean of the labels on end vertices are all distinct.
Hence the Umbrella graph has an odd mean labeling.

Example 2.10: In the following figure we exhibit vertex odd mean labeling for U(5,3)



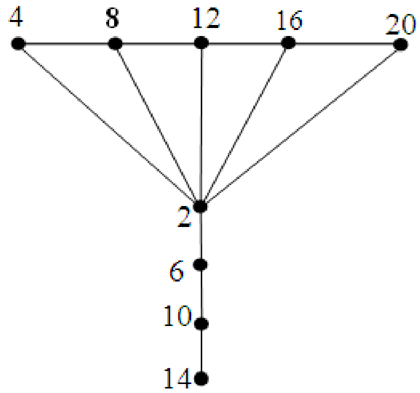
Theorem 2.11: For any integer $m > 2$ and $n > 1$,the umbrella graph U(m,n) has vertex even meanlabeling.

Proof: The graph $U(m,n)$ has $m+n$ vertices and $2m+n-2$ edges.
Define a vertex labeling $f : V(U(m,n)) \longrightarrow \{2,4,6,\ldots\ldots,2q\}$ by

$$f(x_i) = 4i \qquad i = 1,2,\ldots\ldots,m$$
$$f(y_i) = 4i - 2 \qquad i = 1,2,\ldots\ldots,n$$

Clearly labels of the edges received by the mean of the labels on end vertices are all distinct.
Hence the Umbrella graph has an vertex even mean labeling.

Example 2.12: In the following figure we exhibit vertex even mean labeling for U(5,4)



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