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Modified Variational Iteration Method of Solution the Fractional Partial Differential Equation Model

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Abstract: *In this paper, we presented modification of variational iteration method for solving partial differential equation. Tested for some examples and the obtained results demonstrate efficiency of the proposed method. he results were presented in tables and figure using the MathCAD 12 and Matlab software package.*
Keywords: *Modified variational iteration method, partial differential equation, lagrange multiplier*.

I. Introduction

In the last decades, fractional calculus found many applications in various fields of engineering and physical sciences such as physics, chemistry, biology, economy, viscoelasticity, electrochemistry, electromagnetic, relaxation processes, diffusion, control, porous media, and many more; see, for example, [1–7]. In most of these equations analytical solutions are either quite difficult or impossible to achieve, so approximations and numerical techniques must be used. Variational iteration method [8–11] are relatively new approach to provide an analytical approximation to linear and nonlinear problems, and they are particularly valuable as tools for scientists and applied mathematicians, because it provides immediate and visible symbolic terms of analytic solutions, as well as numerical approximate solutions to both linear and nonlinear differential equations.

In this paper, we present computationally efficient numerical method for solving the fractional partial differential equation.

II. Modified Variational Iteration Method

In this section, we will clarify modified variational iteration method for solving partial differential equation. Consider the general nonlinear differential equation

$$Lu(x,t) + Nu(x,t) = g(x,t),$$

where L is a linear differential operator, N is a nonlinear operator, and g an inhomogeneous term. According to modified variational iteration method, we can construct a correct functional as follows:

$$u_0(x,t) = \sum_{i=0}^{2} k_i(x)t^i$$

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^t \lambda(Lu_n(\tau) + Nu_n(\tau) - g(\tau))d\tau$$

and λ is a Lagrange multiplier which can be identified optimally via the variational theory. The subscript n indicates the n th approximation and $\tilde{u}_n$ is considered as a restricted variation $\delta\tilde{u}_n = 0$.

III. The Fractional Partial Differential Equation Model and Its Solution by Modified Variational Iteration Method

We present the analytical solution of the equation of the form:

$$\frac{\partial u(x,t)}{\partial t} = c(x,t)\frac{\partial^{\alpha} u(x,t)}{\partial x^{\alpha}} + s(x,t) \tag{1}$$

subject to the initial condition

u (x,0) = f(x), L < x < R (2)

and the boundary conditions

u (L,t) = w_1(t), $0 \le t \le T$

(3)

u (R,t) = w_2(t), $0 \le t \le T$

where c and s are known functions of x and t, f is a known function of x, w_1 and w_2 are known function of t and α is a given fractional number

Applying modified variational iteration method in (1):

First we will construct new initial condition is: $u_0(x,t) = \sum_{i=0}^{2} k_i(x) t^i$ (4)

Second step: $u_{n+1}(x,t) = u_n(x,t) + \int_0^t \lambda\left(\frac{\partial u_n(x,\tau)}{\partial \tau} - c(x,t)\frac{\partial^\alpha u_n(x,\tau)}{\partial x^\alpha} - s(x,\tau)\right)d\tau$ (5)

Where $\lambda = \frac{(-1)^m (\tau - t)^{m-1}}{(m-1)!}$ (6)

IV. Numerical Illustration

In this section, we apply modified variational iteration method for finding the analytical solution of fractional partial differential equation:

$$\frac{\partial u(x,t)}{\partial t} = c(x,t)\frac{\partial^{1.8} u(x,t)}{\partial x^{1.8}} + s(x,t) \quad (7)$$

subject to the initial condition u (x,0) = x^3, 0 <x <1, and the boundary conditions u (0,t) = 0, u (1, t) = e^{-t}, t > 0 with the coefficient function

, t >0 $c(x,t) = 0.183634x^{2.8}$

and the source function: $s(x,t) = -e^{-t} x^3 (1+x)$

Note that the exact solution to this problem is: u(x,t) = x^3e^{-t}.

Table 1 shows part the analytical solutions for fractional partial differential equation obtained for different values and comparison between exact solution and analytical solution. Figure 1 show the plot of the exact and the numerical solution surface for fractional partial differential equation respectively.

Table1. Some of comparison between exact solution and analytical solution For example when t = 6,7

x	t	Exact Solution	Modified Variational Iteration Method	$\lvert u_{ex}-u_{MVIM}\rvert$
0	6	0.000000000	0.000000000	0.000000000
0.1	6	0.000002479	0.000002479	0.000000000
0.2	6	0.000019830	0.000019830	0.000000000
0.3	6	0.000066930	0.000066930	0.000000000
0.4	6	0.000158600	0.000158600	0.000000000
0.5	6	0.000309800	0.000309800	0.000000000
0.6	6	0.000535400	0.000535400	0.000000000
0.7	6	0.000850200	0.000850200	0.000000000
0.8	6	0.001269000	0.001269000	0.000000000
0.9	6	0.001807000	0.001807000	0.000000000
1	6	0.002479000	0.002479000	0.000000000
0	7	0.000000000	0.000000000	0.000000000
0.1	7	0.000000912	0.000000912	0.000000000
0.2	7	0.000007295	0.000007295	0.000000000
0.3	7	0.000024620	0.000024620	0.000000000
0.4	7	0.000058360	0.000058360	0.000000000
0.5	7	0.000114000	0.000114000	0.000000000
0.6	7	0.000197000	0.000197000	0.000000000
0.7	7	0.000312800	0.000312800	0.000000000
0.8	7	0.000466900	0.000466900	0.000000000
0.9	7	0.000664800	0.000664800	0.000000000
1	7	0.000911900	0.000911900	0.000000000

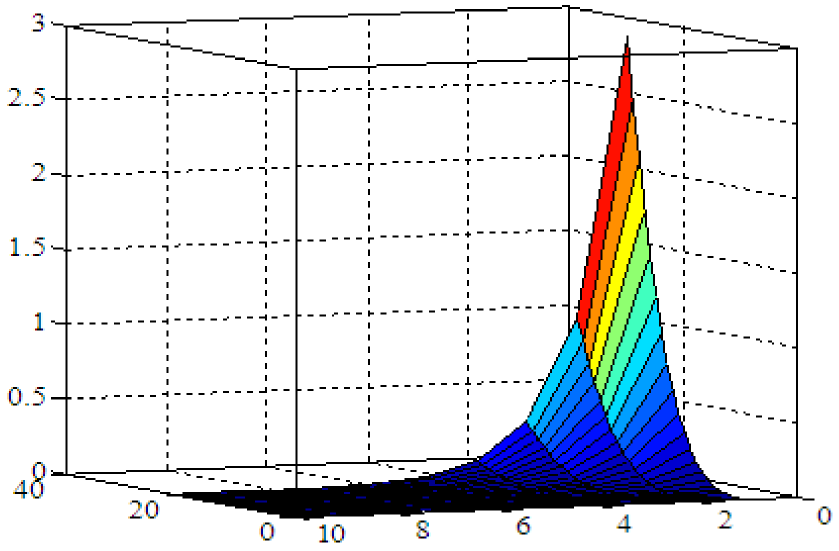

Fig. 1: Exact solution

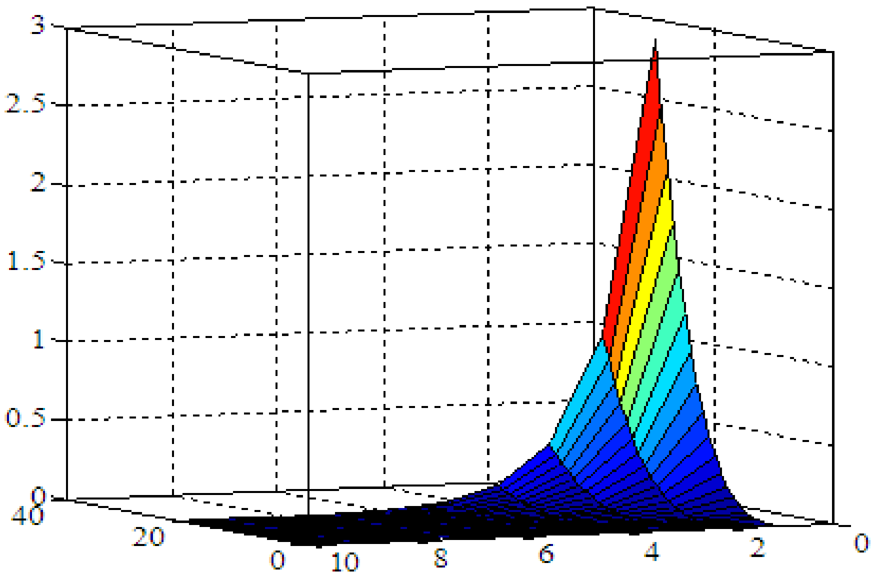

Fig. 2: Numerical solution

V.Conclusion

In this paper, we have applied the modified variational iteration method for the solution of the fractional partial differential equation. This algorithm is simple and easy to implement. The obtained results confirmed a good accuracy of the method. On the other hand, the calculations are simpler and faster than in traditional techniques.

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