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MHD Free Convective Flow through a Porous Medium with Periodic Permeability

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I. Introduction

The problem of free convection flow through a porous medium has attracted the attention of a number of scholars because of its possible application to several geophysical application. Singh et al. (2001) have studied free convection in MHD flow of a rotating viscous liquid in porous medium past a vertical porous plate. Sengupta and Basak (2002) have investigated unsteady flow of visco-elastic Maxwell fluid through porous straight tube under the uniform magnetic field. Singh et al. (2003) have reported heat and mass transfer in MHD flow of a viscous fluid past a vertical plate under oscillatory suction velocity**.** Cookey and Sigalo(2003) have studied unsteady MHD free-convection and mass transfer flow past an infinite heated porous vertical plate with time dependant suction. Das et al. (2004) have reported free convection flow and mass transfer of an elastico-viscous fluid past an infinite vertical porous plate in a rotating porous medium**.** Chaudhary and Chand (2005) have investigated hydromagnetic flow past a long vertical channel embedded in porous medium with transpiration cooling**.** Kurtcebe and Erim (2005) have studied heat transfer of a visco-elastic fluid in a porous channel.

Ogulu and Amos (2005) have reported asymptotic approximation for the flow field in a free convective flow of a non-Newtonian fluid past a vertical porous plate. Panda et al. (2006) have investigated free convection of conducting viscous fluid between two vertical walls filled with porous material. Sharma and Yadav (2006) have studied steady MHD boundary layer flow and heat transfer between two long vertical wavy walls. Das et al. (2006) have reported free Convective and Mass transfer flow of a viscous incompressible fluid through a porous medium in presence of source/ sink with constant suction and heat flux. Panda et al. (2008) have studied unsteady MHD flow of visco-elastic Maxwell fluid through rectangular porous tube. Das and Panda (2009) have reported Magnetohydrodynamic steady free convective flow and mass transfer in a rotating elastico-viscous fluid past an infinite vertical porous flat plate with constant suction.

Most of the investigators have restricted themselves to two dimensional flows only by assuming either constant or time dependent permeability of the porous medium. However, there may arise situations where the flow fields may be essentially three dimensional, for example, when variation of the permeability distribution is transverse to the potential flow.. Singh and Sharma (2002) have discussed three dimensional free convective flow and heat transfer through a porous medium with periodic permeability.

The objective of this paper is to study the effect of mass transfer on three dimensional free convective flow of viscous incompressible fluid through a highly porous medium in the presence of uniform transverse magnetic field. The porous medium is bounded by infinite vertical porous plate. We have assumed here the free stream velocity to be uniform. Our aim is to study the effect of variable permeability on the flow in the presence of magnetic field and heat transfer phenomena.

II. Mathematical Formulation

The physical configuration consists of flow of an electrically conducting and incompressible viscous fluid with simultaneous heat and mass transfer along an infinite vertical non-conducting porous plate with constant suction. The plate lying vertically on the $x^* - z^*$ plane with x^* - axis is taken along the plate in the upward direction. The y^* - axis is taken normal to the plane of the plate and directed along the fluid flowing laminarly with uniform free stream velocity U. Uniform magnetic field B_0 is applied in y^* - direction. $K^*(z^*) = K_P^* / \left(1 + \varepsilon \cos \frac{\pi z^*}{L}\right)$ where K_p^* is the mean permeability of the medium. L is the wave length of permeability distribution and ε (<< 1) is the amplitude of the permeability variation. The problem becomes three dimensional due to such a permeability variation. In this problem the following assumption are made.

(i) Molecular transport properties are constant
(ii) Density variation due to temperature and concentration difference is approximated by Boussinesq approximation.

(iii) Mass fraction of diffusing species is low compare to that of other species in the binary mixture.
(iv) Viscous dissipation in energy equation is negligible and chemical reactions are neglected.

Thus denoting velocity components by u^*, v^*, w^* in the directions of x^*, y^*, z^* respectively and temperature by T^* and concentration by C^*, the flow through highly porous medium is governed by the following equations

$$\frac{\partial v^*}{\partial y^*}+\frac{\partial w^*}{\partial z^*}=0, \tag{1}$$

$$v^*\frac{\partial u^*}{\partial y^*}+w^*\frac{\partial u^*}{\partial z^*}=g\beta\left(T^*-T_\infty^*\right)+g\beta^*\left(C^*-C_\infty^*\right), +\nu\left(\frac{\partial^2 u^*}{\partial y^{*2}}+\frac{\partial^2 u^*}{\partial z^{*2}}\right)-\frac{\nu}{K^*}\left(u^*-U\right)-\frac{\sigma B_0^2\left(u^*-U\right)}{\rho}, \tag{2}$$

$$v^*\frac{\partial v^*}{\partial y^*}+w^*\frac{\partial v^*}{\partial z^*}=-\frac{1}{\rho}\frac{\partial p^*}{\partial y^*}+\nu\left(\frac{\partial^2 v^*}{\partial y^{*2}}+\frac{\partial^2 v^*}{\partial z^{*2}}\right)-\frac{\nu\, v^*}{K^*} \tag{3}$$

$$v^*\frac{\partial w^*}{\partial y^*}+w^*\frac{\partial w^*}{\partial z^*}=-\frac{1}{\rho}\frac{\partial p^*}{\partial y^*}+\nu\left(\frac{\partial^2 w^*}{\partial y^{*2}}+\frac{\partial^2 w^*}{\partial z^{*2}}\right)-\frac{\nu w^*}{K^*}-\frac{\sigma B_0^2 w^*}{\rho}, \tag{4}$$

$$v^*\frac{\partial T^*}{\partial y^*}+w^*\frac{\partial T^*}{\partial z^*}=\frac{k}{\rho C_p}\left(\frac{\partial^2 T^*}{\partial y^{*2}}+\frac{\partial^2 T^*}{\partial z^{*2}}\right) \tag{5}$$

$$v^*\frac{\partial C^*}{\partial y^*}+w^*\frac{\partial C^*}{\partial z^*}=D\left(\frac{\partial^2 C^*}{\partial y^{*2}}+\frac{\partial^2 C^*}{\partial z^{*2}}\right) \tag{6}$$

where

g : acceleration due to gravity, β : coefficient of volume expansion,
β^*: coeficent of mass expansion, p^* : pressure,
ρ: density, ν : kinematic viscosity, μ : viscosity,
k : thermal conductivity, C_p : specific heat at constant pressure,
σ : electrical conductivity, D : concentration diffusivity

The boundary conditions are

$y^* = 0$; $u^* = 0$, $v^* = -V$, $w^* = 0$, $T^* = T_w^*$, $C^* = C_w^*$

$$y^*\to\infty;\quad u^*\to U, w^*\to 0, p^*\to p_\infty^*, T^*\to T_\infty^*, C^*\to C_\infty^* \tag{7}$$

V is a constant and the negative sign indicates that suction is towards the plate and suffix w and ∞ represent the conditions on the wall and at farawy from the wall.

Introducing the following non-dimensional quantities

$$y=\frac{y^*}{L}, z=\frac{z^*}{L}, u=\frac{u^*}{U}, v=\frac{v^*}{V}, w=\frac{w^*}{V}$$

$$p=\frac{p^*}{\rho V^2}, \theta=\frac{T^*-T_\infty^*}{T_w^*-T_\infty^*}, \phi=\frac{C^*-C_\infty^*}{C_w^*-C_\infty^*} \tag{8}$$

Equations (1) to (6) reduces to

$$\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 , \tag{9}$$

$$v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=GrR\theta+GmR\phi+\frac{1}{R}\left(\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)-\frac{(u-1)(1+\in\cos\pi z)}{RKp}-\frac{M^2(u-1)}{R}, \tag{10}$$

$$v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=-\frac{\partial p}{\partial y}+\frac{1}{R}\left(\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)-\frac{(1+\in\cos\pi z)v}{RKp}, \tag{11}$$

$$v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{\partial p}{\partial z}+\frac{1}{R}\left(\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)-\frac{(1+\in\cos\pi z)w}{RKp}-\frac{M^2 w}{R}, \tag{12}$$

$$v\frac{\partial \theta}{\partial y}+w\frac{\partial \theta}{\partial z}=\frac{1}{R\,Pr}\left(\frac{\partial^2 \theta}{\partial y^2}+\frac{\partial^2 \theta}{\partial z^2}\right), \tag{13}$$

$$v\frac{\partial \phi}{\partial y}+w\frac{\partial \phi}{\partial z}=\frac{1}{RSc}\left(\frac{\partial^2 \phi}{\partial y^2}+\frac{\partial^2 \phi}{\partial z^2}\right). \tag{14}$$

where

Gr = $\frac{vg\beta\left(T_w^* - T_\infty^*\right)}{UV^2}$ (Grashof number), Pr = $\frac{\mu C_p}{k}$ (Prandtl number),

Gm = $\frac{vg\beta\left(C_w^* - C_\infty^*\right)}{UV^2}$ (Modified Grashof number), R = $\frac{VL}{\nu}$ (Reynolds number),

Sc = $\frac{\nu}{D}$ (Schmidt number), K_p = $\frac{K_p^*}{L^2}$ (Permeability parameter) and

M = $\left(\frac{\sigma}{\mu}\right)^{1/2} B_0 L$ (Hartmann number)

The corresponding boundary conditions become

$y = 0;$ $\quad u = 0,\ v = -1,\ w = 0,\ \theta = 1,\ \phi = 1,$

$y \to \infty;$ $\quad u \to 1,\ w \to 0,\ p \to p_\infty,\ \theta \to 0,\ \phi \to 0$ (15)

III. Method of Solution

In order to solve the problem we assume the solutions of the following form because the amplitude $\in$ (< < 1) of the permeability variation is very small.

$s(y, z) = s_0(y) + \in s_1(y, z) + \in^2 s_2(y, z) + \ldots\ldots$

where s represents u, v, w, p, θ and ϕ (16)

when $\in = 0$, the problem reduces to the two dimensional free convective flow through a porous medium with constant permeability which is governed by following equations.

$$\frac{dv_0}{dy}=0 \tag{17}$$

$$\frac{d^2u_0}{dy^2} - v_0 R\frac{du_0}{dy} - \frac{u_0}{Kp} - M^2u_0 = -GrR^2\theta_0 - GmR^2\phi_0 - \left(\frac{1}{Kp} + M^2\right) \tag{18}$$

$$\frac{d^2\theta_0}{dy^2} - v_0 R\,Pr\,\frac{d\theta_0}{dy} = 0 \tag{19}$$

$$\frac{d^2\phi_0}{dy^2} - v_0 RSc\frac{d\phi_0}{dy} = 0 \tag{20}$$

The corresponding boundary conditions

$y = 0;\quad u_0 = 0,\ v_0 = -1,\ \theta_0 = 1,\ \phi_0 = 1,$

$y \to \infty;\quad u_0 \to 1,\ p_0 \to p_\infty,\ \theta_0 \to 0,\ \phi_0 \to 0$ (21)

The solutions of these equations are

$$u_0 = 1 + (GrA_0 + Gm\,A_1 - 1)\,e^{-R_1 y} - GrA_0e^{-R\,Pr\,y} - GmA_1e^{-RScy} \tag{22}$$

$$\theta_0 = e^{-RPry} \tag{23}$$

$$\phi_0 = e^{-RScy} \tag{24}$$

with $v_0 = -1$, $w_0 = 0$ and $p_0 = p_\infty$ (25)

where $R_1 = \frac{R}{2} + \sqrt{\frac{R^2}{4} + \frac{1}{Kp} + M^2}$

$A_0 = \frac{R^2}{R^2\Pr(\Pr-1) - \left(M^2 + 1/K_P\right)}$

$A_1 = \frac{R^2}{R^2Sc(Sc-1) - \left(M^2 + 1/K_P\right)}$

When $\in \neq 0$, the periodic permeability enters the equation (9) to (14) and comparing the coefficients of identical power of $\in$, neglecting those $\in^2$, $\in^3$ etc., we get the following first order equation the help of equation (25).

$$\frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0 \tag{26}$$

$$v_1\frac{\partial u_0}{\partial y} - \frac{\partial u_1}{\partial y} = GrR\theta_1 + GmR\phi_1 + \frac{1}{R}\left(\frac{\partial^2u_1}{\partial y^2} + \frac{\partial^2u_1}{\partial z^2}\right) - \frac{(u_0-1)\cos\pi z + u_1}{RKp} - \frac{M^2u_1}{R} \tag{27}$$

$$-\frac{\partial v_1}{\partial y} = -\frac{\partial p_1}{\partial y} + \frac{1}{R}\left(\frac{\partial^2v_1}{\partial y^2} + \frac{\partial^2v_1}{\partial z^2}\right) - \frac{v_1 - \cos\pi z}{RKp} \tag{28}$$

$$-\frac{\partial w_1}{\partial y} = -\frac{\partial p_1}{\partial z} + \frac{1}{R}\left(\frac{\partial^2w_1}{\partial y^2} + \frac{\partial^2w_1}{\partial z^2}\right) - \frac{w_1}{RKp} - \frac{M^2w_1}{R} \tag{29}$$

$$v_1\frac{\partial\theta}{\partial y} - \frac{\partial\theta_1}{\partial y} = \frac{1}{R\,Pr}\left(\frac{\partial^2\theta_1}{\partial y^2} + \frac{\partial^2\theta_1}{\partial z^2}\right) \tag{30}$$

$$v_1 \frac{\partial \phi_0}{\partial y} - \frac{\partial \phi_1}{\partial y} = \frac{1}{RSc}\left(\frac{\partial^2 \phi_1}{\partial y^2} + \frac{\partial^2 \phi_1}{\partial z^2}\right) \tag{31}$$

The corresponding boundary conditions are

y = 0: $u_1 = 0,\ v_1 = 0,\ w_1 = 0,\ \theta_1 = 0,\ \phi_1 = 0$

$y \to \infty$: $u_1 \to 0,\ w_1 = 0,\ p_1 \to 0,\ \theta_1 \to 0,\ \phi_1 \to 0$ (32)

Equations (26) to (31) are the linear partial differential equation which describe free convective, three dimensional flow.

For solution we shall first consider (26), (28) and (29) being independent of the main flow and the temperature field. We assume v_1, w_1 and p_1 of the following form

$$v_1(y, z) = v_{11}(y) \cos \pi z \tag{33}$$

$$w_1(y, z) = \frac{-1}{\pi} v'_{11}(y) \sin \pi z \tag{34}$$

$$p_1(y, z) = p_{11}(y) \cos \pi z \tag{35}$$

where the prime in $v'_{11}(y)$ denote the differentiation with respect to y. Expression for $v_1(y, z)$ and $w_1(y, z)$ have been chosen so that the equation of continuity (26) is satisfied. Substituting the expression (33) to (35) into (28) to (29) we have

$$v''_{11} + Rv'_{11} - \left(\pi^2 + \frac{1}{Kp}\right)v_{11} = Rp'_{11} - \frac{1}{Kp} \tag{36}$$

$$v'''_{11} + Rv''_{11} - \left(\pi^2 + M^2 + \frac{1}{Kp}\right)v'_{11} = R\pi^2 p_{11} \tag{37}$$

and the boundary conditions are

y = 0: $v_{11} = 0,\ v'_{11} = 0$

$y \to \infty$: $v_{11} = 0,\ v'_{11} = 0,\ p_{11} = 0$ (38)

On solving equations (36) and (37) under the boundary condition (38), we get

$$v_1(y, z) = -\frac{1}{(\pi - R_2)(\pi^2 Kp + 1)}\left(\pi e^{-R_2 y} - R_2 e^{-\pi y} - \pi + R_2\right)\cos \pi z \tag{39}$$

$$w_1(y, z) = -\frac{R_2}{(\pi - R_2)(\pi^2 Kp + 1)}\left(e^{-\pi y} - e^{-R_2 y}\right)\sin \pi z \tag{40}$$

$$p_1(y, z) = -\frac{R_2\left(R\pi + \frac{1}{Kp}\right)}{R\pi(\pi - R_2)(\pi^2 Kp + 1)} e^{-\pi y} \cos \pi z \tag{41}$$

where

$$R_2 = \frac{R}{2} + \sqrt{\frac{R^2}{4} + \pi^2 + \frac{1}{Kp}}$$

For the main flow, the temperature and the concentration field solutions we assume u_1, θ_1 and ϕ_1 as

$$u_1(y, z) = u_{11}(y) \cos \pi z \tag{42}$$

$$\theta_1(y, z) = \theta_{11}(y) \cos \pi z \tag{43}$$

$$\phi_1(y, z) = \phi_{11}(y) \cos \pi z \tag{44}$$

Substitution of (42) to (44) into the partial differential equations (27), (30) and (31), reduce them to the ordinary differential equations as $u''_{11} + Ru'_{11} - \left(\pi^2 + M^2 + \frac{1}{Kp}\right)u_{11}$

$$= Rv_{11}u'_0 - GrR^2\theta_{11} - GmR^2\phi_{11} + \frac{u_0 - 1}{Kp} \quad (45)$$

$$\theta''_{11} + R\,Pr\,\theta'_{11} - \pi^2\theta_{11} = R\,Pr\,v_{11}\theta'_0 \quad (46)$$

$$\phi''_{11} + RSc\phi'_{11} - \pi^2\phi_{11} = RScv_{11}\phi'_0$$

(47)

with corresponding boundary conditions

y = 0; $u_{11} = 0$, $\theta_{11} = 0$, $\phi_{11} = 0$

$y \to \infty$; $u_{11} \to 0$, $\theta_{11} \to 0$, $\phi_{11} \to 0$ (48)

Solving equations (45) to (47) under boundary conditions (48) and using equations (42) to (44) we get the results for u_1, θ_1 and ϕ_1 as

$$u_1 = \Bigg[\frac{R}{(\pi - R_2)(\pi^2Kp + 1)}\Big\{A_{26}e^{-R_5y} + A_6e^{-(R_1+R_2)y}$$

$$- A_7e^{-(R\,Pr+R_2)y} - A_8e^{-(RSc+R_2)y} - A_9e^{-(\pi+R_1)y} + A_{10}e^{-(\pi+R\,Pr)y}$$

$$+ A_{11}e^{-(\pi+RSc)y} - A_{12}e^{-R_1y} + A_{13}e^{-R\,Pr\,y} + A_{14}e^{-RScy}\Big\}$$

$$+ \frac{GrR^4\,Pr^2}{(\pi - R_2)(\pi^2Kp + 1)}\Big\{A_{27}e^{-R_5y} - A_{15}e^{-R_3y} - A_{16}e^{-(R_2+R\,Pr)y}$$

$$+ A_{17}e^{-(\pi+R\,Pr)y} - A_{18}e^{-R\,Pr\,y}\Big\}$$

$$+ \frac{GmR^4Sc^2}{(\pi - R_2)(\pi^2Kp + 1)}\Big\{A_{28}e^{-R_5y} - A_{19}e^{-R_4y} - A_{20}e^{-(R_2+RSc)y}$$

$$+ A_{21}e^{-(\pi+RSc)y} - A_{22}e^{-RScy}\Big\} + A_{23}\left(e^{-R_1y} - e^{-R_5y}\right)$$

$$+ A_{24}\left(e^{-R_5y} - e^{-R\,Pr\,y}\right) + A_{25}\left(e^{-R_5y} - e^{-RScy}\right)\Bigg]\cos\pi z \quad (49)$$

$$\theta_1 = \frac{R^2\,Pr^2}{(\pi - R_2)(\pi^2Kp + 1)}\Bigg\{\left(A_2 - A_3 - \frac{1}{\pi} + \frac{R_2}{\pi^2}\right)e^{-R_3y}$$

$$+ A_3e^{-(R_2+R\,Pr)y} - A_2e^{-(\pi+R\,Pr)y} + \frac{\pi - R_2}{\pi^2}e^{-R\,Pr\,y}\Bigg\}\cos\pi z \quad (50)$$

$$\phi_1 = \frac{R^2Sc^2}{(\pi - R_2)(\pi^2Kp + 1)}\Bigg\{\left(A_4 - A_5 - \frac{1}{\pi} + \frac{R_2}{\pi^2}\right)e^{-R_4y}$$

$$+ A_5e^{-(R_2+RSc)y} - A_4e^{-(\pi+RSc)y} + \frac{\pi - R_2}{\pi^2}e^{-RScy}\Bigg\}\cos\pi z \quad (51)$$

IV. Skin friction

The non-dimensional form of the skin friction (omitting the detail expression) is given by

$$\tau = \frac{\tau^*}{\rho UV} = \frac{\upsilon}{VL}\left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{1}{R}\left[\frac{du_0}{dy} + \in \frac{du_{11}}{dy} \cos \pi z\right]_{y=0}$$

V. Result and Discussion

The computation has been carried out in respect of air (Pr = 0.71) and water (Pr = 7.0) on a cooling surface (Gr > 0).Main objective of this discussion is to study the effect of magnetic field on the flow characteristic in the presence of periodic permeability. It is interesting to observe from Fig.1 that sudden rise in the velocity is marked in the layers near the plate then the velocity attains a constant value. Velocity profiles represent a two-layer character one for water (Curve VII) others for air. Maximum rise is marked incase of curve IV (Gm=10.0) due to increase in the value of Gm, depicting buoyancy effect due to mass diffusion. Thus, it is concluded that buoyancy force due to mass transfer has a significant contribution to enhance the velocity. On careful observation it is revealed that further increase in magnetic parameter (M), mass transfer coefficient (Sc) and Prandtl number (Pr) lead to decrease the velocity at all points but the reverse effect is observed due to buoyancy parameter Gm, Gr and permeability Parameter (K_p) . Thus, it is concluded that Lorentz force, thermo and mass diffusion opposes the motion where as boundary forces and permeability enhances it.

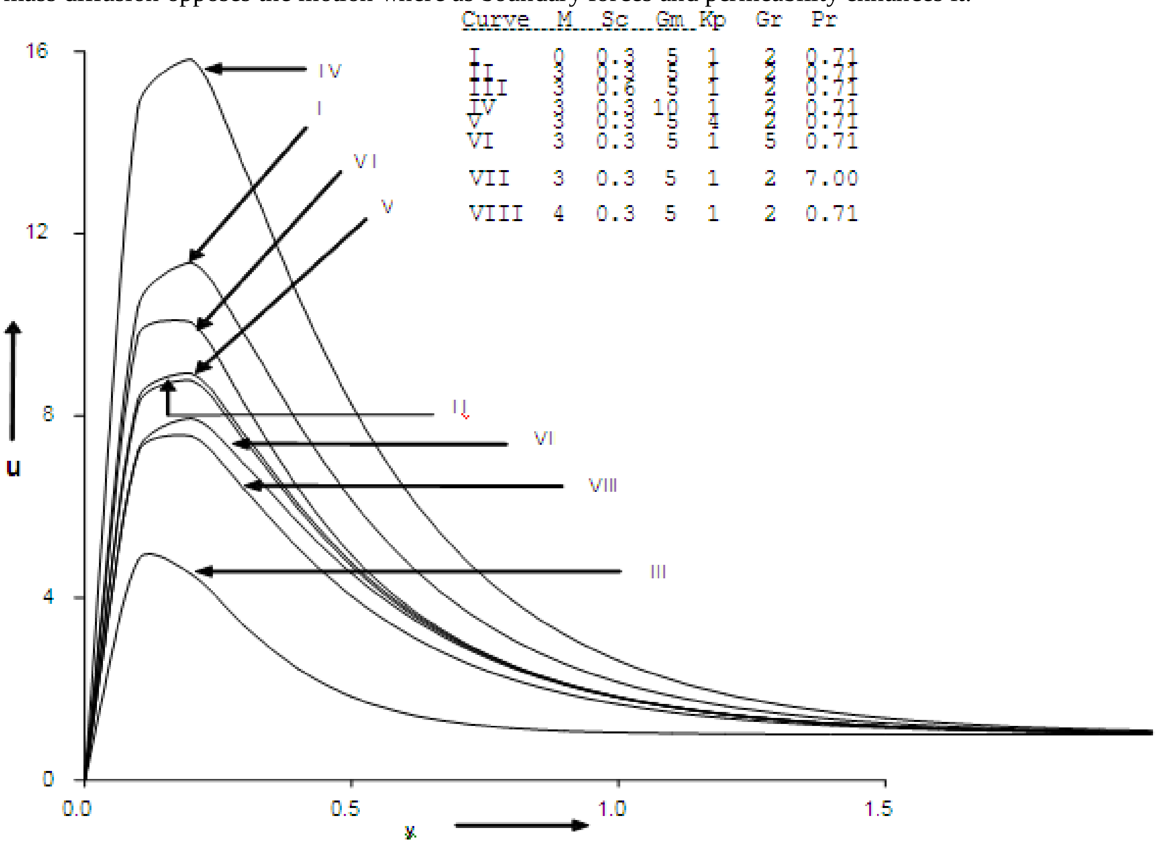

Fig .1 Effect of different parameter on Velocity profile when R = 10, ε = 0.1, Z = 0

Fig. 2 exhibits the temperature profile for different values of Prandtl number (Pr) and Reynolds number (R). Sudden fall is marked in case of water (Pr = 7.0) and for air the fall is quite smooth. An increase in Pr and R leads to an decrease in temperature at all points.

Fig.3 shows that fall of concentration in all the layers is marked due to rise in Sc i.e, for heavier species the concentration falls rapidly at all the layers. Computation is made for hydrogen (Sc = 0.22), helium (Sc = 0.30) and water vapour (Sc = 0.60) through air.

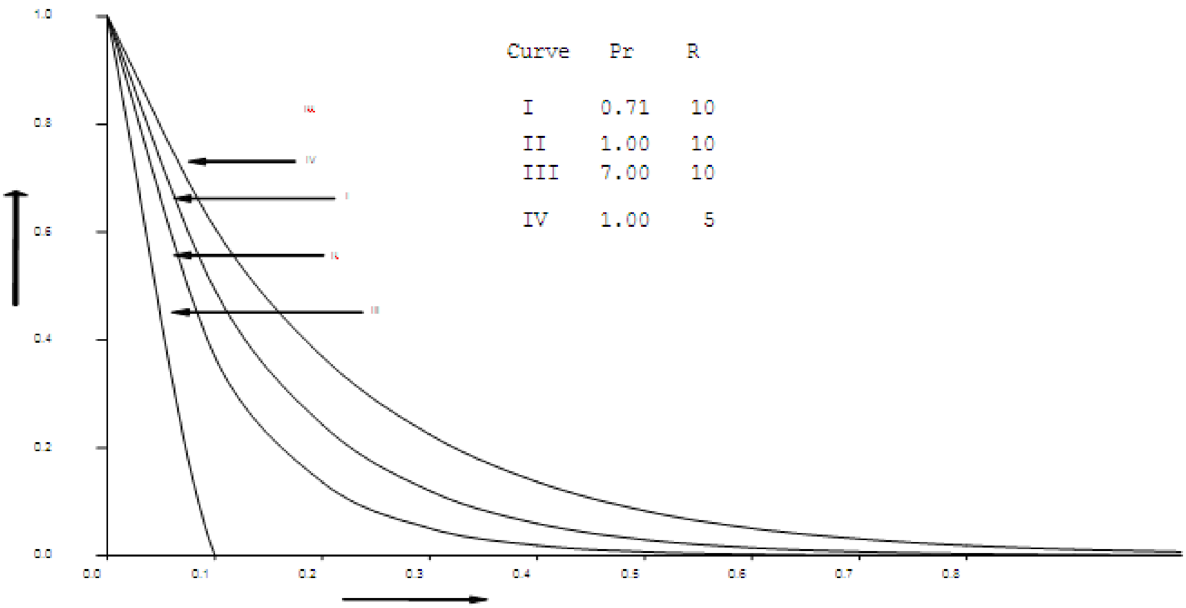

Fig. 2 Effect of Pr & R on temperature profiles when Kp = 1, e = 0.2 , z = 0 , Gr = 2, Gm = 5, Sc = 0.3

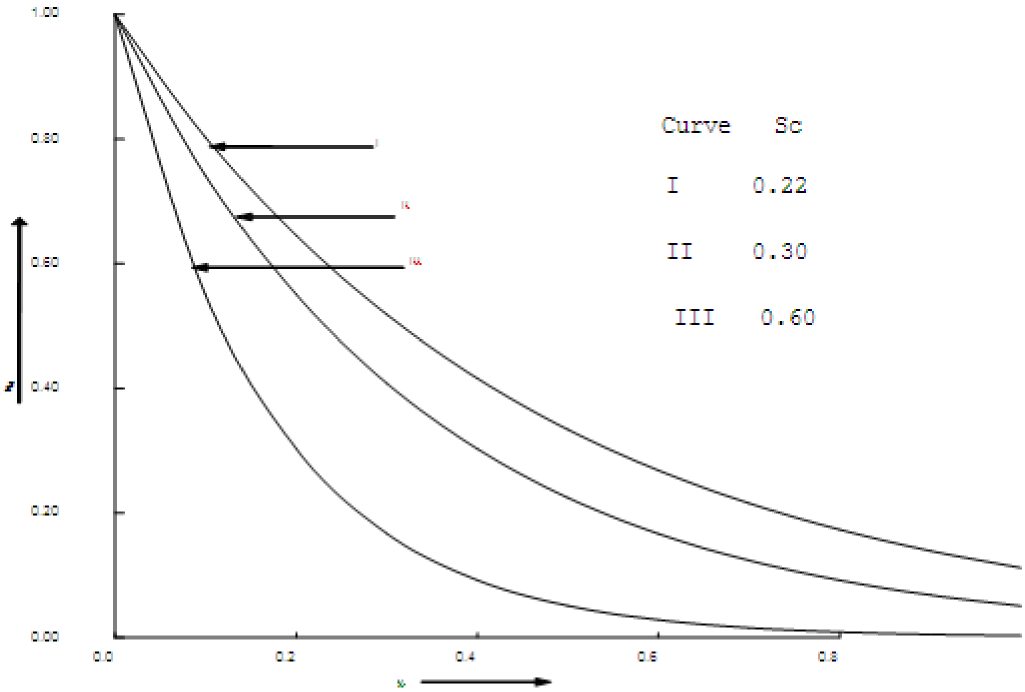

Fig. 3 Effect of Sc on concentration profiles when Kp = 1, e = 0.2, z = 0 , Gr = 2, Gm = 5, Sc = 0.3

VI. Conclusion

1. The bouancy effect due to mass transfer has a significant contribution leading to attain the maximum velocity.
2. The thermal bouancy force enhance the velocity at all points.
3. An increase in magnetic parameter (M), mass transfer coefficient (Sc) and Prandtl number (Pr) leads to decrease the velocity at all points.
4. High Prandtl number fluid with higher Reynolds number (R = 10.0) is responsible to decelerate the fluid motion.

Appendix

$$R_3 = \frac{R\,Pr}{2} + \sqrt{\frac{R^2\ Pr^2}{4} + \pi^2}\ , \quad R_4 = \frac{RSc}{2} + \sqrt{\frac{R^2 Sc^2}{4} + \pi^2}\ ,$$

$$R_5 = \frac{R}{2} + \sqrt{\frac{R^2}{4} + \pi^2 + M^2 + \frac{1}{K_p}}\ , \quad A_2 = \frac{R_2}{\pi R\,Pr},$$

$$A_3 = \frac{\pi}{R_2^2 + R_2 R\,Pr - \pi^2}\ , A_4 = \frac{R_2}{\pi RSc}\ , A_5 = \frac{\pi}{R_2^2 + R_2 RSc - \pi^2}\ ,$$

$$A_6 = \frac{\pi R_1 (GrA_0 + GmA_1 - 1)}{R_1\left(R_1 + 2R_2 - R\right) - M^2}\ , A_7 = \frac{\pi GrA_0 R\,Pr}{R\,Pr(R\,Pr + 2R_2 - R) - M^2}\ ,$$

$$A_8 = \frac{\pi GmA_1 RSc}{RSc(RSc + 2R_2 - R) - M^2},$$

$$A_9 = \frac{R_1 R_2 (GrA_0 + GmA_1 - 1)}{(\pi + R_1)^2 - R(\pi + R_1) - \left(\pi^2 + M^2 + 1/K_p\right)}\ ,$$

$$A_{10} = \frac{GrRR_2 \Pr A_0}{R^2 \Pr(\Pr - 1) + \pi R(2\Pr - 1) - M^2 - 1/K_p}$$

$$, A_{11} = \frac{GmRR_2 ScA_1}{R^2 Sc(Sc-1) + \pi R(2Sc-1) - M^2 - 1/K_p}\ ,$$

$$A_{12} = \frac{R_1(\pi - R_2)(GrA_0 + GmA_1 - 1)}{R_1^2 - RR_1 - \left(\pi^2 + M^2 + M^2 + 1/K_p\right)}, \quad A_{13} = \frac{(\pi - R_2)GrA_0 R\Pr}{(R\Pr)^2 - R(R\Pr) - \left(\pi^2 + M^2 + 1/K_p\right)}\ ,$$

$$A_{14} = \frac{(\pi - R_2)GmA_1 RSc}{(RSc)^2 - R(RSc) - \left(\pi^2 + M^2 + 1/K_p\right)}\ , A_{15} = \frac{A_2 - A_3 - \frac{1}{\pi} + \frac{R_2}{\pi^2}}{R_3^2 - RR_3 - \left(\pi^2 + M^2 + 1/K_p\right)}\ ,$$

$$A_{16} = \frac{A_3}{R\,Pr\left(R\,Pr + 2R_2 - R\right) - M^2}, \quad A_{17} = \frac{A_2}{R^2 \Pr(\Pr - 1) + \pi R(2\Pr - 1)) - M^2 - 1/K_p}\ ,$$

$$A_{18} = \frac{\pi - R_2}{\pi^2 \left\{(R\,Pr)^2 - R(R\,Pr) - \left(\pi^2 + M^2 + 1/K_p\right)\right\}},$$

$$A_{19} = \frac{A_4 - A_5 - \frac{1}{\pi} + \frac{R_2}{\pi^2}}{R_4^2 - RR_4 - \left(\pi^2 + M^2 + 1/K_p\right)}\ , A_{20} = \frac{A_5}{RSc(RSc + 2R_2 - R) - M^2},$$

$$A_{21} = \frac{A_4}{R^2 Sc(Sc-1) + \pi R(2Sc-1)) - M^2 - 1/K_p},$$

$$A_{22} = \frac{\pi - R_2}{\pi^2\left\{(RSc)^2 - R(RSc) - \left(\pi^2 + M^2 + 1/K_p\right)\right\}},$$

$$A_{23}=\frac{(GrA_6+GmA_1-1)/K_P}{R_1^2-RR_1-(\pi^2+M^2+1/K_p)}$$

$$,A_{24}=\frac{(GrA_0)/K_p}{(R\,Pr)^2-R(R\,Pr)-(\pi^2+M^2+1/K_p)},$$

$$A_{25}=\frac{(GmA_1)/K_p}{(RSc)^2-R(RSc)-(\pi^2+M^2+1/K_p)},$$

$A_{26} = A_7 + A_8 - A_6 + A_9 - A_{10} - A_{11} + A_{12} - A_{13} - A_{14},$

$A_{27} = A_{15} + A_{16} - A_{17} + A_{18}$, $A_{28} = A_{19} + A_{20} - A_{21} + A_{22}$.

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