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On generalized n-derivation in prime near – rings

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Abstract: *The main purpose of this paper is to show that zero symmetric prime left near-rings satisfying certain identities are commutative rings . As a consequence of the results obtained ,we prove several commutativity theorems about generalized n-derivatios for prime near-rings.*

I. Introduction

A right near – ring (resp.left near ring) is a set N together with two binary operations (+) and (.) such that (i)(N,+) is a group (not necessarily abelian).(ii)(N,.) is a semi group.(iii)For all a,b,c ϵ N ; we have (a+b).c = a.c + b.c (resp. a.(b+c) = a.b + b.c . Through this paper, N will be a zero symmetric left near – ring (i.e., a left near-ring N satisfying the property 0.x=0 for all x∈ N). we will denote the product of any two elements x and y in N ,i.e.; x.y by xy . The symbol Z will denote the multiplicative centre of N, that is Z={$x \in N \mid xy = yx\ for\ all\ y \in N$. For any x,y $\in$ N the symbol [x,y]=xy-yx stands for multiplicative commutator of x and y, while the symbol x∘y will denote xy+yx . N is called a prime near-ring if xNy = {0} implies either x = 0 or y = 0 . A nonempty subset U of N is called semigroup left ideal (resp. semigroup right ideal) if NU⊆U (resp.UN⊆U)and if U is both a semigroup left ideal and a semigroup right ideal, it will be called a semigroup ideal. A normal subgroup (I,+) of (N, +) is called a right ideal (resp. left ideal) of N if (x + i)y – xy ∈ I for all x,y ∈ N and i ∈ I(resp. xi ∈ I for all i∈I and x∈N). I is called ideal of N if it is both a left ideal as well as a right ideal of N . For terminologies concerning near-rings , we refer to Pilz [11].

An additive endomorphism $d : N \longrightarrow N$ is said to be a derivation of N if $d(xy)$ = xd(y) + d(x)y , or equivalently , as noted in [5 , lemma 4] that d(xy) = d(x)y + xd(y) for all $x,y \in N$.

A map d:$\underbrace{N \times N \times \ldots \times N}_{n-times} \longrightarrow$N is said to be permuting if the equation $d(x_1, x_2, \ldots, x_n)$=$d(x_{\pi(1)}, x_{\pi(2)}, \ldots, x_{\pi(n)})$ holds for all $x_1, x_2, \ldots, x_n \in$ N and for every permutation $\pi \in S_n$ where S_n is the permutation group on $\{1,2,\ldots,n\}$.

Let n be a fixed positive integer . An n-additive(i.e.; additive in each argument) mapping d:$\underbrace{N \times N \times \ldots \times N}_{n-times} \longrightarrow$N is said to be n-derivation if the relations

$$d(x_1 x_1', x_2, \ldots, x_n) = d(x_1, x_2, \ldots, x_n)x_1' + x_1 d(x_1', x_2, \ldots, x_n)$$
$$d(x_1, x_2x_2', \ldots, x_n) = d(x_1, x_2, \ldots, x_n)x_2' + x_2 d(x_1, x_2', \ldots, x_n)$$
$$\vdots\ d(x_1, x_2, \ldots, x_nx_n') = d(x_1, x_2, \ldots, x_n)x_n' + x_n d(x_1, x_2, \ldots, x_n')$$

hold for all $x_1, x_1', x_2, x_2', \ldots, x_n, x_n' \in$ N. If in addition d is a permuting map then d is called a permuting n-derivation of N . For terminologies concerning n-derivation of near-rings , we refer to [4]

An n-additive mapping f:$\underbrace{N \times N \times \ldots \times N}_{n-times} \longrightarrow$N is called a right generalized n-derivation of N with associated n-derivation d if the relations

$$f(x_1 x_1', x_2, \ldots, x_n) = f(x_1, x_2, \ldots, x_n)x_1' + x_1 d(x_1', x_2, \ldots, x_n)$$
$$f(x_1, x_2x_2', \ldots, x_n) = f(x_1, x_2, \ldots, x_n)x_2' + x_2 d(x_1, x_2', \ldots, x_n)$$
$$\vdots$$
$$f(x_1, x_2, \ldots, x_nx_n') = f(x_1, x_2, \ldots, x_n)x_n' + x_n d(x_1, x_2, \ldots, x_n')$$

hold for all $x_1, x_1', x_2, x_2', \ldots, x_n, x_n' \in$ N. If in addition both f and d is a permuting maps then f is called a permuting right generalized n-derivation of N associated permuting n-derivation d . An n-additive mapping f:$\underbrace{N \times N \times \ldots \times N}_{n-times} \longrightarrow$N is called a left generalized n-derivation of N with associated n-derivation d if the relations

$$f(x_1 x_1', x_2, \ldots, x_n) = d(x_1, x_2, \ldots, x_n)x_1' + x_1 f(x_1', x_2, \ldots, x_n)$$
$$f(x_1, x_2x_2', \ldots, x_n) = d(x_1, x_2, \ldots, x_n)x_2' + x_2 f(x_1, x_2', \ldots, x_n)$$
$$\vdots\ f(x_1, x_2, \ldots, x_nx_n') = d(x_1, x_2, \ldots, x_n)x_n' + x_n f(x_1, x_2, \ldots, x_n')$$

hold for all $x_1, x_1', x_2, x_2', \ldots, x_n, x_n' \in$ N. If in addition both f and d is a permuting maps then f is called a permuting left generalized n-derivation of N with associated permuting n-derivation d . Lastly an n-additive mapping d:$\underbrace{N \times N \times \ldots \times N}_{n-times} \longrightarrow$N is called a generalized n-derivation of N with associated n-derivation d if it is both a right generalized n-derivation as well as a left generalized n-derivation of N with associated n-derivation d . If in addition both f and d are permuting maps then f is called a permuting generalized n-derivation of N with associated permuting n-derivation d . For terminologies concerning generalized n-derivation of near-rings, we refer to [3].

Many authors studied the relationship between structure of near – ring N and the behaviour of special mapping on N. There are several results in the existing literature which assert that prime near-ring with certain constrained derivations have ring like behaviour . Recently several authors (see [2–10] for reference where further references can be found) have investigated commutativity of near-rings satisfying certain identities. Motivated by these results we shall consider generalized n-derivation on a near-ring N and show that prime near-rings satisfying some identities involving generalized n-derivations and semigroup ideals or ideals are commutative rings. In fact, our results generalize some known results proved in [3],[4] and [10] .

II. Preliminaries

The following lemmas are essential for developing the proofs of our main results

Lemma 2.1 [6] If U is non-zero semi group right ideal (resp, semi group left ideal) and x is an element of N such that Ux = {0} (resp, xU = {0}) then x = 0 .

Lemma 2.2[6] Let N be a prime near-ring and U a nonzero semigroup ideal of N .If $x, y \in N$ and $xUy = \{0\}$ then $x = 0$ or $y = 0$.

Lemma 2.3[2] Let N be a prime near-ring . then d is permuting n-derivation of N if and only if

$$d(x_1 x_1', x_2, \dots, x_n) = x_1 d(x_1', x_2, \dots, x_n) + d(x_1, x_2, \dots, x_n) x_1'$$

for all $x_1, x_1', x_2, \dots, x_n \in N$.

Lemma 2.4[2] Let N be a near-ring and is a permuting n-derivation of N . Then for every $x_1, x_1', x_2, \dots, x_n, y \in N$,

(i) $(x_1 d(x_1', x_2, \dots, x_n) + d(x_1, x_2, \dots, x_n) x_1') y = x_1 d(x_1', x_2, \dots, x_n) y + d(x_1, x_2, \dots, x_n) x_1' y$,

(ii) $(d(x_1, x_2, \dots, x_n) x_1' + x_1 d(x_1', x_2, \dots, x_n)) y = d(x_1, x_2, \dots, x_n) x_1' y + x_1 d(x_1', x_2, \dots, x_n) y$.

Remark 2.1 It can be easily shown that above lemmas 2.3 and 2.4 also hold if d is a nonzero n-derivation of near-ring N .

Lemma 2.5 [4] Let d be an n-derivation of a near ring N , then $d(Z,N,\dots,N) \subseteq Z$.

Lemma 2.6[4] Let N be a prime near ring , d a nonzero n-derivation of N , and $U_1,U_2,\dots,U_n$ be anonzero semigroup ideals of N .If $d(U_1,U_2,\dots,U_n) \subseteq Z$, then N is a commutative ring .

Lemma 2.7 [4] Let N be a prime near ring ,d a nonzero n-derivation of N .and $U_1,U_2,\dots,U_n$ be a nonzero semigroup ideals of N such that $d([x,y],u_2,\dots,u_n) = 0$ for all $x,y \in U_1, u_2 \in U_2,\dots,u_n \in U_n$, then N is a commutative ring .

Lemma 2.8[3]. f is aright generalized n-derivation of N with associated n-derivation d if and only if

$$f(x_1x'_1,x_2,\cdots,x_n)=x_1d(x'_1,x_2,\cdots,x_n)+f(x_1,x_2,\cdots,x_n)x'_1$$
$$f(x_1,x_2x'_2,\cdots,x_n)=x_2d(x_1,x'_2,\cdots,x_n)+f(x_1,x_2,\cdots,x_n)x'_2$$
$$\vdots$$
$$f(x_1,x_2,\cdots,x_nx'_n)=x_nd(x_1,x_2,\cdots,x'_n)+f(x_1,x_2,\cdots,x_n)x'_n$$

hold for all $x_1, x'_1, x_2, x'_2,\cdots,x_n, x'_n \in N$.

Lemma 2.9[3]. Let N be a near-ring admitting a right generalized n-derivation f with associated n-derivation d of N . Then

$$(f(x_1, x_2, \dots, x_n)x_1' + x_1 d(x_1', x_2, \dots, x_n))y = f(x_1, x_2, \dots, x_n)x_1'y + x_1 d(x_1', x_2, \dots, x_n)y,$$
$$(f(x_1, x_2, \dots, x_n)x_2' + x_2 d(x_1, x_2', \dots, x_n))y = f(x_1, x_2, \dots, x_n)x_2'y + x_2 d(x_1, x_2', \dots, x_n)y,$$
$$\vdots$$
$$(f(x_1, x_2, \dots, x_n)x_n' + x_n d(x_1, x_2, \dots, x_n'))y = f(x_1, x_2, \dots, x_n)x_n'y + x_n d(x_1, x_2, \dots, x_n')y,$$

for all $x_1, x'_1, x_2, x'_2,\cdots,x_n, x'_n, y \in N$

Lemma 2.10[3]. Let N be a near-ring admitting a right generalized n-derivation f with associated n-derivation d of N . Then ,

$$(x_1d(x'_1,x_2,\cdots,x_n)+f(x_1,x_2,\cdots,x_n)x'_1)y= x_1d(x'_1,x_2,\cdots,x_n)y+f(x_1,x_2,\cdots,x_n)x'_1y$$
$$(x_2d(x_1,x'_2,\cdots,x_n)+f(x_1,x_2,\cdots,x_n)x'_2)y=x_2d(x_1,x'_2,\cdots,x_n)y+f(x_1,x_2,\cdots,x_n)x'_2y$$
$$\vdots$$
$$(x_nd(x_1,x_2,\cdots,x'_n)+f(x_1,x_2,\cdots,x_n)x'_n)y=x_nd(x_1,x_2,\cdots,x'_n)y+f(x_1,x_2,\cdots,x_n)x'_n yy$$

for all $x_1, x'_1, x_2, x'_2,\cdots,x_n, x'_n, y \in N$.

Lemma 2.11[3]. f is a left generalized n-derivation of N with associated n-derivation d if and only if

$$f(x_1x'_1,x_2,\cdots,x_n)=x_1f(x'_1,x_2,\cdots,x_n)+d(x_1,x_2,\cdots,x_n)x'_1$$

$f(x_1,x_2x'_2,\cdots,x_n)=x_2f(x_1,x'_2,\cdots,x_n)+d(x_1,x_2,\cdots,x_n)x'_2$

$\vdots$

$f(x_1,x_2,\cdots,x_nx'_n)=x_nf(x_1,x_2,\cdots,x'_n)+d(x_1,x_2,\cdots,x_n)x'_n$

hold for all $x_1, x'_1, x_2, x'_2,\cdots,x_n, x'_n \in N$.

Lemma 2.12[3]. Let N be a near-ring admitting a left generalized n-derivation f with associated n-derivation d of N . Then

$(d(x_1,x_2,\dots,x_n)x_1'+x_1 f(x_1',x_2,\dots,x_n))y = d(x_1,x_2,\dots,x_n)x_1'y + x_1 f(x_1',x_2,\dots,x_n)y,$

$(d(x_1,x_2,\dots,x_n)x_2'+x_2 f(x_1,x_2',\dots,x_n))y=d(x_1,x_2,\dots,x_n)x_2'y + x_2 f(x_1,x_2',\dots,x_n)y,$

$\vdots$

$(d(x_1,x_2,\dots,x_n)x_n'+x_n f(x_1,x_2,\dots,x_n'))y=d(x_1,x_2,\dots,x_n)x_n' y + x_n f(x_1,x_2,\dots,x_n')y,$

for all $x_1, x'_1, x_2, x'_2,\cdots,x_n, x'_n, y \in N$

Lemma 2.13[3]. Let N be a near-ring admitting a left generalized n-derivation f with associated n-derivation d of N . Then ,

$(x_1f(x'_1,x_2,\cdots,x_n)+d(x_1,x_2,\cdots,x_n) x'_1)y= x_1f(x'_1,x_2,\cdots,x_n)y+d(x_1,x_2,\cdots,x_n) x'_1y$

$(x_2f(x_1,x'_2,\cdots,x_n)+d(x_1,x_2,\cdots,x_n)x'_2)y=x_2f(x_1,x'_2,\cdots,x_n)y+d(x_1,x_2,\cdots,x_n)x'_2y$

$\vdots$

$(x_nf(x_1,x_2,\cdots,x'_n)+d(x_1,x_2,\cdots,x_n)x'_n)y=x_nf(x_1,x_2,\cdots,x'_n)y+d(x_1,x_2,\cdots,x_n)x'_n yy$

for all $x_1, x'_1, x_2, x'_2,\cdots,x_n, x'_n, y \in N$.

Lemma 2.14 [3]. Let N be prime near-ring admitting a generalized n-derivation f with associated n-derivation of N , then $f(Z,N,\dots,N) \subseteq Z$.

Lemma 2.15 . Let N be a prime near-ring with nonzero generalized n-derivations f associated with nonzero n-derivation d. Let $x \in N$, $U_1,U_2,\dots,U_n$ be nonzero semigroup ideals of N.

(i) If $f(U_1, U_2,\dots, U_n)x = \{0\}$, then $x = 0$.

(ii) If $xf(U_1, U_2,\dots, U_n) =\{0\}$, then $x = 0$.

Proof . (i) Given that $f(U_1, U_2,\dots, U_n)x = \{0\}$,i.e.;

$f(u_1, u_2,\dots, u_n)x = 0$, for all $u_1 \in U_1, u_2 \in U_2,\dots, u_n \in U_n$ (1)

Putting r_1u_1 in place of u_1 , where $r_1 \in N$, in relation (1)we get $f(r_1u_1, u_2,\dots, u_n)x = 0$. This yields that $d(r_1, u_2,\dots, u_n)u_1x + r_1f(u_1, u_2,\dots, u_n)x = 0$, by hypothesis we have $d(r_1, u_2,\dots, u_n)u_1x = 0$. Replacing u_1 again by u_1s , where $s \in N$ in preceding relation we obtain $d(r_1, u_2,\dots, u_n)u_1sx = 0$,i.e. ; $d(r_1, u_2,\dots, u_n)u_1Nx = \{0\}$.But N is prime near ring ,then either $d(r_1, u_2,\dots, u_n)u_1 = 0$ or $x = 0$. Our claim is that $d(r_1, u_2,\dots, u_n)u_1 \neq 0$,for some $r_1 \in N$, $u_1 \in U_1, u_2 \in U_2,\dots, u_n \in U_n$. For otherwise if $d(r_1, u_2,\dots, u_n)u_1 = 0$ for all $r_1 \in N, u_1 \in U_1, u_2 \in U_2,\dots, u_n \in U_n$, then $d(r_1, u_2,\dots, u_n)tu_1 = 0$, where $t \in N$, i.e.; $d(r_1, u_2,\dots, u_n)Nu_1 = \{0\}$. As $U_1\neq\{0\}$, primeness of N yields

$d(r_1, u_2,\dots, u_n) = 0$ for all $r_1 \in N, u_1 \in U_1, u_2 \in U_2,\dots, u_n \in U_n$. (2)

Now putting $u_2 r_2 \in U_2$ in place of u_2 ,where $r_2 \in N$, in (2) and using it again we get $u_2d(r_1, r_2,\dots, u_n) = 0$ .Now replacing u_2 again by u_2w , where $w \in N$ in preceding relation we obtain $u_2wd(r_1, r_2,\dots, u_n) = 0$,i.e.; $U_2Nd(r_1, r_2,\dots, u_n) = 0$ .As $U_2\neq\{0\}$, primeness of N yields $d(r_1, r_2,\dots, u_n) = 0$ for all $r_1,r_2 \in N,\dots, u_n \in U_n$ .Preceding inductively as before we conclude that $d(r_1, r_2,\dots, r_n) = 0$ for all $r_1,r_2,\dots,r_n \in N$. This shows that $d(N, N,\dots, N) = \{0\}$, leading to a contradiction as d is a nonzero n-derivation .Therefore , our claim is correct and now we conclude that $x = 0$.

(ii) It can be proved in a similar way .

Lemma 2.15 . Let N be a prime near-ring with nonzero n-derivation d. Let $x \in N$, $U_1,U_2,\dots,U_n$ be nonzero semigroup ideals of N.

(i) If $d(U_1, U_2,\dots, U_n)x = \{0\}$, then $x = 0$.

(ii) If $xd(U_1, U_2,\dots, U_n) =\{0\}$, then $x = 0$.

III. Main Results

Recently Öznur Gölbasi ([7],Theorem2.6) proved that if N is aprime near-ring with a nonzero generalized derivation f such that $f(N) \subseteq Z$ then (N,+) is an abelian group . Moreover if N is 2-torsion free , then N is a commutative ring. Mohammad Ashraf and Mohammad Aslam Siddeeque show that "2-torsion free restriction "in the above resultused by Öznur Gölbasi is superfluous . Mohammad Ashraf and Mohammad Aslam Siddeeque ([3] , Theorem 3.1) proved that if N is a prime near-ring with a nonzero generalized n-derivation f with associated n-derivation d of N such that $f(N,N,\dots,N) \subseteq Z$ then N is a commutative ring . We have extended this result in the setting of generalized n-derivation and semigroup ideals in near rings by proving the following theorem.

Theorem 3.1 . Let N be a prime near-ring admitting a nonzero left generalized n-derivation f with associated n-derivation d of N . Let $U_1, U_2, \ldots, U_n$ be nonzero semigroups right ideals of N. If $f(U_1, U_2, \ldots, U_n) \subseteq Z$, then N is a commutative ring **.**

Proof . For all $u_1, u_1' \in U_1, u_2 \in U_2, \ldots, u_n \in Un$ we get

$f(u_1u_1', u_2, \ldots, u_n) = d(u_1, u_2, \ldots, u_n) u_1' + u_1f(u_1', u_2, \ldots, u_n) \in Z$ (3)

Now commuting equation (3) with the element u_1 we have $(d(u_1, u_2, \ldots, u_n) u_1' + u_1f(u_1', u_2, \ldots, u_n)) u_1 = u_1(d(u_1, u_2, \ldots, u_n) u_1' + u_1f(u_1', u_2, \ldots, u_n))$, by lemma 2.12 we get $d(u_1, u_2, \ldots, u_n) u_1' u_1 + u_1f(u_1', u_2, \ldots, u_n) u_1 = u_1 \; d(u_1, u_2, \ldots, u_n) u_1' +$
$u_1u_1f(u_1', u_2, \ldots, u_n))$, by hypothesis we get $d(u_1,u_2, \ldots, u_n) u_1' u_1 = u_1d(u_1,u_2, \ldots, u_n) u_1'$, replacing u_1' by $u_1' y$, where $y \in N$, in previous relation and using it again we get $d(u_1, u_2, \ldots, u_n) u_1' (u_1 y- yu_1) = 0$ for all $u_1, u_1' \in U_1, u_2 \in U_2, \ldots, u_n \in Un$, $y \in N$, then we have $d(u_1, u_2, \ldots, u_n)U_1(u_1 y- yu_1) = 0$. By using lemma 2.2 ,we conclude that for each $u_1 \in U1$ either $u_1 \in Z$ or $d(u_1,u_2,\ldots,u_n) = 0$, but using lemma 2.5 lastly we get $d(u_1,u_2,\ldots,u_n) \in Z$ for all $u_1 \in U_1 ,u_2 \in U_2,\ldots,u_n \in U_n$, i.e., $d(U_1,U_2,\ldots,U_n) \subseteq Z$. Now by using lemma 2.6 we find that N is commutative ring .

Corollary 3.1 ([3]theorem 3.1) Let N be a prime near-ring admitting a nonzero generalized n-derivation f with associated n-derivation d of N . If $f(N, N, \ldots, N) \subseteq Z$, then N is a commutative ring **.**

Corollary 3.2 ([4] ,**Theorem 3.3**) Let N be a prime near-ring , $U_1, U_2, \ldots, U_n$ be nonzero semigroup right ideals of N and let d be a nonzero n-derivation of N. If $d(U_1, U_2, \ldots, U_n) \subseteq Z$, then N is a commutative ring **.**

Corollary 3.3.([2]theorem 3.2) Let N be a prime near-ring admitting a nonzero permuting n-derivation d . If $d(N, N, \ldots, N) \subseteq Z$, then N is a commutative ring **.**

In the year 2014 , Mohammad Ashraf and Mohammad Aslam Siddeeque ([4] , Theorem 3.1) proved that if N is a prime near-ring with no nonzero divisors of zero and $U_1, U_2, \ldots, U_n$ are nonzero semigroup right ideals of N which admits a nonzero n-derivation d such that $d(u_1 u'_1, u_2, \ldots, u_n) = d(u'_1 u_1, u_2, \ldots, u_n)$ for all $u_1, u'_1 \in U_1 ,u_2 \in U_2,\ldots,u_n \in U_n$, then N is commutative ring**.** We have extended this result in the setting of left generalized n-derivation and u_1, u'_1 belong to different semigroup ideals by proving the following theorem :

theorem 3.2 . Let N be a prime near-ring admitting a nonzero left generalized n-derivation f with associated n-derivation d of N . Let $U, V,U_2, \ldots, U_n$ be nonzero semigroup left ideals of N. If $f(uv, u_2, \ldots, u_n) = f(vu , u_2, \ldots, u_n)$ for all $u \in U , v \in V ,u_2 \in U_2,\ldots,u_n \in U_n$, then N is commutative ring **.**

Proof . We have $f(uv, u_2, \ldots, u_n) = f(vu , u_2, \ldots, u_n)$ for all $u \in U , v \in V ,u_2 \in U_2,\ldots,u_n \in U_n$, , hence we have
$f(uv - vu, u_2, \ldots, u_n) = 0$. (4)

Putting vu for u in (4) we get $f(v(uv – vu), u_2, \ldots, u_n) = 0$, hence we get
$d(v, u_2, \ldots, u_n)(uv –vu) + vf (uv - vu, u_2, \ldots, u_n) = 0$, using (4) again we find
$d(v, u_2, \ldots, u_n)(uv –vu) = 0$,i.e.; $d(v, u_2, \ldots, u_n)uv = d(v, u_2, \ldots, u_n)vu$ Replacing u by ur ,where $r \in N$ we get $d(v, u_2, \ldots, u_n) urv = d(v, u_2, \ldots, u_n)vur = d(v, u_2, \ldots, u_n)uvr$, hence we have $d(v, u_2, \ldots, u_n)u[v, r] = 0$. i.e ; $d(v, u_2, \ldots, u_n)U[v , r]=\{0\}$. By lemma 2.2 , we conclude that for each $v \in V$ either $d(v,u_2,\ldots,u_n) = 0$ or $v \in Z$ but using lemma 2.5 lastly we get $d(v,u_2,\ldots,u_n) \in Z$ for all $v \in V ,u_2 \in U_2,\ldots,u_n \in U_n$, i.e., $d(V,U_2,\ldots,U_n) \subseteq Z$. Now by using lemma 2.6 we find that N is commutative ring .

Corolary 3.4. Let N be a prime near-ring admitting a nonzero left generalized n-derivation f with associated n-derivation d of N. Let $U_1, U_2, \ldots, U_n$ be nonzero semigroup left ideals of N. If $f(u_1 u'_1, u_2, \ldots, u_n) = f(u'_1 u_1 , u_2, \ldots, u_n)$ for all $u_1 , u'_1 \in U_1 ,u_2 \in U_2,\ldots,u_n \in U_n$, then N is commutative ring **.**

Corollary 3.5 Let N be a prime near-ring admitting a nonzero left generalized n-derivation f with associated n-derivation d of N . If $f(x_1 x'_1, x_2, \ldots, x_n) = f(x'_1 x_1 , x_2, \ldots, x_n)$ for all $x_1 , x'_1 ,x_2,\ldots,x_n \in N$, then N is commutative ring **.**

Corollary 3.6 Let N be a prime near-ring admitting a nonzero left generalized derivation f with associated derivation d of N . Let U be nonzero left semigroup ideal of N. If $f(x_1 x'_1) = f(x'_1 x_1)$ for all $x_1 , x'_1 \in U$,then N is commutative ring .

Corollary 3.7 Let N be a prime near-ring admitting a nonzero left generalized derivation f with associated derivation d of N . Let U and V be nonzero semigroup left ideals of N. If $f(u v) = f(vu)$ for all $u \in U$ and $v \in V$, then N is commutative ring **.**

Corollary 3.8 Let N be a prime near-ring admitting a nonzero left generalized derivation f with associated derivation d of N .If $f(x_1 x'_1) = f(x'_1 x_1)$ for all $x_1 , x'_1 \in N$, then N is commutative ring **.**

Recently Ahmed A.M.Kamal and Khalid H.AL-Shaalan([10],proposition4.2) proved that if a prime near-ring N admitting a nonzero generalized derivation f associated with the zero derivation and generalized n-derivation g such that $f(u)g(v) = g(v)f(u)$ for all $u \in U$ and $v \in V$, where U and V are nonzero semigroup ideals of N. Then N is commutative ring**.** We have extended this result in the setting of generalized n-derivation and semigroup ideals in near rings by proving the following theorem

Theorem 3.3 Let N be a prime near-ring with a nonzero generalized n-derivation associated with the zero n-derivation and generalized n-derivation g . Let $U, V,U_2, \ldots, U_n$ be nonzero semigroup ideals of N. If $[f(U, U_2, \ldots, U_n) , g(V, U_2, \ldots, U_n)] = 0$, then N is commutative ring **.**

Proof . For all $v \in V$, $u_1,u \in U$, $u_2\in U_2,...,u_n \in U_n$, we have $f(u u_1, u_2, \dots, u_n)\, g(v, u_2, \dots, u_n) = g(v, u_2, \dots, u_n)f(uu_1, u_2, \dots, u_n)$. It follows that $f(u, u_2, \dots, u_n)\, u_1g(v, u_2, \dots, u_n) = g(v, u_2, \dots, u_n)f(u, u_2, \dots, u_n)\, u_1= f(u, u_2, \dots, u_n)\, g(v, u_2, \dots, u_n)\, u_1$, replacing u_1 by u_1x ,where $x \in N$, we get $f(u, u_2, \dots, u_n)\, u_1xg(v, u_2, \dots, u_n) = f(u, u_2, \dots, u_n)\, g(v, u_2, \dots, u_n)\, u_1x = f(u, u_2, \dots, u_n)\, u_1g(v, u_2, \dots, u_n)x$, thus $f(u, u_2, \dots, u_n)\, u_1(xg(v, u_2, \dots, u_n) - g(v, u_2, \dots, u_n)x) = 0$, so we get $f(u, u_2, \dots, u_n)\, U\, (xg(v, u_2, \dots, u_n) - g(v, u_2, \dots, u_n)x) = 0$,by lemma 2.2 we get either $f(u, u_2, \dots, u_n) = 0$ or $(xg(v, u_2, \dots, u_n) - g(v, u_2, \dots, u_n)x) = 0$.

Assume $f(u, u_2, \dots, u_n) = 0$ for all $u \in U,u_2\in U_2,...,u_n \in U_n$. (5)

Putting r_1u for u ,where $r_1\in N$, in (5) we get $f(r_1, u_2, \dots, u_n)u = 0$. Now replacing u by tu ,where $t\in N$, in previous relation we get $f(r_1, u_2, \dots, u_n)tu = 0$, i.e ; $f(r_1, u_2, \dots, u_n)NU = \{0\}$.But $U\neq \{0\}$ and N is prime near ring , we conclude that

$f(r_1, u_2, \dots, u_n) = 0$ (6)

Now putting $r_2 u_2 \in U_2$ in place of u_2 , where $r_2\in N$, in (5) and proceeding as above we get $f(r_1, r_2, \dots, u_n) = 0$. Proceeding inductively as before we conclude that $f(r_1, r_2, \dots, r_n) = 0$ for all $r_1, r_2, \dots, r_n \in N$, this shows that $f(N, N, \dots, N) =\{0\}$, leading to a contradiction as f is a nonzero generalized derivation .

Now we conclude that $(xg(v, u_2, \dots, u_n) - g(v, u_2, \dots, u_n)x) = 0$,i.e; $g(v, u_2, \dots, u_n) \in Z$ for all $v \in V,u_2\in U_2,...,u_n \in U_n$,therefore $g(V, U_2, \dots, U_n) \subseteq Z$, by theorem 3.1 we find that N is commutative ring .

Corollary 3.9 . Let N be a prime near-ring with a nonzero generalized n-derivation f associated with the zero n-derivation and generalized n-derivation g. Let $U_1,U_2, \dots,U_n$ be nonzero semigroup ideals of N. If $[f(U_1, U_2, \dots, U_n) , g(U_1, U_2, \dots, U_n)] = 0$, then N is commutative ring **.**

Corollary 3.10. ([10]Proposition 4.2) Let N be a prime near-ring with a nonzero generalized derivation f associated with the zero derivation and generalized n-derivation g such that $f(u)g(v) = g(v)f(u)$ for all $u \in U$ and $v \in V$, where U and V are nonzero semigroup ideals of N. Then N is commutative ring **.**

Corollary 3.11. Let N be a prime near-ring with a nonzero generalized n-derivation f associated with the zero n-derivation and generalized n-derivation g. If $[f(N, N, \dots, N) , g(N, N, \dots, N)]= 0$, then N is commutative ring **.**

Corollary 3.12. Let N be a prime near-ring with a nonzero generalized derivation f associated with the zero derivation and generalized n-derivation g such that $[f(N),g(N)] = 0$. Then N is commutative ring **.**

Theorem 3.4. Let N be a prime near-ring admitting a generalized n-derivation f with associated n-derivation d of N, and $U_1,U_2,...,U_n$ be a nonzero semigroup left ideals of N, such that $d(Z,U_2,\cdots,U_n) \neq\{0\}$ and $t \in N$. If $[f(U_1, U_2, \dots, U_n) , t] = 0$, then $t\in Z$.

Proof . Since $d(Z,U_2,\cdots,U_n) \neq \{0\}$, then there exist $z\in Z,u_2\in U_2,...,u_n \in U_n$ all being nonzero such that $d(z,u_2,\cdots,u_n) \neq 0$. Furthermore , by lemma 2.5 we get $d(z,u_2,\cdots,u_n) \in Z$. By hypothesis we get $f(zu_1,u_2,\cdots,u_n)t = t\, f(zu_1,u_2,\cdots,u_n)$ for all $u_1\in U_1$, using lemma 2.12 we get $d(z,u_2,\cdots,u_n)\, u_1t + zf(u_1,u_2,\cdots,u_n)t = td(z,u_2,\cdots,u_n)\, u_1 + tzf(u_1,u_2,\cdots,u_n)$. Since both $d(z,u_2,\cdots,u_n)$ and z are element of Z , using hypothesis in previous equation takes the form $d(z,u_2,\cdots,u_n)[u_1 , t] = 0$ i.e ; $d(z,u_2,\cdots,u_n)N[u_1 , t] = 0$, primeness of N and d $d(z,u_2,\cdots,u_n) \neq 0$ yields $[u_1 , t] = 0$, we conclude that $tu_1 = u_1t$ Now replacing u_1 by u_1r ,where $r \in N$, in preceding relation and using it again we get $u_1[t , r] = 0$,i.e.; $U_1[t , r] = 0$,by lemma 2.1 we conclude $[t , r] = 0$ for all $r \in N$,i.e.;
$t\in Z$.

Corollary 3.13 .([4] theorem 3.12) Let N be a prime near-ring admitting a generalized n-derivation f with associated n-derivation d of N such that $d(Z,N,\cdots,N) \neq\{0\}$ and $t \in N$. If $[f(N, N, \dots, N) , t] = 0$, then $t\in Z$.

Corollary 3.14. Let N be a prime near-ring admitting a generalized derivation f with associated derivation d of N such that $d(Z) \neq\{0\}$, let U be a nonzero semigroup ideal of N and $t \in N$. If $[f(U) , t] = 0$, then $t\in Z$.

Corollary 3.15.([8] Theorem 3.5). Let N be a prime near-ring admitting a generalized derivation f with associated derivation d of N such that $d(Z) \neq\{0\}$ and $t \in N$. If $[f(N) , t] = 0$, then $t\in Z$.

Theorem 3.5. Let N be a prime near-ring admitting a generalized n-derivation f with associated n-derivation d of N and $U_1,U_2,...,U_n$ be a nonzero semigroup left ideals such that $d(Z, U_2, \dots, U_n) \neq \{0\}$. If $g:\underbrace{N \times N \times \dots \times N}_{n-times} \longrightarrow N$ is a map such that $[f(U_1, U_2, \dots, U_n), g(U_1, U_2, \dots, U_n)] = \{0\}$ then $g(U_1, U_2, \dots, U_n) \subseteq Z$.

Proof . Taking $g(U_1, U_2, \dots, U_n)$ instead of t in Theorem 3.4 ,we get required result .

Theorem 3.6. Let N be a prime near-ring admitting a generalized n-derivation f with associated n-derivation d of N and $U_1,U_2,...,U_n$ be a nonzero semigroup left ideals such that $d(Z, U_2, \dots, U_n) \neq\{0\}$. If g is a nonzero generalized n-derivation of N such that $[f(U_1, U_2, \dots, U_n), g(U_1, U_2, \dots, U_n)] = \{0\}$, then N is a commutative ring.

Proof . By Theorem 3.5 , we get $g(U_1, U_2, \dots, U_n) \subseteq Z$, by theorem 3.1 we conclude that N is commutative ring .

Theorem 3.7. Let f and g be generalized n-derivations of prime near-ring N with associated nonzero n-derivations d and h of N respectively such that $f(x_1,x_2,\cdots,x_n)h(y_1,y_2,\cdots,y_n) = -g(x_1,x_2,\cdots,x_n)d(y_1,y_2,\cdots,y_n)$ for all

$x_1,y_1 \in U_1,x_2, y_2 \in U_2, \ldots, x_n, y_n \in Un$, where $U_1,U_2,\ldots,U_n$ be a nonzero ideals . Then (N,+) is an abelian group.

Proof. For all $x_1,y_1 \in U_1,x_2, y_2 \in U_2, \ldots, x_n, y_n \in Un$ we have ,
$f(x_1,x_2,\cdots,x_n)h(y_1,y_2,\cdots,y_n) = -g(x_1,x_2,\cdots,x_n)d(y_1,y_2,\cdots,y_n)$. We substitute $y_1+y'_1$ for y_1, where $y_1' \in U_1$, in preceding relation thereby obtaining ,
$f(x_1,x_2,\cdots,x_n)h(y_1+y'_1,y_2,\cdots,y_n) + g(x_1,x_2,\cdots,x_n)d(y_1+y'_1,y_2,\cdots,y_n) = 0$; hence we get
$f(x_1,x_2,\cdots,x_n)h(y_1,y_2,\cdots,y_n)+f(x_1,x_2,\cdots,x_n)h(y'_1,y_2,\cdots,y_n) + g(x_1,x_2,\cdots,x_n)d(y_1,y_2,\cdots,y_n) + g(x_1,x_2,\cdots,x_n)d(y'_1,y_2,\cdots,y_n) = 0$; that is
$f(x_1,x_2,\cdots,x_n)h(y_1,y_2,\cdots,y_n)+f(x_1,x_2,\cdots,x_n)h(y'_1,y_2,\cdots,y_n) - f(x_1,x_2,\cdots,x_n)h(y_1,y_2,\cdots,y_n) - f(x_1,x_2,\cdots,x_n)h(y'_1,y_2,\cdots,y_n) = 0$; thus we get
$f(x_1,x_2,\cdots,x_n)(h(y_1,y_2,\cdots,y_n) + h(y'_1,y_2,\cdots,y_n) - h(y_1,y_2,\cdots,y_n) - h(y'_1,y_2,\cdots,y_n)) = 0$, hence $f(x_1,x_2,\cdots,x_n)h(y_1 + y'_1 - y_1 + y'_1,y_2,\cdots,y_n) = 0$. Now using lemma 2.15 we get
$h(y_1 + y'_1 - y_1 + y'_1,y_2,\cdots,y_n) = 0$,that is $h((y_1 , y'_1),y_2,\cdots,y_n) = 0$, replacing (y_1 , y'_1) by $y'_1 (y_1 , y'_1)$ in previous relation and used it again we get $h(y'_1, y_2,\cdots,y_n) (y_1 , y'_1) = 0$, by lemma 2.16 we conclude that $(y_1 , y'_1) = 0$,i.e.; $y_1 + y'_1= y'_1 + y_1$ for all $y_1, y_1' \in U_1$. Now let $x,y \in N$ then ux , uy $\in U_1$ for all $u \in U_1$, so we have ux+uy = uy + ux , hence ux + uy −ux − uy = 0 , i.e.; u(x+y-x-y)= 0 for all $u \in U_1$, that is U_1 (x+y-x-y)= 0 ,by lemma 2.1 we conclude x+y-x-y = 0 , then (N ,+) is an abelain group .

Corollary 3.16([3]Theorem 3.15). Let f and g be generalized n-derivations of prime near-ring N with associated nonzero n-derivations d and h of N respectively such that $f(x_1,x_2,\cdots,x_n)h(y_1,y_2,\cdots,y_n) = -g(x_1,x_2,\cdots,x_n)d(y_1,y_2,\cdots,y_n)$ for all $x_1, x_2, \ldots, x_n ,y_1 , y_2, \ldots, y_n \in N$. Then (N,+) is an abelian group.

Corollary 3.17([2]Theorem 3.4). Let d and h be apermutting n-derivations of prime near-ring N such that $d(x_1,x_2,\cdots,x_n)h(y_1,y_2,\cdots,y_n) = -h(x_1,x_2,\cdots,x_n)d(y_1,y_2,\cdots,y_n)$ for all $x_1, x_2, \ldots, x_n ,y_1 , y_2, \ldots, y_n \in N$. Then (N,+) is an abelian group.

Recently Öznur Gölbasi ([8],Theorem 3.2) showed that if f is a generalized derivation of a prime near-ring N with associated nonzero derivation d such thatf $f([x , y]) = \pm[x , y]$ for all $x,y \in N$, then N is a commutative ring Mohammad Ashraf and Mohammad Aslam Siddeeque ([3],Theorem 3.3) extended this result in the setting of left generalized n-derivations in prime near-rings by proving that if N is a prime near-ring admitting a nonzero left generalized n-derivation f with associated n-derivation d of N such that $f([x_1, x'_1], x_2, \ldots, x_n) = \pm[x_1, x'_1]$, for all $x'_1, x_1 ,x_2,\ldots,x_n \in N$, then N is commutative ring**.** We have extended these results in the setting of left generalized n-derivations and semigroup ideals in prime near-rings by establishing the following theorem.

Theorem 3.8 Let N be a prime near-ring admitting a nonzero left generalized n-derivation f with associated n-derivation d of N. Let $U_1, U_2, \ldots,U_n$ be nonzero semigroup ideals of N. If $f([u_1, u'_1], u_2, \ldots, u_n) = \pm[u_1, u'_1]$, for all $u_1, u'_1 \in U_1 ,u_2 \in U_2,\ldots,u_n \in U_n$, then N is commutative ring**.**

Proof . Since $f([u_1, u'_1], u_2, \ldots, u_n) = \pm[u_1, u'_1]$, for all $u_1, u'_1 \in U_1 ,u_2 \in U_2,\ldots,u_n \in U_n$**.** Replacing u'_1 by $u_1u'_1$ in preceding relation and using it again we get $d(u_1, u_2, \ldots, u_n) [u_1, u'_1]=0$,i.e.;

$$d(u_1, u_2, \ldots, u_n)u_1u'_1=d(u_1, u_2, \ldots, u_n)u'_1 u_1 \quad (7)$$

Replacing u'_1 by u'_1r, where $r \in N$, in relation (7) and using it again we get $d(u_1, u_2, \ldots, u_n) u'_1 [u_1, r]= 0$,i.e.; $d(u_1, u_2, \ldots, u_n) U_1 [u_1, r] = \{0\}$, By using lemma 2.1 ,we conclude that for each $u_1 \in U_1$ either $u_1 \in Z$ or $d(u_1,u_2,\ldots,u_n) = 0$, but using lemma 2.5 lastly we get $d(u_1,u_2,\ldots,u_n) \in Z$ for all $u_1 \in U_1 ,u_2 \in U_2,\ldots,u_n \in U_n$, i.e., $d(U_1,U_2,\ldots,U_n) \subseteq Z$. Now by using lemma 2.6 we find that N is commutative ring .

Corollary 3.18 ([3] Theorem 3.3.). Let N be a prime near-ring admitting a nonzero left generalized n-derivation f with associated n-derivation d of N. If $f([x_1, x'_1], x_2, \ldots, x_n) = \pm[x_1, x'_1]$, for all $x'_1, x_1 ,x_2,\ldots,x_n \in N$, then N is commutative ring**.**

Theorem 3.9 Let N be a prime near-ring admitting a nonzero left generalized n-derivation f with associated n-derivation d of N. Let $U_1, U_2, \ldots,U_n$ be nonzero semigroup ideals of N. If $f (u_1 \circ u'_1, u_2, \ldots, u_n) = 0$, for all $u_1, u'_1 \in U_1 ,u_2 \in U_2,\ldots,u_n \in U_n$, then N is commutative ring**.**

Proof . We have $f (u_1 \circ u'_1, u_2, \ldots, u_n) = 0$, for all $u_1, u'_1 \in U_1 ,u_2 \in U_2,\ldots,u_n \in U_n$. Substituting $u_1 u'_1$ for u'_1 we obtain $f(u_1(u_1 \circ u'_1), u_2, \ldots, u_n) = 0$, i.e.; $d(u_1, u_2, \ldots, u_n) (u_1 \circ u'_1) + u_1f((u_1 \circ u'_1), u_2, \ldots, u_n) = 0$. By hypothesis we get $d(u_1, u_2, \ldots, u_n) (u_1 \circ u'_1) = 0$, i.e.; $d(u_1, u_2, \ldots, u_n) u_1 u'_1 = - d(u_1, u_2, \ldots, u_n) u'_1u_1$ (8)

Putting u'_1z for u'_1, where $z \in N$, in (8) we have $d(u_1, u_2, \ldots, u_n) u_1 u'_1z = - d(u_1, u_2, \ldots, u_n) u'_1zu_1$,and using (8) again we get($-d(u_1, u_2, \ldots, u_n) u'_1 u_1)z = - d(u_1, u_2, \ldots, u_n) u'_1zu_1$ that is $d(u_1, u_2, \ldots, u_n) u'_1(-u_1)z + d(u_1, u_2, \ldots, u_n) u'_1zu_1 = 0$. Now replacing u_1 by $-u_1$ in preceding relation we have $d(-u_1, u_2, \ldots, u_n) u'_1u_1z + d(-u_1, u_2, \ldots, u_n) u'_1z(-u_1) =0$,i.e.; $d(-u_1, u_2, \ldots, u_n) u'_1[u_1z,z u_1] = 0$,that is $d(-u_1, u_2, \ldots, u_n) U_1[u_1z,z u_1] = 0$ .For each fixed $u_1 \in U_1$ lemma 2.2 yields either $u_1 \in Z$ or $d(-u_1, u_2, \ldots, u_n) = 0$. If the first case holds then by lemma 2.5 we conclude that $d(u_1, u_2, \ldots, u_n) \in Z$ and second case implies $- d(u_1, u_2, \ldots, u_n) = 0$ that is $0 = d(u_1, u_2, \ldots, u_n) \in Z$. Combining the both case we get $d(u_1, u_2, \ldots, u_n) \in Z$ for all $u_1 \in U_1 ,u_2 \in U_2,\ldots,u_n \in U_n$, i.e.; $d(U_1, U_2, \ldots, U_n) \subseteq Z$, ,thus by lemma 2.6 we find that N is commutative ring.

Corollary 3.18([3] Theorem 3.4). Let N be a prime near-ring admitting a nonzero left generalized n-derivation f with associated n-derivation d of . If $f(x \circ y, r_2, \ldots, r_n) = 0$, for all $x, y, r_2, \ldots, r_n \in N$, then N is commutative ring.

The conclusion of Theorem 3.8 remain valid if we replace the product $[u_1, u'_1]$ by $u_1 \circ u'_1$. In fact , we obtain the following results .

theorem 3.10 Let N be a prime near-ring admitting a nonzero left generalized n-derivation f with associated n-derivation d of N. Let $U_1, U_2, \ldots, U_n$ be nonzero semigroup ideals of N. If $f(u_1 \circ u'_1, u_2, \ldots, u_n) = \pm (u_1 \circ u'_1)$, for all $u_1, u'_1 \in U_1, u_2 \in U_2, \ldots, u_n \in U_n$, then N is a commutative ring.

Proof . We have $f(u_1 \circ u'_1, u_2, \ldots, u_n) = \pm u_1 \circ u'_1$, for all $u_1, u'_1 \in U_1, u_2 \in U_2, \ldots, u_n \in U_n$. Substituting $u_1 u'_1$ for u'_1 we obtain $f(u_1(u_1 \circ u'_1), u_2, \ldots, u_n) = \pm u_1(u_1 \circ u'_1)$, i.e.; $d(u_1, u_2, \ldots, u_n)(u_1 \circ u'_1) + u_1 f((u_1 \circ u'_1), u_2, \ldots, u_n) = \pm u_1(u_1 \circ u'_1)$. By hypothesis we get $d(u_1, u_2, \ldots, u_n)(u_1 \circ u'_1) = 0$, i.e.; $d(u_1, u_2, \ldots, u_n) u_1 u'_1 = - d(u_1, u_2, \ldots, u_n) u'_1 u_1$ which is identical with the relation (8) in theorem 3.9 .Now arguing in the same way in the theorem we conclude that N is a commutative ring.

Corollary 3.19 ([3] Theorem 3.5) Let N be a prime near-ring admitting a nonzero left generalized n-derivation f with associated n-derivation d of N.. If $f(x \circ y, r_2, \ldots, r_n) = \pm (x \circ y)$, for all $x, y, r_2, \ldots, r_n \in N$, then N is a commutative ring.

Theorem 3.11 Let N be a prime near-ring admitting a nonzero left generalized n-derivation f with associated n-derivation d of N. Let $U_1, U_2, \ldots, U_n$ be nonzero semigroup ideals of N. If $f([u_1, u'_1], u_2, \ldots, u_n) = \pm(u_1 \circ u'_1)$, for all $u_1, u'_1 \in U_1, u_2 \in U_2, \ldots, u_n \in U_n$, then N is commutative ring.

Proof . we have $f([u_1, u'_1], u_2, \ldots, u_n) = \pm(u_1 \circ u'_1)$. Substituting $u_1 u'_1$ for u'_1 we obtain $f(u_1 [u_1, u'_1], u_2, \ldots, u_n) = \pm u_1(u_1 \circ u'_1)$, i.e.; $d(u_1, u_2, \ldots, u_n)[u_1, u'_1] + u_1 f([u_1, u'_1], u_2, \ldots, u_n) = \pm u_1(u_1 \circ u'_1)$. By hypothesis we get $d(u_1, u_2, \ldots, u_n)[u_1, u'_1] = 0$, i.e.; $d(u_1, u_2, \ldots, u_n) u_1 u'_1 = d(u_1, u_2, \ldots, u_n) u'_1 u_1$ which is identical with the relation (7) .Now arguing in the same way in the Theorem 3.8 we conclude that N is a commutative ring.

Corollary 3.20([3] Theorem 3.6) Let N be a prime near-ring admitting a nonzero left generalized n-derivation f with associated n-derivation d of N.. If $f([x, y], r_2, \ldots, r_n) = \pm (x \circ y)$, for all $x, y, r_2, \ldots, r_n \in N$, then N is a commutative ring.

Theorem 3.12 . Let N be a prime near-ring admitting a nonzero left generalized n-derivation f with associated n-derivation d of N. Let $U_1, U_2, \ldots, U_n$ be nonzero semigroup ideals of N. If $f((u_1 \circ u'_1), u_2, \ldots, u_n) = \pm[u_1, u'_1]$, for all $u_1, u'_1 \in U_1, u_2 \in U_2, \ldots, u_n \in U_n$, then N is commutative ring.

Proof . We have $f(u_1 \circ u'_1, u_2, \ldots, u_n) = \pm [u_1, u'_1]$, for all $u_1, u'_1 \in U_1, u_2 \in U_2, \ldots, u_n \in U_n$. Substituting $u_1 u'_1$ for u'_1 we obtain $f(u_1(u_1 \circ u'_1), u_2, \ldots, u_n) = \pm u_1[u_1, u'_1]$, i.e.; $d(u_1, u_2, \ldots, u_n)(u_1 \circ u'_1) + u_1 f((u_1 \circ u'_1), u_2, \ldots, u_n) = \pm u_1[u_1, u'_1]$. By hypothesis we get $d(u_1, u_2, \ldots, u_n)(u_1 \circ u'_1) = 0$, i.e.; $d(u_1, u_2, \ldots, u_n) u_1 u'_1 = - d(u_1, u_2, \ldots, u_n) u'_1 u_1$ which is identical with (8) .Now arguing in the same way in the theorem 8.9 we conclude that N is a commutative ring.

Corollary 3.21([3] Theorem 3.7) Let N be a prime near-ring admitting a nonzero left generalized n-derivation f with associated n-derivation d of N. If $f((x \circ y), r_2, \ldots, r_n) = \pm[x, y]$, for all $x, y, r_2, \ldots, r_n \in N$, then N is a commutative ring.

Theorem 3.13 Let N be a prime near-ring admitting a nonzero generalized n-derivation f with associated n-derivation d of N. Let $U_1, U_2, \ldots, U_n$ be nonzero ideals of N. If $f([u_1, u'_1], u_2, \ldots, u_n) \in Z$, for all $u_1, u'_1 \in U_1, u_2 \in U_2, \ldots, u_n \in U_n$, then N is commutative ring.

Proof . We have

$f([u_1, u'_1], u_2, \ldots, u_n) \in Z$ for all $u_1, u'_1 \in U_1, u_2 \in U_2, \ldots, u_n \in U_n$. (9)

(1) If $Z = \{0\}$ then $d([u_1, u'_1], u_2, \ldots, u_n) = 0$ for all $u_1, u'_1 \in U_1, u_2 \in U_2, \ldots, u_n \in U_n$.

By lemma 2.7 , we conclude that N is a commutative ring .

(2) If $Z \neq \{0\}$, replacing u_1' by zu_1' in (9) where $z \in Z$, we get $f([u_1, z u_1'], u_2, \ldots, u_n) = f(z[u_1, u'_1], u_2, \ldots, u_n) \in Z$ for all $u_1, u'_1 \in U_1, u_2 \in U_2, \ldots, u_n \in U_n$, $z \in Z$. That is mean $f(z[u_1, u'_1], u_2, \ldots, u_n) r = r f(z[u_1, u'_1], u_2, \ldots, u_n)$ for all $u_1, u'_1 \in U_1, u_2 \in U_2, \ldots, u_n \in U_n$, $z \in Z$, $r \in N$. By using lemma 2.12 we get

$d(z, u_2, \ldots, u_n)[u_1, u'_1] r + z f([u_1, u'_1], u_2, \ldots, u_n) r = r d(z, u_2, \ldots, u_n)[u_1, u'_1] + r z f([u_1, u'_1], u_2, \ldots, u_n)$

Using (9) the previous equation implies

$[d(z, u_2, \ldots, u_n)[u_1, u'_1], r] = 0$ for all $u_1, u'_1 \in U_1, u_2 \in U_2, \ldots, u_n \in U_n$, $z \in Z$, $r \in N$.

Accordingly , $0 = [d(z, u_2, \ldots, u_n)[u_1, u'_1], r] = d(z, u_2, \ldots, u_n)[[u_1, u'_1], r]$ for all $r \in N$. Then we get $t d(z, u_2, \ldots, u_n)[[u_1, u'_1], r] = 0$ for all $t \in N$, so by lemma 2.5 we get

for all $u_1, u'_1 \in U_1, u_2 \in U_2, \ldots, u_n \in U_n$, $z \in Z$, $r \in N$

$d(z, u_2, \ldots, u_n) N [[u_1, u'_1], r] = 0$ (9)

Primeness of N yields either $d(Z, U_2, \ldots, U_n) = 0$ or $[[u_1, u'_1], r] = 0$ for all $u_1, u'_1 \in U_1$, $r \in N$.

Assume that $[[u_1, u'_1], r] = 0$ for all $u_1, u'_1 \in U_1$, $r \in N$ (10)

Replacing u'_1 by $u_1 u'_1$ in (10) yields

$[[u_1, u_1 u'_1], r] = 0$ and therefore $[u_1[u_1, u'_1], r] = 0$, hence $[u_1, u'_1][u_1, r] = 0$ for all $u_1, u'_1 \in U_1$, $r \in N$, so we get

$[u_1, u'_1]N[u_1, r] = 0$ for all $u_1, u'_1 \epsilon U_1, r \epsilon N$. (11)

Primeness of N implies that either $[u_1, u'_1] = 0$ for all $u_1, u'_1 \epsilon U_1$, or $u_1 \epsilon Z$ for all $u_1 \epsilon U_1$. If $[u_1, u'_1] = 0$ for all $u_1, u'_1 \epsilon U_1$ then we get $d([u_1, u'_1],u_2,...,u_n) = 0$ for all $u_1, u'_1 \epsilon U_1, u_2 \epsilon U_2,...,u_n \epsilon U_n$ and by lemma 2.7 we get the required result , now assume that $u_1 \epsilon Z$ for all $u_1 \epsilon U_1$, then by lemma 2.5 we obtain that $d(U_1,U_2,...,U_n) \subseteq Z$. Now by using lemma 2.6 we find that N is commutative ring .

On the other hand , if $d(Z,U_2,...,U_n) = 0$,then $d(d([u_1, u'_1],u_2,...,u_n),u_2,...,u_n) = 0$ for all $u_1, u'_1 \epsilon U_1, u_2 \epsilon U_2,...,u_n \epsilon U_n$, replace $u_{1'}$ by $u_1 u'_1$ in the previous equation we get

$0 = d(d([u_1,u_1u'_1],u_2,...,u_n),u_2,...,u_n) = d(d(u_1[u_1,u'_1],u_2,...,u_n),u_2,...,u_n) = d(d(u_1,u_2,..., u_n)[u_1, u'_1] + u_1d([u_1, u'_1],u_2,...,u_n),u_2,...,u_n) = d(d(u_1,u_2,...,u_n)[u_1, u'_1], u_2,...,u_n) + d(u_1d([u_1, u'_1],u_2,...,u_n),u_2,...,u_n) = d(d(u_1,u_2,...,u_n), u_2,...,u_n)[u_1, u'_1] + d(u_1,u_2,...,u_n)d([u_1, u'_1], u_2,...,u_n) + d(u_1,u_2,...,u_n)d([u_1, u'_1],u_2,...,u_n) + u_1 d(d([u_1, u'_1],u_2,...,u_n),u_2,...,u_n))$, hence we get for all $u_1, u'_1 \epsilon U_1, u_2 \epsilon U_2,...,u_n \epsilon U_n$

$d(d(u_1,u_2,...,u_n),u_2,...,u_n)[u_1,u'_1] + 2d(u_1,u_2,...,u_n)d([u_1,u'_1],u_2,...,u_n) = 0$ (12)

Replace u_1 by $[x_1,y_1]$ in (12) ,where $x_1,y_1 \epsilon U_1$, we get $2d([x_1,y_1],u_2,...,u_n)d([[x_1,y_1],u_1'], u_2,...,u_n) = 0$ for all $x_1,y_1, u_1' \epsilon U_1, u_2 \epsilon U_2,...,u_n \epsilon U_n$, but N is 2-torsion free so we obtain $d([x_1,y_1],u_2,...,u_n)d([[x_1,y_1], u_1'], u_2,...,u_n) = 0$ for all $x_1,y_1, u_1' \epsilon U_1, u_2 \epsilon U_2,...,u_n \epsilon U_n$.

From(9) we get $d([x_1,y_1],u_2,...,u_n)Nd([[x_1,y_1], u_1'], u_2,...,u_n) = 0$, primeness of N yields either $d([x_1,y_1],u_2,...,u_n) = 0$ for all $x_1,y_1 \epsilon U_1, u_2 \epsilon U_2,...,u_n \epsilon U_n$ and by lemma 2.7 we conclude that N is commutative ring .

or $d([[x_1,y_1], u_1'],u_2,...,u_n) = 0$ for all $x_1,y_1, u_1' \epsilon U_1,u_2 \epsilon U_2,...,u_n \epsilon U_n$, hence $0 = d(([x_1,y_1] u_1' - u_1' [x_1,y_1]), u_2,...,u_n) = d([x_1,y_1] u_1',u_2,...,u_n) - d(u_1' [x_1,y_1],u_2,...,u_n) = [x_1,y_1]d(u_1',u_2,...,u_n) + d([x_1,y_1],u_2,...,u_n) u_1' - (u_1'd([x_1,y_1],u_2,...,u_n) + d(u_1', u_2,...,u_n)[x_1,y_1])$, using(9) in the last equation yields for all $x_1,y_1, u_1' \epsilon U_1, u_2 \epsilon U_2,...,u_n \epsilon U_n$.

$[x_1,y_1]d(u_1', u_2,...,u_n) = d(u_1', u_2,...,u_n)[x_1,y_1]$ (13)

Let $x_2,y_2,t \epsilon U_1$, then $t[x_2,y_2] \epsilon U_1$, hence we can taking $t[x_2,y_2]$ instead of u_1' in (13) to get $[x_1,y_1]d(t[x_2,y_2],u_2,...,u_n)) = d(t[x_2,y_2], u_2,...,u_n)[x_1,y_1]$, hence $[x_2,y_2]d(t[x_2,y_2], u_2,...,u_n) = d(t[x_2,y_2], u_2,...,u_n) [x_2,y_2]$, therefore

$[x_2,y_2](d(t,u_2,...,u_n)[x_2,y_2] + [x_2,y_2]td([x_2,y_2], u_2,...,u_n) = d(t,u_2,...,u_n)[x_2,y_2]^2 + td([x_2,y_2], u_2,...,u_n) [x_2,y_2]$, using (12)and(8) implies

$d([x_2,y_2],u_2,...,u_n) [x_2,y_2]t = d([x_2,y_2],u_2,...,u_n)t[x_2,y_2]$, so we have

$d([x_2,y_2],u_2,...,u_n) [[x_2,y_2],t] = 0$. i.e ; $d([x_2,y_2],u_2,...,u_n)N[[x_2,y_2],t] = \{0\}$ for all $t \epsilon U_1$. Primeness of N yields that $d([x_2,y_2],u_2,...,u_n)=0$ or $[[x_2,y_2],t] = 0$ for all $t \epsilon U_1$, if $d([x_2,y_2],u_2,...,u_n) = 0$ then by lemma 2.7 we conclude that N is commutative ring .

Now , when $[[x_2,y_2],t]=0$ for all $t \epsilon U_1$, Replacing y_2 by x_2y_2 in previous equation yields $[[x_2, x_2y_2], t] = 0$ and therefore $x_2[x_2, y_2], t] = 0$, hence $[x_2,y_2][x_2, t] = 0$ for all $x_2,y_2,t \epsilon U_1$, so we get $[x_2, y_2] U_1 [x_2, t] = 0$,by lemma 2.1 we get $[x_2, y_2] = 0$ for all $x_2,y_2 \epsilon U_1$ so we have $d([x_2,y_2],u_2,...,u_n)=0$ then by lemma 2.7 we find that N is commutative ring .

Corollary 3.22([3] Theorem 3.8) Let N be a prime near-ring admitting n-derivation f with associated n-derivation d of N. If $f([x, y], r_2, . . ., r_n) \epsilon Z$, for all $x ,y, r_2, . . ., r_n \epsilon N$, then N is a commutative ring or $d(Z,N,..,N) =\{0\}$.

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