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# On – Acyclic Domination − Parameter

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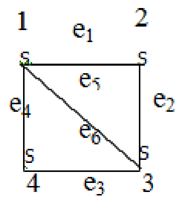
***Abstract:*** *Let G be a graph. The cardinalty of a minimum acyclic dominating set of G, is called the acyclic domination number of G and is denoted by $\gamma_a(G)$. A subset $E_1$ of E(G) is called an edge-vertex dominating set if for every vertex w in G, their exists an edge in $E_1$ which dominates w. The minimum cardinality of an edge-vertex dominating set is called the edge-vertex domination number of G and is denoted by $\gamma_{ev}$. An edge e = uv dominates a vertex $w \in V(G)$ if $w \in N[u] \cup N[v]$.*

## I. Introduction

**Definition:** A subset $E_1$ of E (G) is called an edge-vertex dominating set if for every vertex w in G, their exists an edge in $E_1$ which dominates w.
The minimum cardinality of an edge-vertex dominating set is called the edge-vertex domination number of G and is denoted by $\gamma_{ev}$.



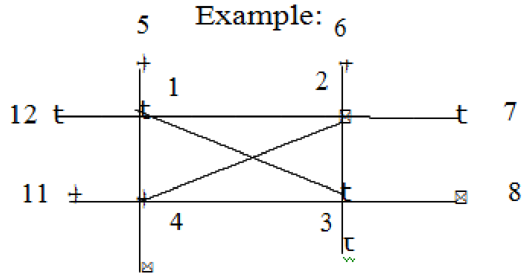

Example:

$K_4$ $\{e_1\}$ is an evd-set of $K_4$, $\gamma_{ev}(K_4) = 1$.

Definition:

et G be a graph.The cardinalty of a minimum acyclic dominating set of G, is called the acyclic domination number of G and is denoted by $\gamma_a(G)$.

Example:




$\gamma = 4$, $\gamma_a = 6$, $\gamma_{ev} = 2$

**Observation**
Let $E_1$ be a minimum evd-set.Then $V(<E_1>)$ is an acyclic dominating set.

Therefore
$\gamma_a(G) \leq |V(<E_1>)|$.
(ie)$|E_1| < |V(<E_1>)|$.
(ie)$\gamma_{ev}(G) < |V(<E_1>)|$.

Observation
In $tk_2$, $\gamma_a(G) = t = \gamma_{ev}(G)$.

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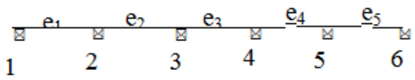
Remark: $P_6$

In $P_6$, 


$\{e_2, e_5\}$ is a minimum evd-set and $\{2, 5\}$ is a minimum acyclic dominat-ing set. Therefore, $\bigcup_{e \in E(H_i)}$
$\gamma_{ev}(G) = \gamma_a(G) = 2$.

**Remark :** In $P_7$,$\gamma_{ev}(G) = 2$ and $\gamma_a(G) = 3$. Therefore $\gamma_{ev}(G) < \gamma_a(G)$.

**Theorem**
Let G be a graph without isolates.Then $\gamma_{ev}(G) \leq \gamma_a(G)$.

**Pf :** Let D be a minimum acyclic dominating set. Let $D = \{u_1, u_2, ......, u_{\gamma a}\}$.Since

G has no isolates, take edges $e_1,e_2,....,e_{\gamma a}$ incident at $u_1,u_2,......,u_{\gamma a}$ respec-tively. Note that $e_1,e_2,.....,e_{\gamma a}$ need not be distinct. clearly the distinct edges from $e_1,e_2,....,e_{\gamma a}$ form a ev-dominating set. Therefore $\gamma_{ev}(G) \leq \gamma_a(G)$.

**Note :** Let D be a minimum acyclic dominating set then D is an independent set.

**Observation :** Let $\gamma_a(G) = \gamma_{ev}(G)$. Let $D = \{u_1, u_2, ....., u_{\gamma a}\}$ be a minimum acyclic domi-nating set. Let $E_1 = \{e_1, e_2, ....., e_{\gamma a}\}$ be a minimum evd-set. Then $< E_1 >$ does not contain $P_4$.

**Pf** :
For suppose

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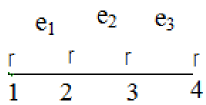
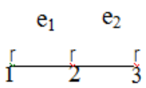
$P_4$




is a subgraph of $< E_1 >$.
Then $N[e_2] \subseteq N[e_1] \cup N[e_3]$. Therefore $E_1 - \{e_2\}$ is a evd-set, a $\Rightarrow\Leftarrow$ . If $< E_1 >$ contains$P_3$:




Then the vertices, 1 and 3 have private neighbours.If $E_1$ contains a star, then each of the non-central vertices must have a private neighbour.

Let $G_1$ be a component of $< E_1 >$. Then $diam(G_1) \leq 2$. For,if $diam(G_1) \geq 3$, then $G_1$ ontains a $P_4$, a $\Rightarrow\Leftarrow$ .Since$G_1$, is connected and $diam(G_1) \geq 2$,
$G_1$ is a star.

Therefore, Every component of $< E_1 >$ is a star. The non-central vertices of every component of $< E_1 >$ must have a private neighbour.

**Theorem**
Let H be any graph with $V(H) = \{u_1, u_2, ...., u_t\}$.Let $u_{i1},u_{i2},..,u_{iki}$ be adjacent to $u_i$, $1 \leq i \leq t$.Let$G_{i1}$, $G_{i2}$,.....,$G_{iki}$, be any graphs in which $u_{i1},u_{i2},....,u_{iri}$,are full degree vertices.Then $\gamma_a(G) = \gamma_{ev}(G)$.

**Pf :** Let $D = \{u_1, u_2, ...., u_k\}$ be a minimum acyclic dominating set of G, where $t = \gamma_a(G)$. Let $H_1, H_2,.....,H_k$ ($k \geq 1$) be the components of $< D >$. If $H_i$
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contains a single-vertex then take any edge passing through that vertex. If $H_i$ contains 2 vertices, then take the edge in $H_i$. If $H_i$ contains 3 or more ver- tices then select set of edges $E_{1i}$ from E(Hi) such that N[Ei]= N[e]. The resulting set of edges is an evd-set of G.

**Therefore,**
$\gamma_G < \gamma_a(G)$, if∃atleastone$H_i$ which contains two or more vertices.

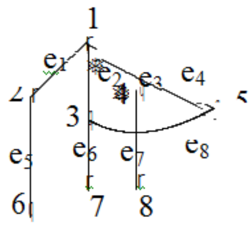
Since $\gamma_{ev}(G) = \gamma_a(G)$, each component of < D > is $k_1$,

Therefore,
$\gamma_{ev}(G) = \gamma_a(G) = i(G)$.

Also, every $\gamma_a$-set of G is independent.

The converse of the above thm is not true.

**Example**



$\gamma_a(G) = 3$, $\gamma_{ev}(G) = 2$

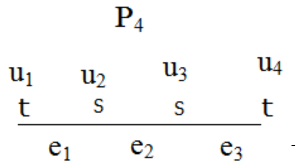
$D_1 = \{2, 3, 4\}$ , $D_2 = \{2, 3, 8\}$ , $D_3 = \{3, 6, 8\}$ , $D_4 = \{2, 7, 8\}$ .
$D_1$, $D_2$, $D_3$, $D_4$ are the only minimum dominating sets of G.
Therefore $\gamma(G) = i(G) = 3$. But $\gamma_{ev}(G) = 2$. Since $\{e_1, e_9\}$ is a minimum
v
evd-set.
Therefore even if every $\gamma_a$-set of a graph is independent, it may not imply that
$\gamma_a(G) = \gamma_{ev}(G)$.(In the above example every $\gamma_a$ − set of G is independent)

**Theorem**

Let G be a simple graph without isolates. $\gamma_{ev}(G) = \gamma_a(G)$ *iff* ∃aminimumevd− set$E_1$, satisfying the following, in each component of < $E_1$ >, the central ver-tices has no private neighbour in V − V (< $E_1$ >).
**Pf :**
Suppose G is a simple graph without isolates satisfying $\gamma_{ev}(G) = \gamma_a(G)$. Since $\gamma_{ev}(G) = \gamma_a(G)$, each component of < D > is $k_1$,
Therefore $\gamma_{ev}(G) = \gamma_a(G) = i(G)$.
Also, every $\gamma_a$-set of G is independent.

Let D = $\{u_1, u_2....., u_\gamma a\}$ be a $\gamma_a$-set of G. Let $E_1 = \{e_1, e_2, ....., e_\gamma a\}$ be a set of edges such that $e_i$ is incident with $u_i$, $1 \leq i \leq \gamma_a$. Let $v_i$ be the other end of $u_i$. Note that $v_1,v_2,......,v_{\gamma a}$ need not be distinct. Now, $E_1$ is a evd-set ofcardinality $\gamma_a$. Since $\gamma_{ev}(G) = \gamma_a(G)$. $E_1$ is a minimum evd-set. Let H be a component of < $E_1$ >.Then ∃some$V_i$, $1 \leq i \leq \gamma_a$ such that H is a star with center $v_i$. Since $\{u_1, u_2, ....., u_{\gamma a}\}$ is a $\gamma_a$-set of G, $V_i$ has no private neighbour in V − V (< $E_1$ >). Therefore G satisfies the condition that G has a minimum evd-set with the condition specified in the thm. Conversely,
Suppose G has a minimum evd-set with the condition specified in the thm. Let
$E_1 \subset E(G)$ be a minimum evd-set with the condition specified in he thm. Let

$G_1$ be a component of (< $E_1$ >). Then diam(G) ≤ 2. For, if diam($G_1$) ≥ 3, then $G_1$, contains a $P_4$, say vi




Then $E_1 - e$ is also an evd-set, a ⇒⇐ totheminimalityof$E_1$.
Therefore, $diam(G_1) \leq 2$. Since $G_1$ is a tree and $|V(G_1)| \geq 2$, $G_1$ is a star. Let D be the set of all non-central vertices from the components of ($< E_1 >$). Then D is an acyclic dominating set of cardinalty $\gamma_{ev}(G)$.

Therefore
$\gamma_a(G) \leq \gamma_{ev}(G)$

But
$\gamma_{ev}(G) \leq \gamma_a(G)$

Therefore
$\gamma_a(G) = \gamma_{ev}(G)$

**Remark :**
Since $\gamma_{ev}(G) \leq \gamma(G) \leq \gamma_G$, if gamma$_{ev}$(G) = $\gamma_a(G)$ then $\gamma_{ev}(G) = \gamma(G) = \gamma_a(G)$.

**Preposition : −1**
For any graph G without isolates $\gamma_{ev}(G) \leq \gamma_a(G) \leq i(G)$. If $\gamma_{ev}(G) = \gamma_a(G)$, then $\gamma_{ev}(G) = \gamma_a(G) = i(G)$.Therefore if $\gamma_{ev}(G) = \gamma_a(G)$ then $\gamma_{ev}(G) =$
$\gamma(G) = \gamma_a(G) = i(G)$.

There are graphs in which $\gamma_{ev}(G) < \gamma_a(G) = i(G)$.
For consider G:

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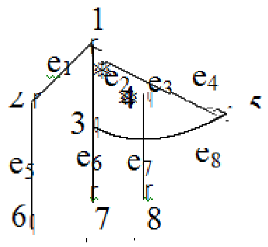


$\gamma_a(G) = 3$, $\gamma_{ev}(G) = 2$.

$D_1 = \{2, 3, 4\}$ , $D_2 = \{2, 3, 8\}$ , $D_3 = \{3, 6, 8\}$ , $D_4 = \{2, 7, 8\}$. $D_1$,

$D_2$, $D_3$, $D_4$ are the only minimum dominating sets of G. Therefore $\gamma(G) = i(G) = \gamma_a(G) = 3$. But $\gamma_{ev}(G) = 2$. Since $\{e_1, e_9\}$ is a minimum evd-set. Therefore, even if every $\gamma_a$-set of a graph is independent, it may not imply that $\gamma_a(G) = \gamma_{ev}(G)$.(In the above examples every $\gamma_a$-set of G is indepen-dent). Observe that the above graph contains $k_{1,3}$ as an induced subgraph and still $\gamma = \gamma_a = i$.

**Preposition − 2 :**
If $\gamma_{ev}(G) = \gamma_a(G)$, then $\gamma_{ev}(G) = \gamma_a(G) = i(G)$ and every $\gamma_a$-set of G is independent. But the converse is not true.
(ie)
∃graphsinwhichevery$\gamma_a$-set is independent, but $\gamma_{ev}(G) < \gamma_a(G)$.
The graph considered

For Preposition-1 is such a graph.

Example : 1
There are graphs in which $\gamma_{ev}(G) < \gamma_a(G) < i(G)$.

For ,let
G:

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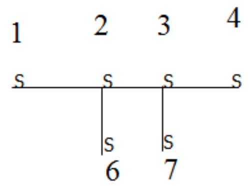
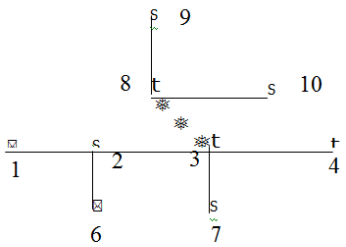



$\gamma$ (G) = 2 , $\gamma_a(G) = 2$ , $\gamma_{ev}(G) = 1$ , i(G) = 3
Therefore,
$\gamma_{ev}(G) < \gamma(G) = \gamma_a(G) < i(G)$.

**Example: 2**
H:




$\gamma$ (H) = 3, $\gamma_a(H) = 4$, $\gamma_{ev}(H) = 2$, i(H) = 5. Therefore
$^{v}2k-1$ $\gamma_{ev}(H) = 2 < \gamma(H) < \gamma_a(H) < i(H)$.

**Observation**
Let G be a graph without isolates. Let V (G) = $\{u_1, u_2....., u_n\}$ , E(G) = $\{e_1, e_2....., e_n\}$. Let H be the graph constructed as follows V (H) = $\{u_1, u_2....., u_n, e_1, e_2....., e_n\}$. $e_i$ is adjacent with $v_j$ if $v_j \in N[ei]$,then H is a bi-paratite graph whose parti-

tions are X = $\{u_1, u_2....., u_n\}$ and Y = $\{e_1, e_2....., e_n\}$. A subset $E_1$ of E(G) is an evd-set iff $E_1 \subseteq Y$ dominates X.

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**Observation**
We have the following chain:

(i) $ir(G) \le \gamma_{ev}(G) \le \gamma(G) \le \gamma_a(G) \le i(G) \le \beta_0(G) \le {}_a(G) \le (G) \le$ IR(G).
(ii) ${}_a(G) \le IR_a(G) \le \beta_a(G)$.
(iii) $ir_a(G) \le \gamma_a(G) \le i_a(G)$.

**Preposition :**
Given a positive integer $k \ge 3$ and a positive integer $m \ge k - 2$ ∃aconnected graphGsuchthat$\gamma_{ev} = \gamma = k$ and $\gamma_a = k + m$.

**Pf :**
Consider $K_{2k}$ with vertex set V = $\{v_1, v_2....., v_{2k}\}$. Attach 2 pendant edges at each of the k vertices, $v_1,v_3,v_7,.....,v_{2k-3}$ and m + 4 − k pendant edges at Let G be the resulting graph. Then $\gamma_{ev}(G) = k = \gamma(G)$.
$\gamma_a(G) = k + m$.

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