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A Note on K_r Excellent Domination Parameter

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Abstract: Let G = (V, E) be a simple graph of order p and size q. A subset S of V is said to be a K_r- dominating set of G if for every vertex v $\in (V - S)$ is K_r- adjacent to atleast one vertex in S. Since v is always a K_r- dominating set, for every r, the existence of K_r-dominating set in G is guaranteed. A K_r- dominating set of minimum cardinality is called a minimum K_r- dominating set and its cardinality is denoted by γ_{kr} . Clearly $\gamma = \gamma_{K2}$ and $\gamma \leq \gamma_{kr}$ for every r > 2

I. Introduction

A vertex v is said to be k_r-adjacent to a vertex u if u and v are contained in a r- clique of G. Let u ∈ V (G),define k_r- neighbourhood, denoted by N_{kr} (v) = {v ∈ V/visk$_r$ − adjacenttou}.IfN_{kr} (u) = φ then u is called a k_r-isolated vertex. Let G = (V, E) be a graph. A subset S ⊆ V is said to be a K_r-dominating set of G if for every vertex v ∈ (V − S) is K_r-adjacent to I atleast one vertex in S. A tree T is said to be γ_{kr} -excellent if for every vertex of T is some γ_{Kr} -set.

Results:

For any n, if G does not contain any r-clique, then γ_{kr} = p. In particular, if r > p then γ_{kr} = p. Therefore we assume that r ≤ p.

(i) $\gamma_{k_r}(K_p) = 1$

(ii) $\gamma_{k_r}(S_{1,p}) = \begin{cases} 1 & \text{if} r = 2 \\ = (p+1) & \text{if} r > 2 \end{cases}$

(iii) $\gamma_{k_r}(W_n) = \begin{cases} \lceil (p/3) \rceil & \text{if} r = 3 \\ = (p+1) & \text{if} r ¿3 \end{cases}$

(iv) $\gamma_{k_r}(P_p) = \begin{cases} [illegible] \\ = (p) & \text{if} r > 2 \end{cases}$

(v) $\gamma_{k_r}(C_p) = \begin{cases} [illegible] \\ = (p) & \text{if} r ¿2 \end{cases}$

(vi) $\gamma_{k_r}(G) = \begin{cases} (p) & \text{if} r = 2 \\ = (p+1) & \text{if} r > 2 \end{cases}$

Where G is the graph obtained from $K_{1,p}$ by dividing each edge exactly once

(vii) $\gamma_{k_r}(K_{p1,p2}) = \begin{cases} 2 & \text{if} r = 2 \\ =(p_1 + p_2) & \text{Otherwise} \end{cases}$

(viii) If r > 2 then any γ_{k_r}-set contains all pendent vertices. Therefore $\gamma_{k_r}(GoK_1) = (p + \gamma_{k_r})(G)$ if r > 2.

Ore's Theorem

II Statement:

A K_r-dominating set S of a graph G is minimal if and only if for every u ∈ S either or both of the following conditions hold.

(i) N_{kr} (u) ∩ S = φ

(ii)∃ a vertex v ∈ (V − S) such that N_{kr} (u) ∩ S = u.

Proof :

Let S be a K_r-dominating set. Then obviously any u ∈ S, Condi-tions (i) (or) (ii) (or) both.
Conversely, assume that for every u ∈ S, conditions (i) (or) (ii) (or) both holds.
Claim : S is a minimal K_r-dominating set.

Suppose not. Then there exists $u \in S$ such that $(S - u)$ is a K_r-dominating set. That is, there exists $v \in S$ such that u is K_r-adjacent to v. (i.e) $N_{kr} \cap S$ 6= φ. (i.e) (i) is not satisfied. Therefore u satisfies condition(ii). (i.e) there exists $v \in (V - S)$ such that
$N_{kr}(v) \cap S = u$. (i.e) $N_{kr}(v) \cap (S - u) = \varphi$. Therefore $(S - u)$ is not a
K_r- dominating, Which is a contradiction. Hence the thm.

Remark:
Let $G = (V, E)$ be a graph with a vertices. Let $r \geq 2$. Then $1 \leq \gamma_{kr} \leq n$ and these bounds are Sharp.

Remark:
Any K_r dominating set with $r \geq 3$ contains all pendent vertices. Also, for a tree T , V (T) is the minimum K_r dominating set for all $r \geq 3$.

iii Result
A graph G has V , as its unique K_r dominating set if and if G contains no
$r -$ clique.

Proof
$=\Rightarrow$
Assume that G contains no r-clique. To prove that G has V as its unique
K_r-dominating set. The proof is Obvious.
$\Leftarrow=$
Assume that graph G has V as its unique K_r dominating set. To prove that

G contains no r-clique. Suppose G contains a r-clique say $\{v_1, v_2, ..., v_r\}$. Then $(V -\{v_2, v_3, ..., v_r\})$ is a K_r-dominating set. Which is a contradiction to our hypothesis.

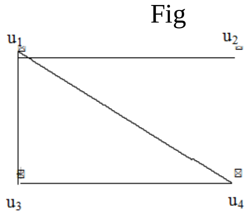
Corollary
If their exists $v \in V$ such that N[V] contains a $r -$ clique. Then $\gamma_{kr}(G) \leq n - r + 1$.

Observation
A graph G has $\gamma_{kr}(G) = 1$ if and only if there exists a point $u \in V(G)$ such that every point is in a r-clique containing u. (i.e) if and only if G is K_r with $r \geq n$ or G is obtained from union of cliques, each of size $\geq n - 1$ and joining every point of each clique to a new point.

Result
Let S be a γ-set. Let the number of points in $(V - S)$ which are not K_r-adjacent to any of the vertices of S be t. Then $\gamma_{kr}(G) = \gamma(G) + t$.

iv
Examples:

Fig



$\gamma = 1 = \gamma_{k2}\ \gamma_{k3} = 1 + 1 = 2$

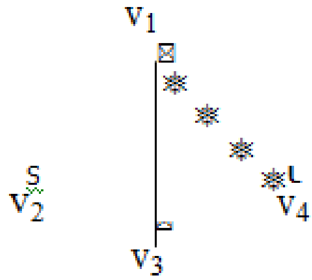
Theorem
Statement
Every graph of order p is an induced subgraph of a γ_{kr} -excellent graph.

Proof
Let G be a graph of order p. Attach at each point v, a complete graph K_{r-1} with v as one of the vertices. The resulting graph is denoted by GoK_r. The graph G is an induced subgraph of GoK_r which is γ_{kr} -excellent. Hence the theorem.

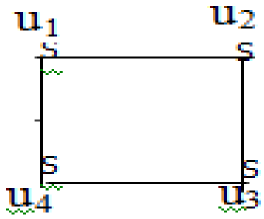
Corollary:

There does not exists a forbidden sub graph characterization of the class of
v γ_{kr} -excellent graphs.

Examples :



Subdivided graph (or) Star is not K_2-excellent.



C_4 is K_2-excellent but not K_3-excellent.

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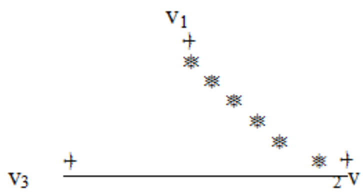

K_3 is K_3-excellent.

K_n-is not K_r-excellent.

Note:
AtreeTisγ_{kr} – excellent, for all $n \geq 3$, γ_{k2} -excellent, tree has already been characterized by sumner.

Definition:
A connected graph G is called a K_r-tree if every vertex is in a K_r-clique and
G does not contain C_m where $m \geq r + 1$.

Remark:

A K_2-tree is simply a tree.

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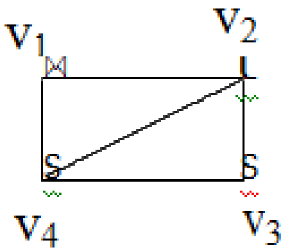
Example:



v_4 v3 K_3-tree.

Definition:

A pendent vertex v of a K_r-tree of a graph G is a vertex which is contain exactly one K_r-clique.

Note:

For any r, if G does not contain any r-clique, then $\gamma_{kr} = p$. In particular if $r > p$ then $\gamma_{kr} = p$. Therefore we assume that $r \leq p$

.

(i)γ_{kr} $(K_p) = 1$

(ii)$\gamma k_r(P_p)=d(p/3)e^{\text{ if } r = 2}$

γ_{kr} $(P_p) = p$ if $r > 2$. Where P is Peterson's graph.

Definition:

A K_r-path is a K_r-tree containing exactly K_r-pendent vertices and every other point is contained in exactly two K_r-cliques.

Note:

A K_3-path is a K_3-Tree in which there are exactly three K_3-pendant vertices.

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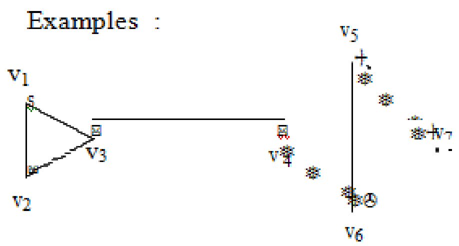

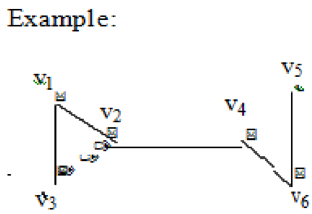

Remark:

(i) d_{kr} (v)= K_r-degree of v = number of r-cliques containing v.

(ii) Length of a K_r –path P denoted by l(P) is the number of K_r's (r-cliques) present in the path.

Theorem:

A k_r-path P is γk_r-excellent $\Longleftrightarrow$ l(P) = 1(or)l(P) = 0(mod3).

Uses:
K_r-domination has application in communication network system for rapid transfer of shared information among the members of the core group.

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