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Proposed Simplex Method For Fuzzy Linear Programming With Fuzziness at the Right Hand Side

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Abstract: *Linear programming (LP) is one of the frequently applied tools in operations research, it plays a vital role for solving real-life problem because of its efficiency and simplicity. However, managers and decision makers may lack information about exact values of most of the parameters used in any of the optimization models, the flexible approach of fuzzy linear programming (FLP) comes up with a powerful tool to deal with such situations. In this paper, the simplex method for imprecise resources has been proposed to solve the parametrized LP. GAMS software can always be use to solve the FLP with fuzziness at the RHS in a simplest way.*
Keyword Phrase: *Fuzzy Linear Programming.*

I. Introduction

Many researchers proposed several approaches and considered various types of Fuzzy Linear Programming (FLP) for solving conventional Linear Programming (LP) with imprecise information. The availability of resources and most of the parameters used in the real world are not certain. Even if the result obtained using linear programming is accepted, neither the expected revenues nor the constraints can be characterized by uncertainty. The most important aspects in dealing with imprecise information is the use of membership functions, the value of the membership function represents the degree of satisfaction (preference) on the constraints. The feasible alternative with the maximum degree of satisfaction is chosen as the optimal solution of the production scheduling problem. The literature on formulating real world problems as LP problems is very rich. Dantzig [1] (1955) proposed that interrelation between activities of a large organization can be viewed as a linear programming type model and optimizing determined by minimizing or maximizing a linear objective function. Since the conventional linear programming deals with a precise data, the study of fuzzy/imprecise data became necessary in order to overcome the shortcoming of linear programming in an uncertain environment. The theory of fuzzy mathematical programming was first proposed by Tanaka et al. [2] (1974) based on the fuzzy decision to address the uncertainty of the parameters in the problems with fuzzy objective functions and constraints. The formulation of FLP was first introduced by Zimmermann [3] (1978). He builds a crisp model of the problem and obtained its results by an existing algorithm. He then used the results and fuzzified the problem by evaluating admissible deviations for the goal and constraints of the subjective constants. Finally, he represented the maximization of the minimization of the deviations on the constraints by defining an equivalent crisp problem using auxiliary variable. Zimmermann[3] (1978, 1987) used Bellman and Zadeh’s [4] (1970) interpretation that a fuzzy decision is a union of goals and constraints. In the past decade, researchers have discussed various properties of FLP proposed various kinds of models. Zhang et al. [5] (2003) proposed a FLP with fuzzy numbers for the coefficients of objective functions. They introduced a number of optimal solutions to the FLP problems to multi-objective optimization problems with four-objective functions. Stanciulescu [6] (2003) proposed a FLP model with fuzzy coefficients for the objectives and the constraints. He used fuzzy decision variables with joint membership function instead of crisp decision variables and linked the decision variables together to sum up to a constant. He considered lower-bounded fuzzy decision variables that also set up the upper bounds of the decision variables. Ganesan and Veeramani [7] (2006) proposed a FLP model with symmetric trapezoidal fuzzy numbers. They also proved analogues of some interesting results which in turn led to the solution for FLP problems without converting them into crisp LP problems. Ebrahimnejad [8] (2011) showed that the method proposed by Ganesan and Veeramani [7] (2006) stops in a finite number of iterations and proposed revised version of their method that was more strong and efficient in practice. He also proved the absence of degeneracy and showed that if an FLP problem has a fuzzy feasible solution, it also has fuzzy basic feasible solution and if an FLP problem has an optimal fuzzy solution, it also has an optimal fuzzy basic solution. Hosseinzadeh Lotfi et al.[9] (2009) considered full fuzzy linear programming problems in which all parameters and variables are triangular fuzzy numbers. According to them there is no single method in the literature in finding the optimal solution of full FLP problems and proposed a new one to find the fuzzy optimal solution of full FLP with equality constraints. They introduced an approach to defuzzify a general fuzzy quantity using the concept of the symmetric triangular fuzzy numbers. They first approximated the fuzzy triangular numbers to its nearest symmetric triangular numbers, under the assumptions that all decision variables were symmetric triangular.

Kumar et al. [17] (2011) further studied the full FLP problems with equality introduced by Hosseinzadeh Lotfi et al. [9] (2009) and proposed a new method for finding the fuzzy optimal solution in these problems. Zhang et al.[10] (2005) discussed the duality theory in fuzzy mathematical programming problems with fuzzy coefficient. Nasseri and Ebrahimnejad [11] (2012) introduce a new approach to duality in FLP, they discussed the linear programming problem with trapezoidal fuzzy variables by use of several ranking functions as LP. Mahdavi-Amiri et al.[12] (2009) consider simplex algorithm of fuzzy primal, he used the method for solving FLP problem. Sanei [13] (2013) restricted his simplex method for solving fuzzy number LPP with bounded variables. Alrefaei et al.[14] used GAMS software to solve Fuzzy Linear Programming for Supply Chain Management in Steel Industry. To make it more general for solving flexible approach of FLP with fuzziness at the RHS, We proposed a new method using existing simplex method of LP, which is more easier than the existing simplex method of FLP.

II. Fuzzy Linear Programming

In many applications the parameters are usually uncertain (Fuzzy). Therefore, the classical LP are not applicable, instead, the FLP is used to model such situations. The problem 1 is equivalent to the crisp parametric programming problem, for a nonsymmetric model, the membership function of the fuzzy constraint can be considered as continuous monotonic functions and it is allowed to trade-off between those fuzzy constraints by applying the $\alpha - level$ cut concept.

$$\mu_i(x) = \begin{cases} 1 & if\ (Ax)_i < b_i\ ; \\ 1 - \dfrac{(Ax)_i - b_i}{p_i} & if\ b_i \le (Ax)_i \le b_i + p_i\ ; \\ 0 & if\ (Ax)_i > b_i + p_i\ . \end{cases}$$

The interpretation of the membership function are

• If $(Ax)_i \le b_i$ then the ith constraint is absolutely satisfied, $\mu_i(x) = 1$

• If $(Ax)_i \ge b_i + p_i$, where p_i is the maximum tolerance from b_i and it can be determined by the decision maker in any way, symmetric or nonsymmetric, then that particular constraint is violated, $\mu_i(x) = 0$

• Lastly, if $(Ax)_i$ belong to $(b_i, b_i + p_i)$; then we have monotonically decreasing membership function, and the real life meaning is that the more resources consumed, the less satisfaction the decision maker feels. If we substitute membership function in to the equivalent $\alpha - level\ \ cut$ equation, the following problem can be obtained

$$\begin{gathered} max\, cx \\ \text{subject to} \\ (Ax)_i \le b_i + (1-\alpha) p_i,\ \forall i \\ x \ge 0 \text{ and } \alpha \in [0,1] \end{gathered} \tag{1}$$

this is equivalent to parametric programming, when we substitute $\theta = 1 - \alpha$.
Therefore, the FLP

$$\begin{gathered} max\ z = cx \\ \text{subject to} \\ (Ax)_i \le \tilde{b}_i \\ i = 1,2,...,m \\ x \ge 0 \end{gathered} \tag{2}$$

can be transformed to crisp parametric LPP when we assumed some proper forms of membership functions of the fuzzy constraint. For each α, we have an optimal solution, the solution with α grade of membership is actually fuzzy. The equation 2 will be equivalent to

$$\max cx \tag{3}$$
subject to
$$X_\alpha = \{x : \mu_i(x) \geq \alpha, \forall i, x \geq 0\}, \text{ for } \alpha \in [0,1]$$

Let $A = (a_{ij})_{m \times n}$, $c = (c_1, c_2, ..., c_n)$, $\tilde{b} = (\tilde{b}_1, \tilde{b}_2, ..., \tilde{b}_m)^T$, $\tilde{x} = (\tilde{x}_1, \tilde{x}_2, ..., \tilde{x}_n)^T$
where a_{ij}, $c_i, (i = 1,2,...,n, j = 1,2,...,m)$ are crisp numbers, $\tilde{b}_j, (j = 1,2...,m)$ are fuzzy constant, $\tilde{x}_i, (i = 1,2,...,n)$ are fuzzy variable, then

$$\max z = c\tilde{x} \tag{4}$$
subject to
$$A\tilde{x} \leq \tilde{b}, \quad \tilde{x} \geq 0$$

is said to be Fuzzy linear programming problem with fuzzy variables and constraints.
The general model formulation of the LP with fuzzy resources available is as follows-:

$$\max z = cx \tag{5}$$
subject to
$$(Ax)_i \leq \tilde{b}_i \quad i = 1,2,...,m,$$
$$x \geq 0$$

where $\tilde{b}_i$, $\forall i$ are in $[b_i, b_i + p_i]$ with given p_i, we may also consider the following fuzzy inequality constraints

$$\max z = cx \tag{6}$$
subject to
$$(Ax)_i \lesssim b_i$$
$$i = 1,2,...,m$$
$$x \geq 0$$

where $\lesssim$ is fuzzy version of inequality and p_i is the tolerance for each fuzzy constraints and it is known. Then $(Ax)_i \leq \tilde{b}_i$ can be transform to $(Ax)_i \leq (\tilde{b}_i + \theta p_i), \forall i$
where θ is in [0,1]
If both two cases have the same membership functions then they are the same.

III. Simplex Method For FLP With Only RHS Fuzziness

When the FLP with fuzzy constraints that involves only fuzzy at the RHS transformed to parametric LP, the simplex method of conventional LP in tableau format is applicable for simplicity; Suppose the transformed parametric LP are as follows;

$$\max z = \sum_{i=1}^{n} c_i x_i \tag{1}$$
subject to
$$\sum_{i=1}^{n} a_{ij} x_i \leq b_j + \theta_j \gamma, \quad j = 1,2,...,m, \quad \gamma \in [0,1]$$
$$x \geq 0$$

For any crisp constraints the value of $\theta_j = 0$

- *Step* 1: Get rid of $\theta_j \gamma$ for any fuzzy constraints so that, the parametric LP will be transformed to simple LP.
- *Step* 2: Construct the initial tableau for minimization problem, the optimality condition calls for selecting the entering as the nonbasic variables with the most positive objective coefficient in the objective equation.

Before pivoting

Table 1: initial tableau

	z	x_{B_1}		x_{B_r}		x_{B_m}		x_{N_1}		x_{N_k}		RHS
z	1	0	$\cdots$	0	$\cdots$	0	$\cdots$	$z_j - c_{N_1}$	$\cdots$	$z_k - c_{N_k}$	$\cdots$	$c_B \bar{b}$
x_{B_1}	0	1	$\cdots$	0	$\cdots$	0	$\cdots$	y_{1j}	$\cdots$	y_{1k}	$\cdots$	$\bar{b}_1$
x_{B_r}	0	0	$\cdots$	1	$\cdots$	0	$\cdots$	y_{rj}	$\cdots$	y_{rk}	$\cdots$	$\bar{b}_r$
x_{B_m}	0	0	$\cdots$	0	$\cdots$	1	$\cdots$	y_{mj}	$\cdots$	y_{mk}	$\cdots$	$\bar{b}_m$

- *Step* 3: If x_{N_k} is the entering variable, select the leaving variable x_{B_r} (basic variable) associated with the smallest nonnegative ratio. (with strictly positive denominator), suppose that the pivot element is y_{rk} (element that correspond to entering variable from non basic and leaving variable from basic variable)
- *Step* 4: (pivot row) (1) Replace the leaving variable in the basic column with entering variable.

(2) If x_{N_k} enters the basis and x_{B_r} leaves the basis, then pivoting on y_{rk} can be done as follows:-multiply row r by $\dfrac{1}{y_{rk}}$

(3) for $i = 1,2,...,m$ and $i \neq r$, the ith row can be updated by adding $-y_{ik}$ to it, times the new rth row.

(4) update row zero by adding to it $c_k - z_k$ times the new rth row.

- *Step* 5: Determine the new basic solution by using appropriate Gauss-Jordan computation. If all objective coefficient are non positive final tableau is obtained.

After pivoting

Table 2: Current Tableau

	z	x_{B_1}		x_{B_r}		x_{B_m}		x_j		x_k		RHS
z	1	0	$\cdots$	$\dfrac{c_k - z_k}{y_{rk}}$	$\cdots$	0	$\cdots$	$z_j - c_j - \dfrac{y_{rj}(z_k - c_k)}{y_{rk}}$	$\cdots$	0	$\cdots$	$c_B \bar{b} - \dfrac{(z_k - c_k)\bar{b}_r}{y_{rk}}$
x_{B_1}	0	1	$\cdots$	$\dfrac{-y_{1k}}{y_{rk}}$	$\cdots$	0	$\cdots$	$y_{1j} - \dfrac{y_{rj} y_{1k}}{y_{rk}}$	$\cdots$	0	$\cdots$	$\bar{b}_1 - \dfrac{y_{1k}\bar{b}_r}{y_{rk}}$
x_k	0	0	$\cdots$	$\dfrac{1}{y_{rk}}$	$\cdots$	0	$\cdots$	$\dfrac{y_{rj}}{y_{rk}}$	$\cdots$	1	$\cdots$	$\dfrac{\bar{b}_r}{y_{rk}}$
x_{B_m}	0	0	$\cdots$	$\dfrac{-y_{mk}}{y_{rk}}$	$\cdots$	1	$\cdots$	$y_{mj} - \dfrac{y_{rj} y_{mk}}{y_{rk}}$	$\cdots$	0	$\cdots$	$\bar{b}_m - \dfrac{y_{mk}\bar{b}_r}{y_{1k}}$

Otherwise Go to Step 2.

Main steps:

Let $z_k - c_k = max\{z_j - c_j : j \in R\}$. If $z_k - c_k \leq 0$, then stop; the current solution is optimal. Otherwise, examine the row $y_{\bullet k}$, If $y_{jk} \le 0$, then stop, the optimal solution is unbounded along the ray

$\{\begin{pmatrix} B^{-1}b \\ 0 \\ \end{pmatrix} + x_k \begin{pmatrix} -y_k \\ e_k \\ \end{pmatrix} : x_k \ge 0\}$ Where e_k is a vector of zero's except for a 1 at the kth position. If $\exists$ j such that $y_{jk} > 0$, the index r can be determined as follows

$\frac{\bar{b}_r}{y_{rk}} = min\{\frac{\bar{b}_i}{y_{ik}} : y_{ik} > 0\}$. *Step* 6: In the final tableau we will obtain the new column vector at the right hand side.

$$\dot{b} = \begin{pmatrix} \dot{b}_1 \\ \dot{b}_2 \\ \vdots \\ \dot{b}_m \end{pmatrix} \quad and \quad z = C_B b$$

The corresponding $m \times m$ entiries of slack and surplus are as follows;

$$\begin{matrix} (e_{11}, e_{12}, \dots\dots, e_{1m}) \\ (e_{21}, e_{22}, \dots\dots, e_{2m}) \\ \dots\dots\dots\dots\dots \\ \dots\dots\dots\dots\dots \\ (e_{m1}, e_{m2}, \dots\dots, e_{mm}) \end{matrix}$$

Multiplying each raw by culumn vector $\begin{pmatrix} \gamma\theta_1 \\ \gamma\theta_2 \\ \vdots \\ \gamma\theta_m \end{pmatrix}$

$$(e_{i1}, e_{i2}, \dots\dots, e_{im}) \times \begin{pmatrix} \gamma\theta_1 \\ \gamma\theta_2 \\ \vdots \\ \gamma\theta_m \end{pmatrix} = \gamma \sum_{j=1}^{m} e_{ij}\theta_j, \quad \forall i = 1,2,\dots,n, \text{ let } d_i = \sum_{j=1}^{m} e_{ij}\theta_j$$

Step 7: The R.H.S. of final tableau becomes

$$RHS = \begin{pmatrix} \dot{b}_1 + \gamma d_1 \\ \dot{b}_2 + \gamma d_2 \\ \vdots \\ \dot{b}_m + \gamma d_m \end{pmatrix} \qquad (2)$$

and $z = c_B(\dot{b} + \gamma d)$

$$\text{where } d = \begin{pmatrix} d_1 \\ d_2 \\ \vdots \\ d_m \end{pmatrix} \qquad (3)$$

The optimal solution depends on the parameter $\gamma \in [0,1]$

IV. Application

In this section, we show the reliability, simplicity and accuracy of the proposed simplex method of FLP with fuzziness at the RHS.

Example 1

$$\begin{aligned}
&max 4x_1 + 5x_2 + 9x_3 + 11x_4 \\
&s.t. \\
&x_1 + x_2 + x_3 + x_4 \le 15 \\
&7x_1 + 5x_2 + 3x_3 + 2x_4 \le 120 \\
&3x_1 + 5x_2 + 10x_3 + 15x_4 \le 100 \\
&x_1, x_2, x_3 \\
&and \\
&x_4 \ge 0
\end{aligned} \qquad (1)$$

Applying the simplex method of conventional LP. The final tableau is shown in table 3, the optimal solution is: $x^* = (\frac{50}{7}, 0, \frac{55}{7}, 0)$ and $z = \frac{695}{7}$. Let us assume that the first, second and third RHS are imprecise and their maximum tolerances are 3, 10 and 20 units respectively.

Table 3: Final tableau of mix-problem

Basic and non basic variables	x_1	x_2	x_3	x_4	x_5	x_6	x_7	RHS
$z_j - c_j$	0	$\frac{3}{7}$	0	$\frac{11}{7}$	$\frac{13}{7}$	0	$\frac{5}{7}$	$\frac{695}{7}$
x_1	1	$\frac{5}{7}$	0	$\frac{-5}{7}$	$\frac{10}{7}$	0	$\frac{-1}{7}$	$\frac{50}{7}$
x_6	0	$\frac{-6}{7}$	0	$\frac{13}{7}$	$\frac{-61}{7}$	1	$\frac{4}{7}$	$\frac{325}{7}$
x_3	0	$\frac{2}{7}$	1	$\frac{12}{7}$	$\frac{-3}{7}$	0	$\frac{1}{7}$	$\frac{55}{7}$

Then the membership function of fuzzy constraints are:

$$\mu_1(x) = \begin{cases} 1 & if \;\; g_1(x) < 15 \,; \\ 1 - \dfrac{g_1(x) - 15}{3} & if \;\; 15 \le g_1(x) \le 18 \,; \\ 0 & if \;\; g_1(x) > 18 \,. \end{cases}$$

$$\mu_2(x) = \begin{cases} 1 & if \;\; g_3(x) < 120 \,; \\ 1 - \dfrac{g_2(x) - 120}{3} & if \;\; 120 \le g_3(x) \le 130 \,; \\ 0 & if \;\; g_3(x) > 120 \,. \end{cases}$$

$$\mu_3(x) = \begin{cases} 1 & if \;\; g_3(x) < 100 \,; \\ 1 - \dfrac{g_3(x) - 100}{3} & if \;\; 100 \le g_3(x) \le 120 \,; \\ 0 & if \;\; g_3(x) > 120 \,. \end{cases}$$

Then we have the following problem:

$$max\, 4x_1 + 5x_2 + 9x_3 + 11x_4 \qquad (2)$$

$$\text{s.t. } x_1 + x_2 + x_3 + x_4 \le 15 + 3(1-\alpha)$$
$$7x_1 + 5x_2 + 3x_3 + 2x_4 \le 120 + 10(1-\alpha)$$
$$3x_1 + 5x_2 + 10x_3 + 15x_4 \le 100 + 20(1-\alpha)$$
$$x_1, x_2, x_3 \text{ and } x_4 \text{ and } \alpha \in [0,1]$$

Set $\theta = 1-\alpha$, the following parametric programming problem is obtained:

$$max\, 4x_1 + 5x_2 + 9x_3 + 11x_4 \qquad (3)$$

$$\text{s.t. } x_1 + x_2 + x_3 + x_4 \le 15 + 3\theta$$
$$7x_1 + 5x_2 + 3x_3 + 2x_4 \le 120 + 10\theta$$
$$3x_1 + 5x_2 + 10x_3 + 15x_4 \le 100 + 20\theta$$
$$x_1, x_2, x_3 \text{ and } x_4$$

where $\theta \in [0,1]$ is a parameter. By the use of the parametric technique and the final table of the simplex method shown in Table 3 and using Equations in step 7, we can obtain the right hand side as follows:

$$z = \frac{695}{7} + (13/7, 0, 5/7) \times \begin{pmatrix} 3\theta \\ 10\theta \\ 20\theta \end{pmatrix} = \frac{695}{7} + \frac{139\theta}{7}$$

$$\dot{b}_1 + \gamma d_1 = \frac{50}{7} + (10/7, 0, -1/7) \times \begin{pmatrix} 3\theta \\ 10\theta \\ 20\theta \end{pmatrix} = \frac{50}{7} + \frac{10\theta}{7}$$

$$\dot{b}_2 + \gamma d_2 = \frac{325}{7} + (-61/7, 1, 4/7) \times \begin{pmatrix} 3\theta \\ 10\theta \\ 20\theta \end{pmatrix} = \frac{325}{7} - \frac{33\theta}{7}$$

$$\dot{b}_3 + \gamma d_3 = \frac{55}{7} + (-3/7, 0, 1/7) \times \begin{pmatrix} 3\theta \\ 10\theta \\ 20\theta \end{pmatrix} = \frac{55}{7} + \frac{11\theta}{7}$$

Adding these results to RHS of the final tableau, we get the final parametric tableau as in Table 4, the optimal solution of the parametric LP is: $x^* = (\frac{50}{7} + \frac{10\theta}{7}, \frac{325}{7} - \frac{33\theta}{7}, \frac{55}{7} + \frac{11\theta}{7}, 0)$ and $z^* = \frac{695}{7} + \frac{139\theta}{7}$.

Table 4: Final Tableau Of Mix-Problem

Basic and non basic variables	x_1	x_2	x_3	x_4	x_5	x_6	x_7	RHS
$z_j - c_j$	0	$\frac{3}{7}$	0	$\frac{11}{7}$	$\frac{13}{7}$	0	$\frac{5}{7}$	$\frac{695}{7}+\frac{139\theta}{7}$
x_1	1	$\frac{5}{7}$	0	$\frac{-5}{7}$	$\frac{10}{7}$	0	$\frac{-1}{7}$	$\frac{50}{7}+\frac{10\theta}{7}$
x_6	0	$\frac{-6}{7}$	0	$\frac{13}{7}$	$\frac{-61}{7}$	1	$\frac{4}{7}$	$\frac{325}{7}-\frac{33\theta}{7}$
x_3	0	$\frac{2}{7}$	1	$\frac{12}{7}$	$\frac{-3}{7}$	0	$\frac{1}{7}$	$\frac{55}{7}+\frac{11\theta}{7}$

V. Conclusion

The main aim of this paper, is to propose a more general simplex method for solving the fuzzy linear programming with fuzziness at the RHS and General Algebraic Modeling System (GAMS) software can be use to solve this type of FLP by introducing another constraint that belong to [0,1].

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