IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 11, Issue 3 Ver. II (May - Jun. 2015), PP 66-72
www.iosrjournals.org

Fuzzy and Rough Approximations Operations on Graphs

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Abstract: *In this paper, we introduce a covering model on connected graph for modified lower and upper approximations operation on graph vertices with respect to connected sub graph. We obtain new classes induced by covering every sub graphs by lower and upper approximation operations using fuzzification set of vertices of main graph . Also, properties of these classes studied and illustrate that with examples.*
Keywords: *Graph theory, Rough set theory, Topology, Fuzzy set theory and Data mining.*

I. Introduction

Many extensions of rough set theory have been discussed for approximations methods in information systems and other applications . Ordinary rough set is a new approach proposed by Pawalak [11,18,19,20,21] to deal with incomplete or imperfect information.

Rough set analysis is based on relation that describes in distinguish ability of objects. The concepts are represented by their lower and upper approximations. In applications, rough sets focus on approximate reasoning. Rough sets are applied to data mining, machine learning, finance, industry, multimedia, medicine, control theory, pattern recognition, and most recently bioinformatics[5,11].Rough set is non-statistical tool for analysis of imperfect Data, data dependencies, fuzzy set and decision rules [3,4,5,8, 12,24].

Graph theory plays an important core for transformation any engineering ,chemical, physical , social sciences, game theories, networks, preferences, and mathematication of discrete spaces.We may find new method for classification vertices of graph G=(V,E) , where V is the set of vertices and E the set of edges , elements of the V depended on the form of link each vertex in V with others ,this form can determined by neighborhoods which classified V to sets have elements are the same or belong to the same elementary set if they have the same link value.

In [17,18] neighborhoods of graph vertices used to studied some inequalities, which joint the number of each incident edge and number of vertices in open sets and closed sets in topological structures formed by this neighborhoods, then applied this on blood circulation in the human body and nervous system in some medical application of neighborhoods of graph vertices. This paper may be help us to knows the maximum and minimum numbers of vertices used to determined any path in connected graph with respect to part of this graph .We introduce some important new properties of graph in sense of rough set concepts , we also tried to used classification by neighborhoods of each vertices to obtained upper and lower sub graph of any part of graph by new mathematical methods.

This paper is organized as follows: In Sec.2 we introduced the main concepts of rough set theory. By neighborhoods on connected graph .In Sec.3 we introduced new definitions of lower and upper approximation operators by membership function and precision rough set model

II. Rough Set Concepts On Connected Graph

We try investigate rough set concepts on vertices of graph as neighbors[3,8,9,15,17].Upper and lower approximate of connected sub graph vertices with respect to the main graph determined by neighborhoods, which cover every sub graph of main graph, lower approximation concept make all vertices and edges lies completely in sub graph, upper approximation concept make all vertices and edges possibly lies in sub graph . some new rough concepts on this classes discussed.

Definition 2.1

Let $G \equiv (V, E)$be connected graph, the neighborhood of any vertex is defined as

$N(v_i) = \{v_i\} \cup \{v_j : v_j \in V(G), \overrightarrow{v_i v_j} \in E\}$. The covering graph is defined as

$C(G) = \{G(N_{vi}), v_i \in V(G), \overrightarrow{v_i v_j} \in E(G)\}$. The minimal neighborhood of any vertex is defined as

$$MN(v_i) = \cap \{V\left(G\left(N_{v_j}\right)\right) : G\left(N_{v_j}\right) \in C(G), v_i \in G\left(N_{v_j}\right)\}.$$

Definition 2.2

Let $G_1 \equiv (V_1, E_1)$ be a connected subgragh of a connected graph $G \equiv (V, E)$, covering lower and covering upper approximation operation are define on the graph an follows

$$L_{G_1}, U_{G_1}: P(V) \to P(V)$$

$L_{G_1}(X) = \{y: y \in MN(v_i) \subseteq X, \overrightarrow{yv_i} \in E_1, X \subseteq V \text{and } v_i \in V\}$

$U_{G_1}(X) = \{y: y \in MN(v_i) \cap X \neq \emptyset, \overrightarrow{yv_i} \in E_1, X \subseteq V \text{and } v_i \in V\}$.

Definition 2.3

Let $G_1 \equiv (V_1, E_1)$ be a connected subgragh of a connected graph G=(V,E). Rough degree of $G_1(V_1, E_1)$ with respect to $G \equiv (V, E)$,denoted by $\rho_{G_1}(X)$,is defined as $\rho_{G_1}(X) = 1 - \left|\frac{L_{G_1}(X)}{U_{G_1}(X)}\right|$

Definition 2.4

Let $G \equiv (V, E)$ be asimple connected graph and $G_1 \equiv (V_1, E_1), G_2 \equiv (V_2, E_2)$ are subgraraphs of G The positive region, negative region and boundary region of G_2 with respect to G_1 define as $pos_{G_1}(G_2) = L_{G_1}(V_2)$, $neg_{G_1}(G_2) = V - U_{G_1}(V_2)$ and $bd_{G_1}(G_2) = U_{G_1}(V_2) - L_{G_1}(V_2)$

Definition 2.5

Let $G \equiv (V, E)$ be a simple connected graph and $G_1 \equiv (V_1, E_1), G_2 \equiv (V_2, E_2)$ are subgraraphs of G. The positive region, negative region and boundary region of E_2 with respect to G_1 define as $pos_{G_1}(E_2) = \cup_{V_2 \in K_2} pos_{G_1}(G_2)$ and $neg_{G_1}(E_2) = \cup_{V_2 \in K_2} neg_{G_1}(G_2)$, Where $K_2 = \{N(V_i): V_i \in V_2\}$

Example2.1: Consider the Connected graph G=(V,E)

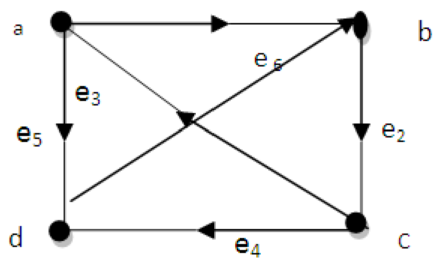


Fig (2.1)

The neighborhoods of vertices are $N(a) = \{a, b, d\}, N(b) = \{b, c\}, N(c) = \{a, c, d\}, N(d) = \{b, d\}$
The graphs of the neighborhoods are connected sub graphs of G=(V,E).

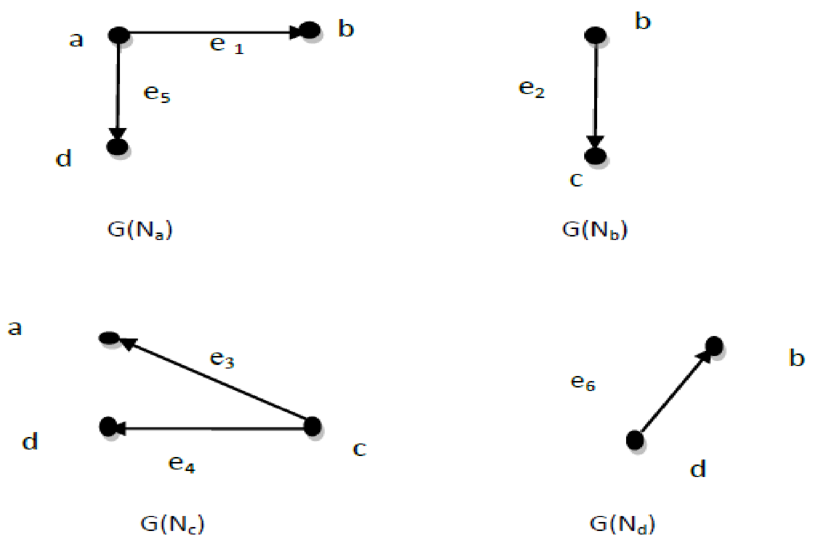
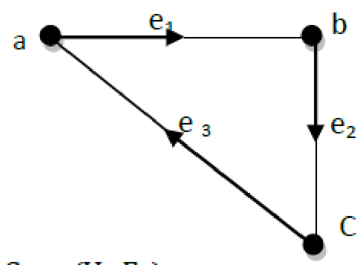


Fig (2.2)

Then the covering of graph is $C(G) = \{G(N_a), G(N_b), G(N_c), G(N_d)\}$and
$MN(a) = \{a\}, MN(b) = \{b\}, MN(c) = \{c\}, MN(d) = \{d\}$.Consider the sub graph $G_1(V_1, E_1)$



$G_1 \equiv (V_1, E_1)$

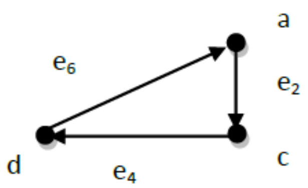
Fig (2.3)

The set of sub graph is $V_1 = \{a, b, c\}$. The covering lower approximation of each set of vertices in $G_1(V_1, E_1)$ with respect to sets of neighborhoods of all vertices of G=(V,E)are formed as.

$L_{G_1}(\{a\}) = \{a\}$, $L_{G_1}(\{b\}) = \{b\}$, $L_{G_1}(\{c\}) = \{c\}$,
$L_{G_1}(\{a, c\}) = \{a, c\}$, $L_{G_1}(\{a, b\}) = \{a, b\}$,
$L_{G_1}(\{b, c\}) = \{b, c\}$, $L_{G_1}(V_1) = \{a, b, c\}$.

The covering upper approximation of each set of vertices in $G_1(V_1, E_1)$ with respect to sets of neighborhoods of all vertices of G=(V,E)are formed as.

$U_{G_1}(\{a\}) = \{a\}$, $U_{G_1}(\{b\}) = \{b\}$, $U_{G_1}(\{c\}) = \{c\}$,
$U_{G_1}(\{a, c\}) = \{a, c\}$, $U_{G_1}(\{a, b\}) = \{a, b\}$,
$U_{G_1}(\{b, c\}) = \{b, c\}$, $U_{G_1}(V_1) = \{a, b, c\}$.



$G_2 \equiv (V_2, E_2)$

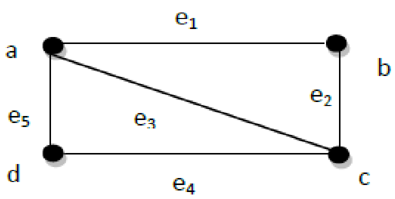
Fig (2.4)

$pos_{G_1}(G_2) = \{b, c, d\}, neg_{G_1}(G_2) = \{d\}$

We can obtainthe upper and lower approximation of any path vertices in $G_1 \equiv (V_1, E_1)$ with respect to $G \equiv (V, E)$.Let P1=ae_1be_2c be a path form a to c so, the set of vertices of the path is $V(P_1)$= {a,b,c}has lower and upper approximations as: $L_{G1}(V(P_1)) = \{a, b, c\}$, $U_{G1}(V(P_1)) = \{a, b, c\}$.

From above we notice that the pervious of determine can apply the above definition on undirected connected graph by use the direction of every edge in two direction.

Example 2.2: Consider the undirected connected graph



$G \equiv (V, E)$

Fig (2.5)

$N(a) = V, N(b) = \{a,b,c\}, N(c) = V, N(d) = \{a,c,d\}$

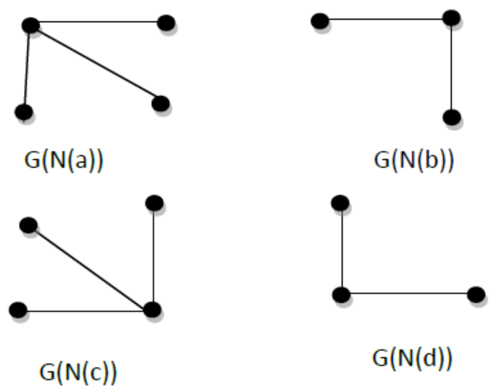


Fig (2.6)

$C(G) = \{G(N_a), G(N_b), G(N_c), G(N_d),$ $MN(a) = \{a,c\}, MN(b) = \{a,b,c\},$
$MN(c) = \{a,c\}$ *and* $MN(d) = \{a,c,d\}$

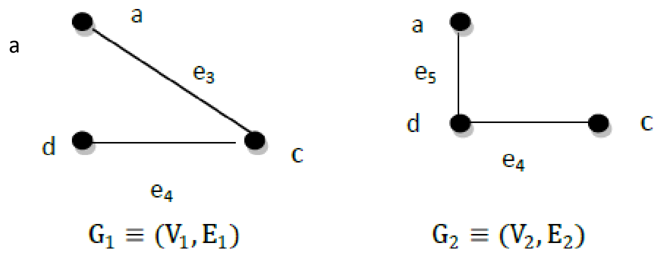


Fig (2.7)

$L_{G_1}(\{a\}) = L_{G_1}(\{b\}) = L_{G_1}(\{c\}) = \emptyset, L_{G_1}(\{a,c\}) = \{a,c\}, L_{G_1}(\{a,b\}) = \emptyset,$
$L_{G_1}(\{b,c\}) = \emptyset, L_{G_1}(V_1) = \{a,c\}. U_{G_1}(\{a\}) = \{a,c\}, U_{G_1}(\{b\}) = \emptyset, U_{G_1}(\{c\}) = \{a,c\},$
$U_{G_1}(\{a,c\}) = \{a,c,d\}, U_{G_1}(\{a,b\}) = \{a,c\}, U_{G_1}(\{b,c\}) = \{a,c\}, U_{G_1}(V_1) = \{a,b,c\}.$
$pos_{G_1}(G_2) = \{a,c\}$, $neg_{G_1}(G_2) = \{b\}$ and $bd_{G_1}(G_2) = \{d\}$

III. Rough Set Based On Rough Membership Function

In this section we introduce the concept of rough set theory by fuzzifications crisp set of sub graphs vertices using its neighborhoods $N_{G_j}(v_i)$ vertices , we obtained rough membership function indicated the dependence of graph G=(V,E) vertices with sub graphs neighborhoods . The important question be , what is the different between graph operations (deletions, adding, unions, …)and the same operations on the new classes and parameters. To answer this question we must introduced some new results.

Definition 3.1

Let G=(V,E) and G1= (V1,E1) be a subgraph. Then the membership function of any vertex with respect to the sub graph G_1 is defined as

$$\mu^G_{G_1}(v_i) = \frac{|\{\cap N_G(v_i)\} \cap V(G_1)|}{|\{\cap N_G(v_i)\}|},\ v_i \in N_G(v_i) \subseteq V(G)$$

We can redefine the definition of lower and upper covering by using rough membership function.

Proposition 3.1

Let $G \equiv (V,E)$ be a simple connected graph and $G_1 \equiv (V_1, E_1), G_2 \equiv (V_2, E_2)$ are subgraraphs of G and $G_2 \subseteq G_1$ then $\mu^G_{G_2}(v_i) \le \mu^G_{G_1}(v_i)$ for all $v_i \in V(G)$

proof

From the definition of membership and $|\{\cap N_G(v_i)\} \cap V(G_2)| \le |\{\cap N_G(v_i)\} \cap V(G_1)|$

We obtained $\mu_{G_2}^{G}(v_i) \leq \mu_{G_1}^{G}(v_i)$

Proposition3.2

Let $G \equiv (V, E)$ be a simple connected graph , $G_1 \equiv (V_1, E_1)$ a subgraraph of G and G_1^C its complement With respect to G then$\mu_{G_1}^{G}(v_i) + \mu_{G_1^C}^{G}(v_i) = 1$ for all $v_i \in V(G)$

proof

From the definition of membership and $\frac{|\{\cap N_G(v_i)\}\cap(V(G_1)\cup V(G_1^C))|}{|\{\cap N_G(v_i)\}|} = 1$

Proposition 3.3

Let $G \equiv (V, E)$ be a simple connected graph and $G_1 \equiv (V_1, E_1), G_2 \equiv (V_2, E_2)$ are subgraraphs of G and $G_2 \subseteq G_1$

(1) $\mu_{G_1}^{G}(v_i) = 1$ iff $v_i \in L_{G_1}(V_2)$

(2) $\mu_{G_1}^{G}(v_i) = 0$ iff $v_i \in (V - U_{G_1}(V_2))$

(3) $0 \leq \mu_{G_1}^{G}(v_i) \leq 1$ iff $v_i \in (U_{G_1}(V_2) - L_{G_1}(V_2))$

Proof: obviously

From above we can fuzzyfication sub graph of a simple connected graph, by using membership fuction as characteristic function of all vertices of a sub graph , we can obtained a fuzzy set of graph vertices as $F_{G_1}^{G}(v_i) = \{\left(v_i, \mu_{G_1}^{G}(v_i)\right)$ for every $G_1 \subseteq G$ and $v_i \in V(G)\}$.

Proposition 3.4

Let $G \equiv (V, E)$ be a simple connected graph and $G_1 \equiv (V_1, E_1), G_2 \equiv (V_2, E_2)$ are subgraraphs

(i) $F_{G_{1\cup G_2}}^{G} \supseteq F_{G_1}^{G} \cup F_{G_2}^{G}$

(ii) $F_{G_{1\cup G_2}}^{G} = F_{G_1}^{G} \cup F_{G_2}^{G}$ if either $G_1 \subseteq G_2$ or $G_2 \subseteq G_1$

Proof

(i) From $\mu_{G_{1\cup G_2}}^{G}(v_i) \geq max_{v_i \epsilon V}(\mu_{G_1}^{G}(v_i), \mu_{G_2}^{G}(v_i))$

Pawlak forms of rough set theory applied on a special cases of graph theory in [9,18] Thus, our new work may be Redefined another forms in the sense of precision coefficient k (k∈ (0.5, 1]) .

Definition 3.1

Let $G \equiv (V, E)$ be a simple connected graph and $G_1 \equiv (V_1, E_1)$ subgraraphs of G, k-lower and k-upper approximation of X based on rough membership function may be defined as

$L_{G_1}^{k}(X) = \{v_i \in X\colon \mu_{G_1}^{G}(v_i) \geq k , X \subseteq V_1\}$
$U_{G_1}^{k}(X) = \{v_i \in X\colon \mu_{G_1}^{G}(v_i) \geq 1 - k , X \subseteq V_1\}$

From above X is k-exact if and only if $L_{G_1}^{k}(X) = U_{G_1}^{k}(X)$Otherwise X is k-rough. We try to obtain some properties between the new definitions by comparing rough sub graphs model with each others

Theorem 3.1

Let $G \equiv (V, E)$ be a simple connected graph , $G_1 \equiv (V_1, E_1)$, subgraraphs of G and $X \subseteq Y \subseteq V$

1) $L_{G_1}^{k}(\emptyset) = \emptyset$, $U_{G_1}^{k}(\emptyset) = \emptyset$
2) $L_{G_1}^{k}(V) = V$, $U_{G_1}^{k}(V) = V$
3) $L_{G_1}^{k}(L_{G_1}^{k}(X)) = V - (U_{G_1}^{k}(V - X)$
4) $U_{G_1}^{k}(U_{G_1}^{k}(X)) = V - (L_{G_1}^{k}(V - X)$
5) $L_{G_1}^{k}(X \cup Y) \supseteq L_{G_1}^{k}(X) \cup L_{G_1}^{k}(X)$
6) $U_{G_1}^{k}(X \cup Y) \supseteq U_{G_1}^{k}(X) \cup U_{G_1}^{k}(X)$

7) $L^k_{G_1}(X \cap Y) \subseteq L^k_{G_1}(X) \cap L^k_{G_1}(X)$
8) $U^k_{G_1}(X \cap Y) \subseteq U^k_{G_1}(X) \cap U^k_{G_1}(X)$
9) If $k_1 \leq k_2$ then $L^{k_2}_{G_1}(X) \subseteq L^{k_1}_{G_1}(X)$
10) If $k_1 \leq k_2$ then $U^{k_1}_{G_1}(X) \subseteq U^{k_2}_{G_1}(X)$
11) If $X \subseteq Y$ then $L^k_{G_1}(X) \subseteq L^k_{G_1}(Y)$
12) If $X \subseteq Y$ then $U^k_{G_1}(X) \subseteq U^k_{G_1}(Y)$
13) $L^k_{G_1}(X) \subseteq U^k_{G_1}(X)$

Proof

From Definition 3.1 and membership properties

Definition 3.2

Let $G \equiv (V, E)$ be a simple connected graph , $G_1 \equiv (V_1, E_1)$, subgraraphs of G and k∈(0.5,1].For any $X \subseteq V$ k- positive region, k-negative region and k-boundary region of X with respect to G_1 denoted respectively by $pos^k_{G_1}(X)$, $neg^k_{G_1}(X) and\ bnd^k_{G_1}(X)$ define as :

$$pos^k_{G_1}(X) = \{v_i \in X:\ \mu^G_{G_1}(v_i) \geq k\ , X \subseteq V_1\}$$
$$neg^k_{G_1}(X) = \{v_i \in X:\ \mu^G_{G_1}(v_i) \geq 1 - k\ , X \subseteq V_1\}$$
$$bnd^k_{G_1}(X) = \{v_i \in X:\ 1 - k\ \leq \mu^G_{G_1}(v_i) \leq\ k\ , X \subseteq V_1\}$$

Proposition 3.5

Let $G \equiv (V, E)$ be a simple connected graph,$G_1 \equiv (V_1, E_1)\ and\ \ G_2 \equiv (V_2, E_2)$are sub graphs of G, k∈(0.5,1] .Te following satisfies:

1) $G_2 \subseteq G_1$ is k-exact with respect to G_1 if and only if $bnd^k_{G_1}(V_2) = \emptyset$.
2) If $k_1 \leq k$ and G_2 is k-exact then G_2 is k_1-exact. If $k_2 \geq k$ and G_2 is k-exact then G_2 is k_2-exact.

Proof

From above definition

Proposition 3.6

Let $G \equiv (V, E)$ be a simple connected graph,$G_1 \equiv (V_1, E_1)\ and\ \ G_2 \equiv (V_2, E_2)$are subgraraphs of G, k∈(0.5,1] and $G_2 \subseteq G_1$the following statements satisfied

1) $pos^k_{G_1}(V_2) = neg^k_{G_1}(V - V_2)$.
2) $bnd^k_{G_1}(V_2) = bnd^k_{G_1}(V - V_2)$.
3) If $bnd^k_{G_1}(V_2) = \emptyset$ then $\mathrm{pos}^k_{G_1}(V_2) \cup \mathrm{neg}^k_{G_1}(V_2) = V$.

Proof
Obviously

Proposition3.7

Let $G \equiv (V, E)$ be a simple connected graph,$G_1 \equiv (V_1, E_1)$ and $G_2 \equiv (V_2, E_2)$are subgraraphs of G, k∈(0.5,1] and $G_2 \subseteq G_1$the following statements satisfied

1) $L_{G_1}(V_2) \subseteq L^k_{G_1}(V_2)$
2) $U^k_{G_1}(V_2) \subseteq U_{G_1}(V_2)$
3) $bnd^k_{G_1}(V_2) \subseteq bnd_{G_1}(V_2)$
4) $neg_{G_1}(V_2) \subseteq neg^k_{G_1}(V_2)$

Example 3.1

From Example 2.2 $\mu^G_{G_1}(a) = 1$, $\mu^G_{G_1}(b) = \frac{2}{3}$, $\mu^G_{G_1}(c) = 1$, $\mu^G_{G_1}(d) = 1$ .Consider X={a,b}, Y={a} $L^{0.7}_{G_1}(X) \subseteq L^{0.6}_{G_1}(X)$ and $U^{0.6}_{G_1}(X) \subseteq U^{0.7}_{G_1}(X)$, $L^{0.6}_{G_1}(Y) \subseteq L^{0.6}_{G_1}(X)$ and $U^{0.6}_{G_1}(Y) \subseteq U^{0.6}_{G_1}(X)$

IV. Conclusion

Rough set theory has been regarded as tool to deal with the uncertainty or imprecision information, the graded rough set model based on two distinct but related universes was proposed .But it is still restrictive for many applications, the rough membership function based type graded rough set in any graph which based on covering neighborhood of all vertices. We have some interesting properties and conclusions about rough set model on many sub graphs and paths, which can help us understand the approximations structure of graphs. In the future we will further study other types of operations on graphs such that deletions vertices or edges by using rough sets and so on.

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