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Rayleigh-Bénard Convection in A Horizontal Layer Of Porous Medium Saturated With A Thermally Radiating Dielectric Fluid

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Abstract: *The classical linear stability analysis is used to analyze the effect of thermal radiation on the onset of Darcy electroconvection in a horizontal porous layer heated from below. The boundaries are assumed to be black bodies and the optical properties of the transparent dielectric fluid are independent of the wavelength of radiation. The principle of exchange of stabilities is shown to be valid. The critical values pertaining to the stationary instability are obtained by means of the higher order Galerkin method. It is found that basic temperature profile becomes exponential and symmetric as the radiative parameters increase and that the effect of thermal radiation is to delay the onset of Darcy electroconvection.*

Keywords: *Thermal Radiation, Electroconvection, Integro-differential equations, Milne- Eddington approximation*

I. Introduction

There exist situations in which thermal radiation is important even though the temperature may not be high. In fact, even under some of the most unexpected situations, the radiative heat transfer could account for a non-negligible amount of total heat transfer. Earlier works on heat transfer in Newtonian fluids paid attention to both convection and conduction but overlooked the effect of thermal radiation.

The available literature barely delineates the part played by convection in a fluid combined with radiation. The formulation of heat transfer by conduction and convection leads to differential equations while that by radiation leads to integral equations. Thus the complexity involved in the solution of the integro-differential equations resulting from the coupled convection and radiation problem warrants the use of several simplifying assumptions.

Combined heat transfer processes such as convection-radiation play a significant role in several chemical processes involving combustion, drying, fluidization, MHD flows, and so forth. In general, the radiative process either occurs at the boundaries or as a term in the energy equation. In the latter case, the radiative term is usually approximated as a flux in such a way that the term corresponding to radiation in the heat transfer equation appears as a gradient term similar to Fourier's conduction term. This method has found considerable favour among many researchers. Alternatively, radiation effects can be incorporated at the boundaries through appropriate assumptions. Free surface flows present a challenging problem to engineers as the combined convection-radiation at the boundaries has major applications in many industries.

Goody [1] estimated the radiative transfer effects in the conventional natural convection problem with free boundaries using a variational method. He solved the problem for optically thin and optically thick cases and showed that there could be very large variations near the boundaries. Goody's radiative transfer model has been extended and modified by subsequent investigators to take into account the effects of magnetic field, rotation and fluid non-grayness (Spiegel [2]; Murgai and Khosla [3]; Khosla and Murgai [4]; Christophorides and Davis [5]; Arpaci and Gozum [6]; Yang [7]; Bdeoui and Soufiani [8]).

Larson [9] studied linear and nonlinear stability properties of Goody's model analytically. When thermal diffusivity is zero, the energy method is used to rule out subcritical instabilities. When thermal diffusivity is nonzero, the energy method is used to find a critical threshold below which all infinitesimal and finite amplitude perturbations are stable.

Shobha Devi *et al*. [10] studied the problem of Rayleigh-Bénard convection in an anisotropic porous medium in the presence of radiation. A linear stability analysis is performed and the Milne-Eddington approximation is employed for obtaining the initial static state. The Galerkin method is used to obtain the critical Rayleigh numbers. It is shown that radiation is to stabilize the system for both transparent and opaque media. It is found that opaque media releases heat for convection more slowly than transparent media and that the cell size gets affected by radiation only in the case of transparent media.

Maruthamanikandan [11] analyzed the effect of radiative transfer on the onset of thermal convection in a ferromagnetic fluid layer bounded by two parallel plates and heated from below. The Milne-Eddington approximation is employed to convert radiative heat flux into thermal heat flux. It is found that radiation inhibits the onset of convection in both transparent and opaque media. Furthermore, the opaque medium is shown to release heat for convection more slowly than the transparent medium. It is also shown that radiation affects the cell size at the onset of convection only in the case of transparent medium.

Anwar *et al*. [12] studied the effects of thermal radiation and porous drag forces on the natural convection heat and mass transfer of a viscous, incompressible, gray, absorbing, emitting fluid flowing past an impulsively started moving vertical plate adjacent to a non-Darcian porous regime. The Rosseland diffusion approximation is employed to analyze the radiative heat flux and is appropriate for non-scattering media. Increasing Darcy number is seen to accelerate the flow; the converse is apparent for an increase in Forchheimer number. Thermal radiation is seen to reduce both velocity and temperature in the boundary layer.

Shateyi *et al.* [13] sought to investigate the influence of a magnetic field on heat and mass transfer by mixed convection from vertical surfaces in the presence of Hall, radiation, Soret thermal diffusion, and Dufour diffusion-thermo effects. Similarity solutions were obtained using suitable transformations. The numerical results for some special cases were compared with the exact solution and were found to be in good agreement.

Jafarunnisa *et al.* [14] analyzed the effect of chemical reaction and radiation absorption on unsteady convective heat and mass transfer flow of a viscous fluid through a porous medium in a vertical channel in the presence of heat generating sources. The nonlinear coupled governing equations are solved by a regular perturbation technique. The effect of chemical reaction and radiation absorption on all flow characteristics are discussed with the help of graphs.

Most investigations of electroconvection in dielectric fluids consider only thermally conducting fluids, though it is of geophysical importance to understand the effects of radiative transfer on electrically induced convection. In this chapter we extend Goody's model to take into account the electric force and porous medium. In fact, we study qualitatively the effect of thermal radiative transfer on the onset of electroconvection in a porous medium in the presence of a uniform vertical ac electric field. We also restrict our attention to the case in which the absorption coefficient of the fluid is the same at all wavelengths and is independent of the physical state (the so-called gray medium approximation). The equation of radiative transfer is developed in optically thin approximation and the effect of scattering is ignored.

II. Mathematical formulation

Consider a horizontal constant porosity layer of a dielectric fluid confined between two parallel infinite boundaries heated from below. In addition to a vertical temperature gradient, a vertical electric field is also applied across the layer. A Cartesian coordinate system is taken with the lower surface in the xy - plane and z - axis vertically upwards. The lower surface at $z = -\frac{d}{2}$ and the upper surface at $z = \frac{d}{2}$ are maintained at constant temperatures T_0 and T_1 respectively. The boundaries are assumed to be perfect conductors of heat and the fluid between the boundaries absorbs and emits thermal radiation. We treat the two boundaries as black bodies. The absorption coefficient of the fluid is assumed to be the same at all wavelengths and to be independent of the physical state. Moreover, it is assumed that local thermal equilibrium exists between the solid matrix and the saturated fluid.

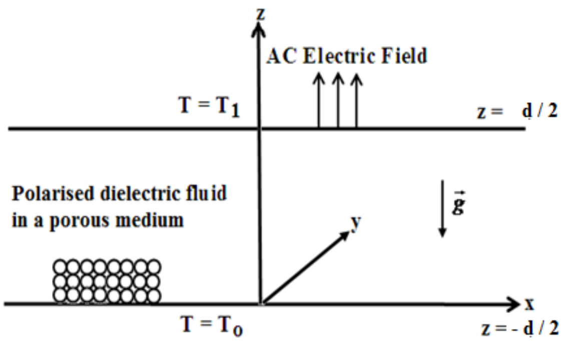

Figure 1: Configuration of the problem.

The system of equations describing the problem at hand is the following

$$\nabla \cdot \vec{q} = 0 \qquad (1)$$

$$\frac{\rho_0}{\varepsilon}\left[\frac{\partial \vec{q}}{\partial t}+\frac{1}{\varepsilon}\left(\vec{q}\bullet\nabla\right)\vec{q}\right]=-\nabla p+\rho\vec{g}-\frac{\mu_f}{k}\vec{q}+\left(\vec{P}\bullet\nabla\right)\vec{E} \tag{2}$$

$$M\frac{\partial T}{\partial t}+\left(\vec{q}\bullet\nabla\right)T=\kappa\nabla^2 T+\frac{G}{s_r} \tag{3}$$

$$\rho=\rho_0\left[1-\alpha_T\left(T-T_0\right)\right] \tag{4}$$

where $\vec{q}=\left(u,v,w\right)$ is the fluid velocity, t is the time, p is the pressure, $\vec{P}$ is the dielectric polarization, $\vec{E}$ is the electric field, ρ is the fluid density, ρ_o is the density at a reference temperature at $T=T_0$, $\vec{g}$ is the acceleration due to gravity, μ_f is the effective fluid viscosity, ε is the porosity, k is the permeability of the porous medium, κ is the effective thermal diffusivity, M is the ratio of heat capacities, α_T is the thermal expansion coefficient, T is the temperature, G is the rate of radiative heating per unit volume, s_r is the heat content of the fluid per unit volume, ∇ is the vector differential operator and $\left(x,y,z\right)$ are the spatial coordinates.

The relevant Maxwell equations are

$$\vec{P}=\varepsilon_o\left[\varepsilon_r-1\right]\vec{E} \tag{5}$$

$$\nabla\bullet\left(\varepsilon_o\varepsilon_r\vec{E}\right)=0 \tag{6}$$

$$\nabla\times\vec{E}=0 \quad \text{or} \quad \vec{E}=\nabla\phi \tag{7}$$

where ε_0 is the electric permittivity, ε_r is the relative permittivity or dielectric constant and ϕ is the electric potential. The dielectric constant is assumed to be a linear function of temperature in the form

$$\varepsilon_r=\varepsilon_r^o-e\left(T-T_o\right) \tag{8}$$

where $e\left(>0\right)$ is the dielectric permittivity and $\varepsilon_r^o=1+\chi$ with χ being the electric susceptibility.

The equation of radiative heat transfer is

$$\frac{dI(\vec{s})}{ds}=K_a\left[P_B-I(\vec{s})\right] \tag{9}$$

where $I(\vec{s})$ is the intensity of radiation along the direction of the vector $\vec{s}$, ds is an infinitesimal displacement in the $\vec{s}$ direction, K_a is the absorption coefficient of the fluid and P_B is the Planck black-body intensity. The radiative heating rate is given by

$$G=-\int\frac{dI(\vec{s})}{ds}\,d\omega_s \tag{10}$$

where the integral is taken over the solid angle 4π and ω_s is the element of solid angle.

The basic state is quiescent and is described by

$$\left.\begin{aligned}&\vec{q}=\vec{q}_b=\left(0,0,0\right),\ p=p_b(z),\ \rho=\rho_b(z),\ T=T_b(z),\\&\vec{E}=\vec{E}_b=\left(0,0,E_b(z)\right),\ \vec{P}=\vec{P}_b=(0,\,0,\,P_b(z)),\\&\varepsilon_r=\varepsilon_{rb}(z),\ \phi=\phi_b(z),\ G=G_b\left(z\right)\end{aligned}\right\} \tag{11}$$

where the subscript b denotes the basic state. The quiescent basic state has a solution in the form

$$-\nabla p_b + \rho_b \vec{g} + \left(\vec{P}_b \bullet \nabla\right)\vec{E}_b = \vec{0} \tag{12}$$

$$\varepsilon_{rb} = (1+\chi) + e\beta z \tag{13}$$

$$E_b = \frac{E_0(1+\chi)}{(1+\chi)+e\beta z} \tag{14}$$

$$\vec{P}_b = \varepsilon_0\left[\varepsilon_{rb} - 1\right]\vec{E}_b \tag{15}$$

where $\beta = -\dfrac{dT_b}{dz}$ and E_0 root mean square value of the electric field at the lower surface. In the quiescent basic state, the equation of radiative transfer (9) takes the form

$$\mu_3 \frac{dI}{dz} = K_a\left[P_B - I\right] \tag{16}$$

where μ_3 is the directional cosine of $\vec{s}$ in the z-direction. Equation (16) is suggestive of the fact that the intensity of radiation is increased by emission and decreased by absorption.

The energy equation (3) in the basic state becomes

$$\frac{G_b}{s_r} + \kappa\frac{d^2T_b}{dz^2} = 0\,. \tag{17}$$

Equation (17) indicates that the heat transfer in the basic state is essentially by conduction and radiation. If F_z is the z-component of the radiative heat flux, then we may write

$$G_b = -\frac{dF_z}{dz} \tag{18}$$

and we may write Eq. (17) in the integrated form

$$F_z + \kappa s_r\,\beta = C_1 \tag{19}$$

where C_1 is the constant of integration.

Iterative solutions of one-dimensional radiative equilibrium problems all show that remarkably accurate results can be obtained by assuming a simple form for the angular distribution of radiative intensity. Assuming the Milne-Eddington approximation, and using the radiative heat transfer equation (16), the differential equation associated with the heat flux F_z can be obtained in the form (Goody, [1])

$$\frac{d^2F_z}{dz^{*2}} - \lambda^2 F_z = -\frac{\lambda^2\psi}{1+\psi}C_1 \tag{20}$$

where $z^* = \dfrac{z}{d}$, $\lambda^2 = 3K_a^2 d^2(1+\psi)$, $\psi = \dfrac{4\pi Q_r}{3\kappa K_a s_r}$, $Q_r = \dfrac{4\sigma_s}{\pi}T_0^3$ and

σ_s is the Stefan-Boltzmann constant.

Solving Eq. (20) using the following dimensionless radiative boundary conditions

$$\left.\begin{aligned} \frac{dF_z}{dz^*} &= -2K_a F_z d \quad \text{at} \quad z^* = 1/2 \\ \frac{dF_z}{dz^*} &= 2K_a F_z d \quad \text{at} \quad z^* = -1/2 \end{aligned}\right\} \tag{21}$$

we obtain the solution in the form

$$f\left(z^*\right) = \frac{\beta}{\bar{\beta}} = L_1 \cosh\left(\lambda z^*\right) + L_2 \tag{22}$$

where

$$L_1 = \psi \left[\frac{2\psi}{\lambda} + \frac{1}{2}\sqrt{3+3\psi}\,\sinh\left(\lambda/2\right) + \cosh\left(\lambda/2\right) \right]^{-1}$$

and

$$L_2 = \frac{L_1}{\psi}\left[\frac{1}{2}\sqrt{3+3\psi}\,\sinh\left(\lambda/2\right) + \cosh\left(\lambda/2\right) \right]$$

and $\bar{\beta}$ is the mean value of β throughout the medium. The radiative boundary conditions (21) are obtained using the fact that the molecular conduction ensures continuity of temperature at the two surfaces. It is worth mentioning that $f(z^*)$ tends to unity if either λ or ψ tends to zero independently. Moreover, if λ and ψ are both greater than unity, the variation of the basic state temperature is exponential. In other words, the basic state temperature is no longer linear if the radiation effect is accounted for. In what follows we study the stability of the quiescent state within the framework of the linear theory.

2.1 Stability Analysis

Let the components of the perturbed physical quantities be

$$\left.\begin{aligned} &\vec{q} = \vec{q}_b + \vec{q}\,' = \left(u', v', w'\right),\ p = p_b + p',\ \rho = \rho_b + \rho',\ T = T_b + T', \\ &\vec{E} = \vec{E}_b + E',\ \vec{P} = \vec{P}_b + \vec{P}',\ \varepsilon_r = \varepsilon_{rb} + \varepsilon'_r,\ \phi = \phi_b + \phi',\ G = G_b + G' \end{aligned}\right\} \tag{23}$$

where the primes indicate infinitesimally small perturbations. Substituting (23) into the governing equations, neglecting the nonlinear terms, incorporating the quiescent state solutions and eliminating the pressure term gives the following equations

$$\frac{\rho_0}{\varepsilon}\frac{\partial}{\partial t}\left(\nabla^2 w'\right) = \alpha_T \rho_0 g \nabla_1^2 T' - \frac{\mu_f}{k}\nabla^2 w' + \varepsilon_0 e \beta E_0 \frac{\partial}{\partial z}\left(\nabla_1^2 \phi'\right) + \frac{\varepsilon_0 e^2 E_0^2 \beta}{(1+\chi)}\nabla_1^2 T' \tag{24}$$

$$M\frac{\partial T'}{\partial t} - \beta w' = \kappa \nabla^2 T' + \frac{G'}{s_r} \tag{25}$$

$$(1+\chi)\nabla^2 \phi' - eE_0 \frac{\partial T'}{\partial z} = 0 \tag{26}$$

where $\nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$. Since Eq. (25) is an integro-differential equation, we adopt the approximation which is valid when the fluid medium is optically thin (known as transparent approximation). For the transparent approximation (Goody, [1]), the relation becomes

$$\nabla_1^2 G' = -4\pi Q_r K_a \nabla_1^2 T'. \tag{27}$$

Equation (25), corresponding to the transparent approximation, after making use of (27), becomes

$$M\frac{\partial}{\partial t}\left(\nabla_1^2 T'\right)-\beta\,\nabla_1^2 w'=\kappa\nabla^2\left(\nabla_1^2 T'\right)-\frac{4\pi Q_r\,K_a}{s_r}\,\nabla_1^2 T'. \tag{28}$$

We next make Eqs. (24), (26) and (28) dimensionless using the following transformations

$$w^*=\frac{d\,w'}{\kappa},\quad \phi^*=\frac{(1+\chi)}{eE_0\,\bar{\beta}d^2}\,\phi',\quad \left(x^*,\,y^*,\,z^*\right)=\left(\frac{x}{d},\,\frac{y}{d},\,\frac{z}{d}\right),\, t^*=\frac{t}{\left(\dfrac{M\,d^2}{\kappa}\right)},\quad T^*=\frac{T'}{\bar{\beta}d} \tag{29}$$

where the quantities with asterisks are dimensionless. Eqs. (24), (26) and (28), using (29), can be written (after dropping the asterisks)

$$\frac{1}{Va}\frac{\partial}{\partial t}\left(\nabla^2 w\right)=-\,\nabla^2 w+(R+L)\nabla_1^2 T-L\,\frac{\partial}{\partial z}\left(\nabla_1^2\phi\right) \tag{30}$$

$$\frac{\partial}{\partial t}\left(\nabla_1^2 T\right)-f(z)\,\nabla_1^2 w=\nabla^2\left(\nabla_1^2 T\right)-\frac{\lambda^2\psi}{1+\psi}\,\nabla_1^2 T \tag{31}$$

$$\nabla^2\phi-\frac{\partial T}{\partial z}=0 \tag{32}$$

where $Va=\dfrac{\varepsilon\,\mu_f\,d^2}{\rho_0\,k\,\kappa}$ is the Vadasz number, $R=\dfrac{\alpha_T\,\rho_0\,g\,\bar{\beta}\,k d^2}{\mu_f\,\kappa}$ is the Darcy-Rayleigh number, $L=\dfrac{\varepsilon_0\left(eE_0 d\right)^2\,\beta\bar{\beta}\,k}{\mu_f\,\kappa\,(1+\chi)}$ is the Darcy-Roberts number for the radiative heat transfer, $\psi=\dfrac{4\pi Q_r}{3\kappa K_a s_r}$ is the conduction-radiation parameter, $\lambda=K_a\,d\sqrt{3\,(1+\psi)}$ is the absorptivity parameter and $f(z)$ is given in Eq. (22).

The boundary conditions are

$$w=T=\frac{\partial\phi}{\partial z}=0\quad\text{at}\quad z=0,1. \tag{33}$$

We use the normal mode solution for the dependent variables in the form

$$\left[w,\,T,\,\phi\right]=\left[W(z),\,\Theta(z),\,\Phi(z)\right]\exp\left\{i(lx+my)+\sigma t\right\} \tag{34}$$

where l and m are the dimensionless wave numbers in the x and y directions respectively and σ is the growth rate.

2.2 Stationary Instability

Substituting (34) into Eqs. (30) - (32), we arrive at the following equations for stationary instability (characterized by $\sigma=0$)

$$\left(D^2-\alpha^2\right)W+\left(R+L\right)\alpha^2\Theta-L\alpha^2 D\Phi=0 \tag{35}$$

$$\left[D^2-\alpha^2-\frac{\lambda^2\psi}{1+\psi}\right]\Theta+f(z)W=0 \tag{36}$$

$$\left(D^2-\alpha^2\right)\Phi-D\Theta=0 \tag{37}$$

where $D=\dfrac{d}{dz}$ and $\alpha=\sqrt{l^2+m^2}$ is the overall horizontal wavenumber.

In view of (34), the boundary conditions take the form

$$W=\Theta=D\Phi=0 \quad \text{at} \quad z=\pm\,1/2 \tag{38}$$

2.3 Oscillatory Instability

We now examine the validity of the principle of exchange of stabilities (PES) for the problem under consideration by the Galerkin method. It should be noted that PES has been shown to be valid for the Rayleigh-Bénard problem of dielectric fluids in the absence of chemical reaction (Turnbull, [15]).

Substituting (34) into Eqs. (30) - (32), we arrive at the following equations

$$\frac{\sigma}{Va}\left(D^2-\alpha^2\right)W+\left(R+L\right)\alpha^2\Theta+\left(D^2-\alpha^2\right)W-L\alpha^2 D\Phi=0 \tag{39}$$

$$\sigma\Theta-f(z)W-\left[D^2-\alpha^2-\frac{\lambda^2\psi}{1+\psi}\right]\Theta=0 \tag{40}$$

$$\left(D^2-\alpha^2\right)\Phi-D\Theta=0\,. \tag{41}$$

Multiplying equations (39) - (41) by W, Θ and Φ respectively, integrating the resulting equations with respect z between the limits $z=-1/2$ and $z=1/2$, taking $W(z)=A_1W_1(z)$, $\Theta(z)=B_1\Theta_1(z)$ and $\Phi(z)=C_1\Phi_1(z)$ (in which W_1,Θ_1 and Φ_1 are trial functions) leads to the following system of equations

$$P_1\left(1+\frac{\sigma}{Va}\right)A_1+\left(R+L\right)P_2\,B_1-L\,P_3C_1=0 \tag{42}$$

$$\left[P_4+\left(\sigma+\frac{\lambda^2\psi}{1+\psi}\right)P_5\right]B_1-P_6\,A_1=0 \tag{43}$$

$$P_7\,C_1-P_8\,B_1=0 \tag{44}$$

where

$$P_1=\left\langle W_1D^2W_1\right\rangle-\alpha^2\left\langle W_1^{\,2}\right\rangle,\ P_2=\alpha^2\left\langle W_1\Theta_1\right\rangle,\ P_3=\alpha^2\left\langle W_1D\Phi_1\right\rangle,\ P_4=\alpha^2\left\langle\Theta_1^{\,2}\right\rangle-\left\langle T_1D^2T_1\right\rangle,$$

$$P_5=\left\langle\Theta_1^{\,2}\right\rangle,\ P_6=\left\langle T_1f(z)W_1\right\rangle,\ P_7=\left\langle\Phi_1D^2\Phi_1\right\rangle-\alpha^2\left\langle\Phi_1^{\,2}\right\rangle,\ P_8=\left\langle\Phi_1D\Theta_1\right\rangle.$$

Here the inner product is defined as $\langle f\, g \rangle = \int_{1/2}^{-1/2} f\, g\, dz$. Assuming $\sigma = i\,\omega$ with ω being the frequency of oscillations, the criterion for the existence of the unique solution of the system of equations (42) - (44) leads to the expression $R = R_1 + i\,R_2$

where

$$R_1 = \left[\frac{P_3 P_8}{P_2 P_7} - 1 \right] L - \left[P_4 + \frac{\lambda^2 \psi}{1+\psi} P_5 \right] \frac{P_1}{P_2 P_6} + \frac{P_1 P_5}{P_2 P_6 Va}\, \omega^2$$

$$R_2 = \left[P_4 + \left(Va + \frac{\lambda^2 \psi}{1+\psi} \right) P_5 \right] \frac{P_1}{P_2 P_6 Va}\, \omega\ .$$

As the Rayleigh number R is real, we must have $R_2 = 0$. Clearly $\omega = 0$. This means that the PES is valid for the present problem and the possibility of existence of oscillatory instability is ruled out. Hence the stationary instability is the preferred mode.

2.4 Method of Solution

The system comprising Eqs. (35) - (37) and the homogeneous boundary conditions (38) is an eigenvalue problem, with R being the eigenvalue. An approximate solution of the foregoing eigenvalue problem can be arrived at by means of the Galerkin method. To this end, we let

$$W = \sum A_i W_i\,,\ \ \Theta = \sum B_i \Theta_i\,,\ \ \Phi = \sum C_i \Phi_i \tag{45}$$

where A_i, B_i and C_i are constants and the basis functions W_i, Θ_i and Φ_i are represented by the power series satisfying the respective boundary conditions. Application of the Galerkin method yields

$$\left.\begin{aligned} &D_{ji} A_i + E_{ji} B_i + F_{ji} C_i = 0 \\ &G_{ji} A_i + H_{ji} B_i = 0 \\ &K_{ji} B_i + L_{ji} C_i = 0 \end{aligned}\right\} \tag{46}$$

where

$$D_{ji} = \left\langle W_j D^2 W_i \right\rangle - \alpha^2 \left\langle W_j W_i \right\rangle, \quad E_{ji} = (R+L)\,\alpha^2 \left\langle W_j \Theta_i \right\rangle,$$

$$F_{ji} = -L\alpha^2 \left\langle W_j D\Phi_i \right\rangle, \quad G_{ji} = \left\langle \Theta_j\, f(z)\, W_i \right\rangle,\ H_{ji} = \left\langle \Theta_j D^2 \Theta_i \right\rangle - \alpha^2 \left\langle \Theta_j \Theta_i \right\rangle - \frac{\lambda^2 \psi}{1+\psi} \left\langle \Theta_j \Theta_i \right\rangle,$$

$$K_{ji} = -\left\langle \Phi_j D\Theta_i \right\rangle,\ L_{ji} = \left\langle \Phi_j D^2 \Phi_i \right\rangle - \alpha^2 \left\langle \Phi_j \Phi_i \right\rangle.$$

The trial functions chosen are $W_i = \Theta_i = \cos(2i-1)\pi z$ and $\Phi_i = \sin(2i-1)\pi z$. On applying the Galerkin method to the system (46) of equations, we would obtain the critical Rayleigh number and the corresponding critical wavenumber.

III. Figures

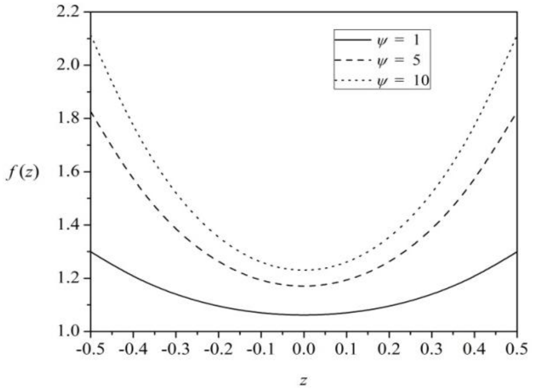

Figure 2: Basic temperature profiles for different values of the conduction-radiation parameter ψ .

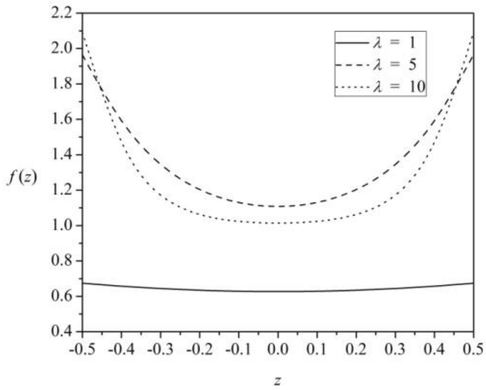

Figure 3: Basic temperature profiles for different Values of the absorptivity parameter λ .

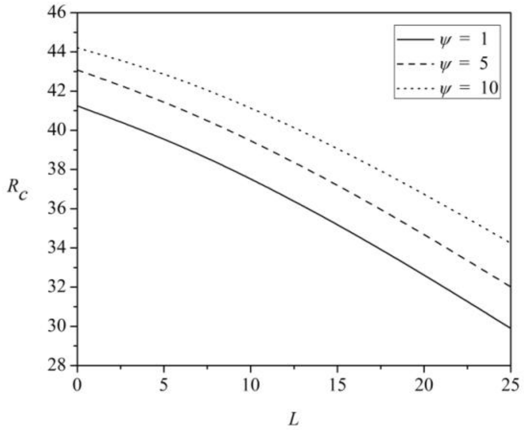

Figure 4: Plot of R_c as a function of L for different values of ψ .

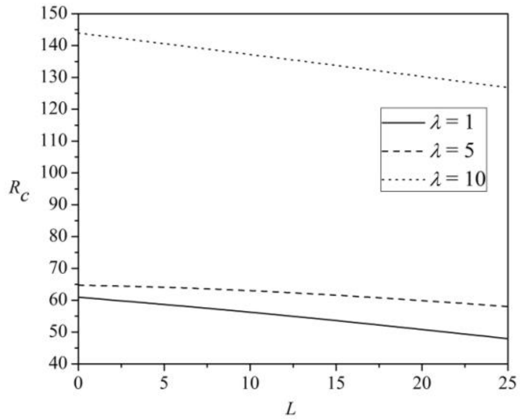

Figure 5: Plot of R_c as a function of L for different values of λ .

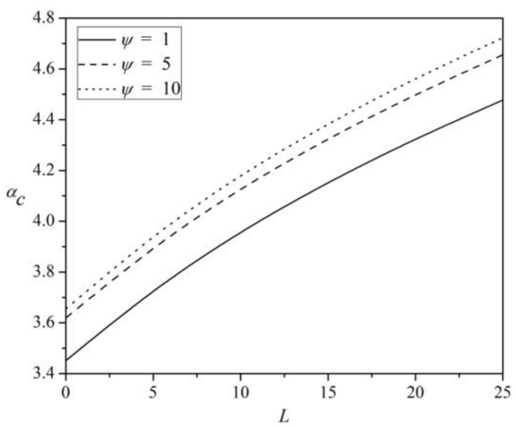

Figure 6: Plot of α_c as a function of L for different values of ψ .

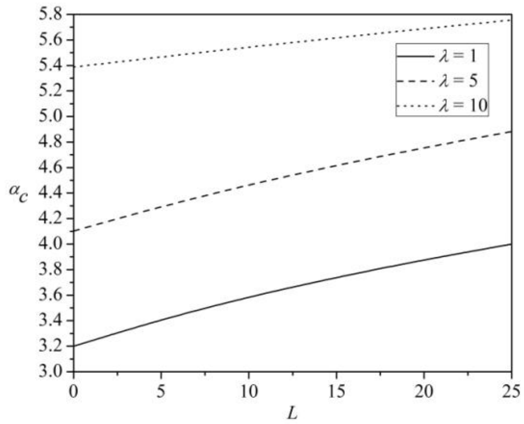

Figure 7: Plot of α_c as a function of L for different values of λ .

IV. Results and Conclusion

The effect of thermal radiation on the onset of Darcy electroconvection in an absorbing and emitting dielectric fluid layer in the presence of a vertical ac electric field is studied. The principle of exchange of stabilities is shown to be valid by means of the single term Galerkin method. The critical values concerning stationary instability are obtained using the higher order Galerkin method. As to the values of radiative parameters ψ and λ, large radiative effects are more likely if a gas rather than a liquid is used as a fluid. In view of this, large values of ψ and λ have been overlooked in this work.

In order to understand the results of the problem better, we examine the basic state temperature distribution which sheds some light on the effect of radiative heat transfer on the stability of the system. Figs.2 and 3 are plots of z versus $f(z)$ for different values of ψ and λ respectively. We notice that the basic state temperature profile turns out to be exponential and nonlinear as ψ and λ increase and it is symmetric about the line z = 0. This symmetry of the basic temperature profiles is largely responsible for the stabilizing effect of both ψ and λ.

The variation of R_c with L for different values of ψ and λ is portrayed in Figs. 4 and 5 respectively. The parameter ψ signifies the temperature in the equilibrium state, while λ is the characteristic of absorption coefficient and distance between the horizontal planes. The stabilizing influence of ψ and λ is evident from Figs. 4 and 5. This is due to the fact that radiative transfer tends to damp out any motions which may arise due to the heat transfer from hotter to colder parts of the dielectric fluid. As a result, the effect of thermal radiation is to inhibit Darcy electroconvection. Further, Figs. 6 and 7 show that α_c increases with increase in L, ψ and λ. Thus convection cell size gets smaller with an increase in L, ψ and λ. In the limiting case of $\psi = L = 0$, one obtains the classical values of $\alpha_c = \pi$ and $R_c = 4\pi^2$ (Nield and Bejan, [16]).

Thus, it may be concluded that basic temperature profiles tend to be nonlinear and symmetric with respect to the variations in the radiative parameters ψ and λ. This symmetry is largely responsible for the stabilizing effect of both ψ and λ; the effect of electric forces and thermal radiation is to contract the convection cell size at the onset of convection and in the presence of radiation effect, stationary mode of instability is preferred to the oscillatory mode.
The model finds applications in solar energy collection systems, porous combustors and also in the design of high temperature chemical process systems.

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