IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 11, Issue 3 Ver. III (May - Jun. 2015), PP 78-85
www.iosrjournals.org

An Unsteady Flow past an Accelerated Infinite Vertical Plate with the Variable Temperature and Uniform Mass Diffusion through the Porous Medium

Prof (Dr) P.T.Hemamalini[1], N.Suresh Kumar[2]
[1](HOD, Department of science and humanities, Karpagam University, Coimbatore- 641021.TN, India)
[2] (Research Scholar in Department of Mathematics, Karpagam University, Coimbatore-641021, TN, India)

Abstract: *The purpose of present analysis is to study the effect of the unsteady flow of an incompressible viscous fluid past an uniformly Accelerated Infinite Vertical Porous Plate through the Porous Medium taking into the account of the presence of the variable temperature, uniform mass diffuse with the Heat and Mass Transfer. The dimensionless governing equations are solved by using the Laplace Transform technique. The Velocity, temperature and concentration fields are studied for the different physical parameters like Thermal Grashof Number, Modified Grashof Number, Permeability Parameter, Prandtl Number, Schmidt Number, and time. The effect of these parameters on the Velocity field, Temperature field and Concentration distribution are studied and the results are presented graphically and discussed quantitatively.*

Keywords: *Accelerated infinite Vertical Plate, Heat Transfer, Mass Transfer, Porous Medium and mass diffusion.*

I. Introduction

Effect of the heat and mass transfer play a vital role in the space craft design in cooling of the liquid metal of the nuclear reaction, pollution of environment etc. Flow through the porous medium has numerous engineering problems such as in the underground water resources, rain water harvesting, the moment of oil and natural gas through oil sand stone reservoirs, purification of crude oil, paper and pulps industry membrane separation process, flow of blood.

The problem of past as an accelerated vertical plate has many technological applications such as filtration process, the drying of porous materials in the textile industries and saturation of the porous materials by the chemicals. The combined effect of the heat and mass transfer plays an important role in the manufacturing industries for designing of fins, steels rolling, nuclear power plants gas turbines and various propulsion devices for aircraft, missile, spacecraft design, solar energy collectors, design of chemical processing equipments, satellites and space vehicles are examples of such engineering applications

II. Literature Review

Free convection effect on the flow past an accelerated vertical plate in an incompressible dissipative fluid studied by Gupta et al [1]. Soundalgekar [2] presented an exact solution on the flow past a vertical oscillating plate with variable temperature. Singh et al [3] analyzed mass transfer on the flow past an accelerated vertical plate with a constant heat Flux. Muthucumaraswamy et al [4] studied the effect of the heat and mass transfer on the flow past an oscillating vertical plate with the variable temperature. Basant kumar et al [5] discussed the free convection and mass transfer effect on the flow past an accelerated vertical plate with the source. Muthucumaraswamy et al [6] studied an unsteady flow past an accelerated infinite vertical plate with the variable temperature and uniform mass diffusion. Chaudhary et al [7] discussed the heat and mass transfer effect on the MHD free convection flow past an oscillating plate embedded in the porous medium. Muthucumaraswamy et al [8] analyzed the heat and mass effect on the moving vertical plate in the presence of the thermal radiation. Muthucumaraswamy et al [9] presented an exact solution for the diffusion and heat transfer effect on the exponentially accelerated vertical plate with the variable temperature Bala Siiddulu Malga et al [10] studied viscous dissipation effect on the unsteady free convection and mass transfer flow past an accelerated vertical porous plate with the suction. Sahin Ahmaed et al [11] analyzed convection laminar radiation flow over an accelerated vertical plate embedded in a porous medium with an external magnetic field. Saraswat Amit et al [12] discussed the heat and mass transfer effect on the flow past an oscillating infinite vertical plate with the variable temperature through the porous media. Jain et al [13] presented the solution for the computational method for the partial differential equation. Muthucumaraswamy et al[14] studied the MHD flow past an exponentially accelerated vertical plate with the variable temperature with the mass diffusion in the presence of chemical reaction. Chenna Kesavaiah et al [15] analyzed the effect of the radiation chemical reaction and soret on an unsteady flow past an accelerated isothermal infinite vertical plate.

The present study deals with the Investigation of an unsteady flow of an incompressible viscous fluid with the heat and mass transfer effects on the Flow Past an Accelerated Infinite Vertical Plate with the variable temperature through the porous medium.

III. Formulation of Problem

The unsteady flow of an incompressible viscous fluid which is initially at rest past an infinite vertical plate with variable temperature through a porous medium is considered. The flow is assumed to be in x-direction which takes vertical plate in the upward direction .The y-axis is taken to be normal to the plate. Initially the plate and the fluid are in same Temperature T' with the same concentration level C' at all points. At time $t' > 0$ the plate accelerated with velocity $u' = \frac{u_0^3 t'}{\nu}$ in its own plane. The plate temperature is raised to T'_w and the level of concentration near the plate is raised to C'_w linearly with the time t. Then by Boussinesq's approximation the unsteady flow is governed by the following equation.

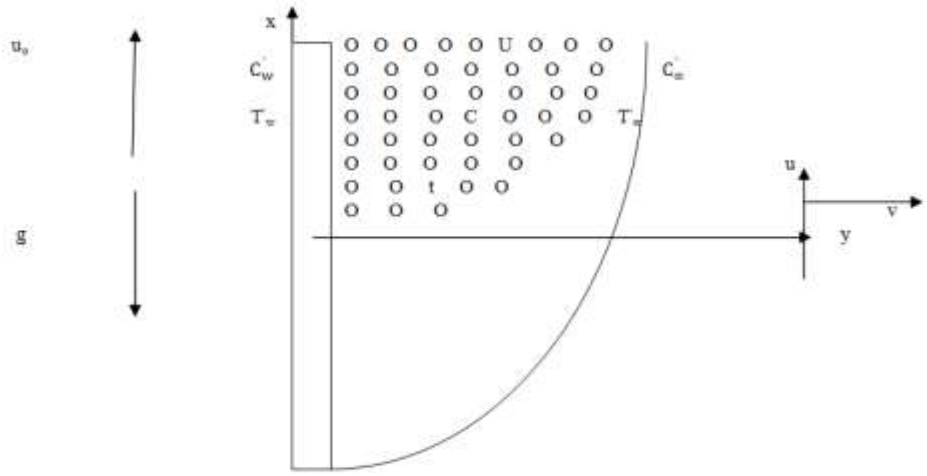

Figure:1 Physical model of the problem

$$\frac{\partial u'}{\partial t'} = g\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) + \upsilon\frac{\partial^2 u'}{\partial y^2} - \upsilon\left(\frac{u'}{k'}\right) \quad (1)$$

$$\rho c_p \frac{\partial T'}{\partial t'} = \kappa \frac{\partial^2 T'}{\partial y^2} \quad (2)$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y^2} \quad (3)$$

with the following initial and boundary conditions

$u = 0,\quad T' = T'_\infty,\quad C' = C'_\infty,\quad t' \le 0,\quad$ for all y.

$u = \frac{u_0^3 t'}{\nu},\ T' = T'_\infty + (T'_w - T'_\infty)\,A\,t',\quad C' = C'_\infty + (C'_w - C'_\infty)\,A\,t',\ t' > 0,$ at $y = 0$,

$u = 0,\quad T' \to T'_\infty,\ C' \to C'_\infty,$ as $y \to \infty$. (4)

Where $A = \left(\frac{u_0^2}{\nu}\right)$

Introduce the following dimensionless quantities,

$$U = \left(\frac{u}{u_0}\right),\ t = \left(\frac{t' u_0^2}{\nu}\right),\ Y = \left(\frac{y u_0}{\nu}\right),\ \Theta = \left[\frac{T' - T'_\infty}{T'_w - T'_\infty}\right],\ Sc = \left(\frac{\nu}{D}\right),\ \frac{1}{K} = \left(\frac{u_0^2 k'}{\nu^2}\right)$$

$$Pr = \left(\frac{\mu C_p}{k}\right),\ C = \left[\frac{C' - C'_\infty}{C'_w - C'_\infty}\right],\ Gr = \left[\frac{g\beta\nu(T'_w - T'_\infty)}{u_0^3}\right],\ Gc = \left[\frac{g\beta^*\nu(C'_w - C'_\infty)}{u_0^3}\right], \quad (5)$$

then Equation (1) –(3) leads to

$$\frac{\partial U}{\partial t} = Gr\,\Theta + Gc\,C + \frac{\partial^2 U}{\partial Y^2} - K\,U \quad (6)$$

$$\frac{\partial \Theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2 \Theta}{\partial Y^2} \quad (7)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} \quad (8)$$

Initial and boundary conditions in the non dimensional form are

$U = 0,\quad \Theta = 0,\quad C = 0,\quad$ for all $\quad Y \le 0,\quad t \le 0$

$U = t,\quad \Theta = t,\quad C = t,\quad$ at $\quad Y = 0,\quad t > 0$

$U = 0,\quad \Theta \to 0,\quad C \to 0,\quad$ as $\quad Y \to \infty$ (9)

IV. Method of Solution

The governing equations By using the Laplace transform technique, the governing equations are solved by taking the Laplace transform of the equations (6),(7),(8) and (9) the results are,

$$\frac{d^2\bar{U}}{dY^2} - (s+k)\bar{U} = -Gr\,\bar{\Theta} - Gc\,\bar{C} \quad (10)$$

$$\frac{d^2\bar{\Theta}}{dY^2} - s\,Pr\,\bar{\Theta} = 0 \quad (11)$$

$$\frac{d^2\bar{C}}{dY^2} - s\,Sc\,\bar{C} = 0 \quad (12)$$

Now the initial and boundary conditions are

$$\begin{aligned} &\bar{U}=0, \quad \bar{\Theta}=0, \quad \bar{C}=0 \quad \text{for all} \quad Y, \quad t\le 0\\ &\bar{U}=\frac{1}{s^2}, \quad \bar{\Theta}=\frac{1}{s^2}, \quad \bar{C}=\frac{1}{s^2}, \quad \text{at} \quad Y=0, \quad t>0.\\ &\bar{U}=0, \quad \bar{\Theta}\to 0, \quad \bar{C}\to 0 \quad \text{as} \quad Y\to\infty, \quad t>0 \end{aligned} \quad (13)$$

On solving the equation (10), (11), (12) with the help of equation (13) the results are,

$$\bar{\Theta} = \frac{e^{-y\sqrt{s\,Pr}}}{s^2} \quad (14)$$

$$\bar{C} = \frac{e^{-y\sqrt{s\,Sc}}}{s^2} \quad (15)$$

$$\bar{U} = \left[\frac{1}{s^2}\left\{1+\frac{Gr}{s(Pr-1)-k}+\frac{Gc}{s(Sc-1)-k}\right\}\right] e^{-Y\sqrt{s+k}} - \frac{1}{s^2}\left\{\frac{Gr\,e^{-Y\sqrt{sPr}}}{s(Pr-1)-k}+\frac{Gc\,e^{-Y\sqrt{s\,Sc}}}{s(Sc-1)-k}\right\} \quad (16)$$

Where s is the laplace transform parameter. On taking inverse laplace transform of equation (14), (15), and (16) the results are,

$$\Theta = t\left[(1+2\eta^2 Pr)\,\mathrm{erfc}\left(\eta\sqrt{Pr}\right) - \frac{2}{\sqrt{\pi}}\,e^{-\eta^2 Pr}\,\eta\sqrt{Pr}\right] \quad (17)$$

$$C = t\left[(1+2\eta^2 Sc)\,\mathrm{erfc}\left(\eta\sqrt{Sc}\right) - \frac{2}{\sqrt{\pi}}\,e^{-\eta^2 Sc}\,\eta\sqrt{Sc}\right] \quad (18)$$

$$\begin{aligned} U = {} & t\left[(1+2\eta^2)\,\mathrm{erfc}(\eta) - \frac{2}{\sqrt{\pi}}e^{-\eta^2}\eta\right]\\ & + e^{Y\sqrt{k}}\,\mathrm{erfc}\left(\eta+\sqrt{kt}\right)\left[\frac{Gr}{Pr-1}\left(\frac{-1}{2C^2}+\frac{t}{2C}+\frac{Y}{4C\sqrt{k}}\right)\right.\\ & \qquad \left.+\frac{Gc}{Sc-1}\left(\frac{-1}{2b^2}+\frac{t}{2b}+\frac{Y}{4b\sqrt{k}}\right)\right]\\ & + e^{-Y\sqrt{k}}\,\mathrm{erfc}\left(\eta-\sqrt{kt}\right)\left[\frac{Gr}{Pr-1}\left(\frac{-1}{2C^2}+\frac{t}{2C}-\frac{Y}{4C\sqrt{k}}\right)\right.\\ & \qquad \left.+\frac{Gc}{Sc-1}\left(\frac{-1}{2b^2}+\frac{t}{2b}-\frac{Y}{4b\sqrt{k}}\right)\right]\\ & + \frac{Gr\,e^{-ct}}{2(Pr-1)C^2}\left[e^{Y\sqrt{-C\,Pr}}\left\{\mathrm{erfc}\left(\eta+\sqrt{-C\,Pr\,t}\right)-\mathrm{erfc}\left(\eta\sqrt{Pr}+\sqrt{-Ct}\right)\right\}\right.\\ & \qquad \left.+e^{-Y\sqrt{-C\,Pr}}\left\{\mathrm{erfc}\left(\eta-\sqrt{-C\,Pr\,t}\right)-\mathrm{erfc}\left(\eta\sqrt{Pr}-\sqrt{-Ct}\right)\right\}\right]\\ & + \frac{Gc\,e^{-bt}}{2(Sc-1)b^2}\left[e^{Y\sqrt{-b\,Sc}}\left\{\mathrm{erfc}\left(\eta+\sqrt{-b\,Sc\,t}\right)-\mathrm{erfc}\left(\eta\sqrt{Sc}+\sqrt{-bt}\right)\right\}\right.\\ & \qquad \left.+e^{-Y\sqrt{-b\,Sc}}\left\{\mathrm{erfc}\left(\eta-\sqrt{-b\,Sc\,t}\right)-\mathrm{erfc}\left(\eta\sqrt{Sc}-\sqrt{-bt}\right)\right\}\right]\\ & - \frac{Gr}{Pr-1}\left[\mathrm{erfc}\left(\eta\sqrt{Pr}\right)\left\{\frac{-1}{C^2}+\frac{1}{C}\left(t+\frac{Y^2 Pr}{2}\right)\right\}-\frac{Y}{C}\,e^{\frac{Y^2 Pr}{4t}}\sqrt{\frac{Prt}{\pi}}\right]\\ & - \frac{Gc}{Sc-1}\left[\mathrm{erfc}\left(\eta\sqrt{Sc}\right)\left\{\frac{-1}{b^2}+\frac{1}{b}\left(t+\frac{Y^2 Sc}{2}\right)\right\}-\frac{Y}{b}\,e^{\frac{Y^2 Sc}{4t}}\sqrt{\frac{Sc\,t}{\pi}}\right] \end{aligned} \quad (19)$$

Where $C = \frac{-k}{Pr-1}$, $b = \frac{-k}{Sc-1}$, $\eta = \frac{Y}{2\sqrt{t}}$

V. Results and Discussions

The problem of the Heat and Mass transfer on the Flow Past an Accelerated Infinite Porous Plate with the variable temperature through the porous medium is formulated and solved analytically. The values of the velocity, temperature and concentration are obtained for the parameters such as Thermal Grashof number (Gr), Modified Grashof Number (Gc), Prandtl Number (Pr), Schmidt Number (Sc), time (t) and Permeability (k). The value of the Prandtl Number Pr is chosen to represent air (Pr = 0.71) and the value of Schmidt Number are chosen to represent presence of spices by Hydrogen (Sc = 0.22), Water vapour (Sc = 0.6), Ammonia (Sc = 0.78), Carbondioxied (Sc = 0.96).

The velocity profiles are studied and presented in the Fig 2 to 7. The velocity profile for the different values of Modified Grashof Number Gc (Gc = 2 , 3, 5) is presented in the Fig:2. It is observed that the velocity decreases with the increasing of the Gc. The velocity profiles for the different values of the Thermal Grashof Number (Gr = 2, 3, 4) is shown in the Fig:3.It is shown that the velocity increases with the increasing of the Gr. The effect of velocity for the different values of the permeability (K = 3, 5, 10) is seen in the Fig:4. And it is noted that the velocity decreases with the increasing of the permeability K. The velocity profile for the different values of time (t = 0.3, 0.4, 0.6) is presented in the Fig:5. It is clearly shown that the velocity decreases with the increasing of the time t. The temperature profiles for the different values of time (t = 0. 2, 0.4, 0.6, 0.8) is shown in the Fig:6.It is shows that the temperature increases of the with the increasing of the time t. The effect of Concentration for the different values of Schmidt Number (Sc = 0.22, 0.6, 0.78, 0.96) is seen in the Fig:7. It is noted that the concentration decreases with the increasing of the Schmidt Number Sc.

VI. Conclusion

An exact solution of the flow past an accelerated infinite vertical plate in the presence of the variable temperature and mass diffusion are studied. The dimensionless governing equations are solved by the usual Laplace transformation technique. The effect of the different parameters like Thermal Grashof number, Mass Grashof number, Schmidt number, and time are studied graphically. It is observed that the velocity decreases with the increasing value of the modified Grashof Number (Gc), Permeability (K), time (t). But the trend is reversed with respect to the thermal Grashof Number (Gr). It is also observed that the temperature increases with the increasing of the time (t) and wall concentration decreases with the increasing of the Schmidt number (Sc).

VII. Appendix – Nomenclature

C'_∞ Concentration in the fluid far away from the plate.
C'_W Concentration of the plate
D Mass diffusion coefficient
A Constant
Y' Coordinate axis normal to the plate
C dimensionless concentration
Y Dimensionless coordinate axis normal to the Plate
U Dimensionless velocity
Gc Modified Grashof Number
Pr Prandtl Number
Sc Schmidt Number
C' Species concentration in the fluid
C_P Specific heat at constant pressure
T'_∞ Temperature of the fluid far away from the Plate
T' Temperature of the fluid near the plate
T'_w Temperature of the plate
K Thermal conductivity of fluid
Gr Thermal Grashof Number
t' Time
t Dimensionless time
x Spatial coordinate along the plate
u' Velocity of the fluid in the x- direction
u_o Velocity of the plate
g Acceleration due to Gravity
κ' Permeability Parameter
μ Coefficient of viscosity
erfc Complementary error function
ρ Density of the fluid

Θ Dimensionless Temperature
σ Electric Conductivity
ν Kinematic Viscosity
β Volumetric coefficient of Thermal expansion
W Condition on the wall
∞ Free stream conditions
β^* Volumetric coefficient of expansion with concentration
η Similarity parameter

VIII. Graphical Representation

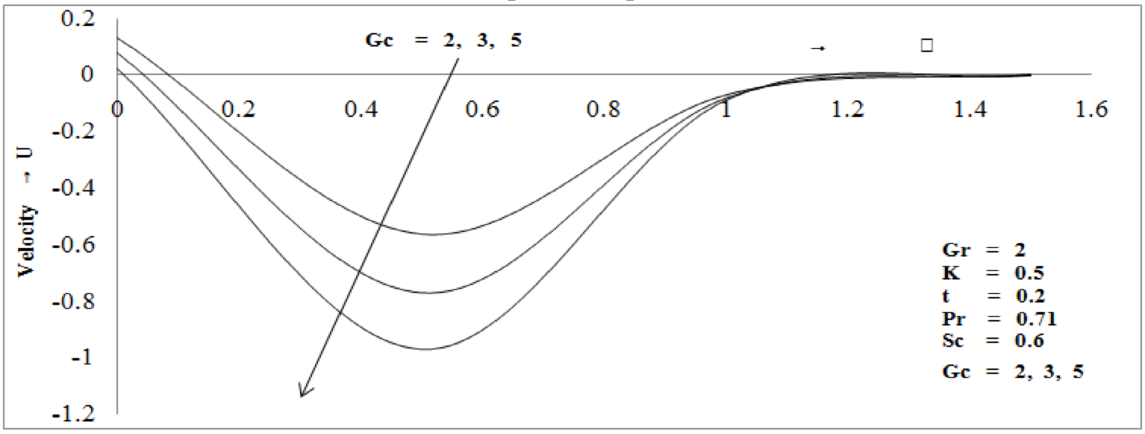

Figure 2: Velocity profile for different values of Gc

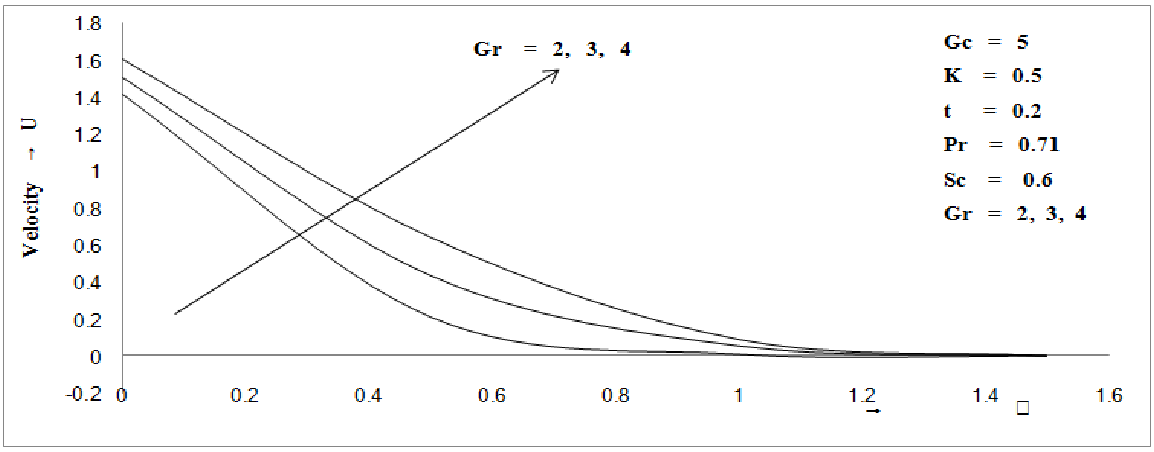

Figure 3 : Velocity profile for different values of Gr

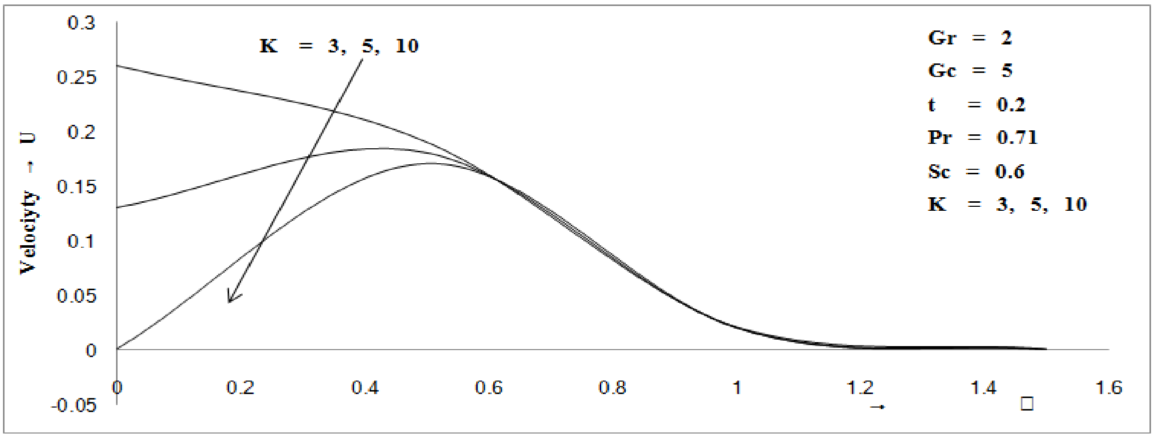

Figure 4: Velocity profile for different values of K

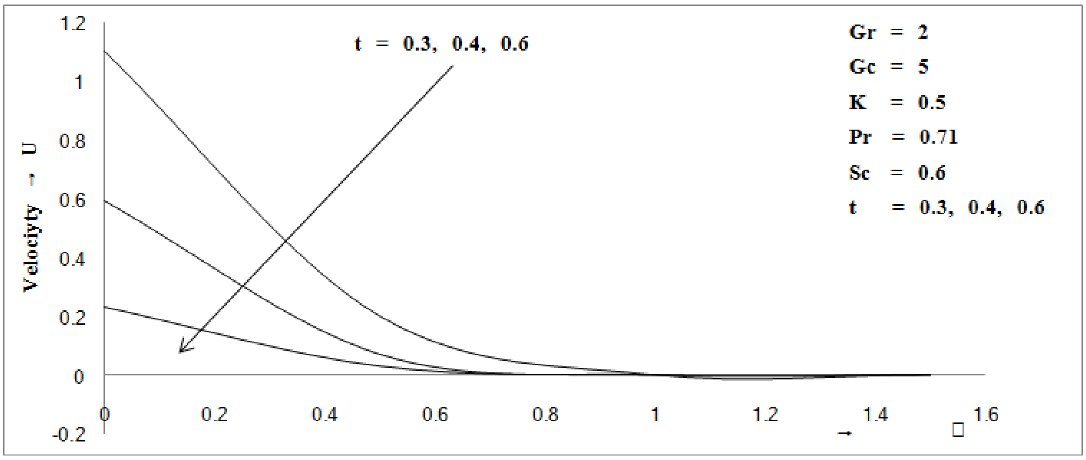

Figure 5: Velocity profile for different values of t

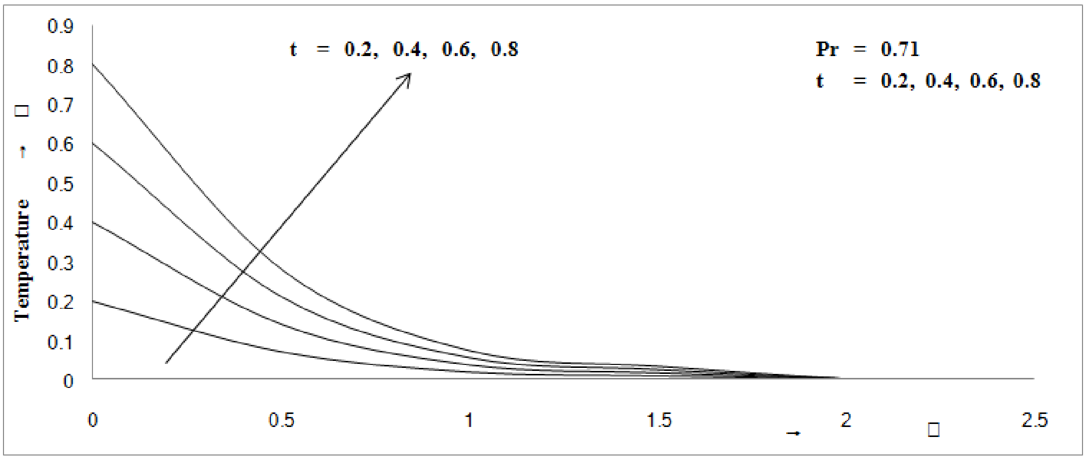

Figure 6: Temperature profile for different values of t

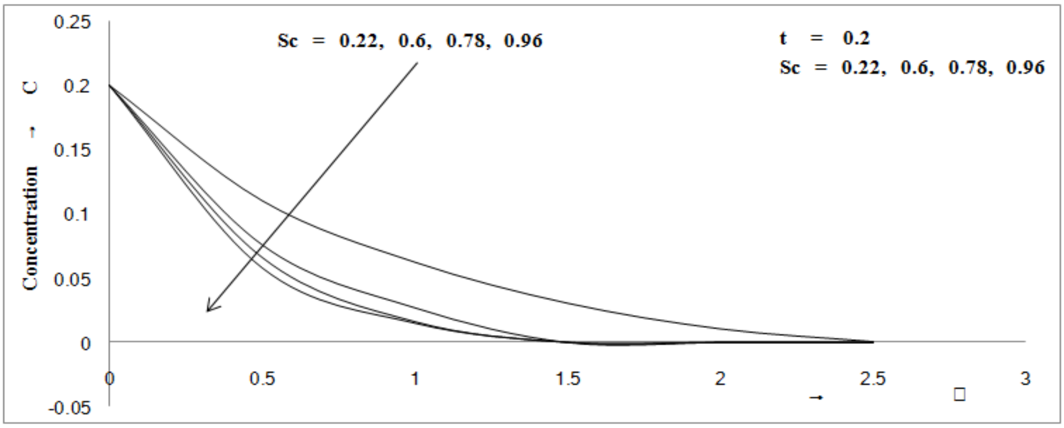

Figure 7: Concentration profiles for different values of Sc

References

[1] Gupta.A.S, I.Soudalgekar.V.M "Free convection effects on the flow past an Accelerated vertical plate in an incompressible dissipative fluid". Rev, Roum Sci Techn-mec- Apl 24, pp 561-568, (1979).

[2] Soudalgekar.V.M and Patel M.R "On flow past a vertical Oscillating plate with variable temperature". Journal of Heat and Mass Transfer (4) pp 143-148, (1980).

[3] Singh.A.K and Singh.J "Mass Transfer Effects on the Flow past an Accelerated vertical plate with constant Heat flux" Astrophysics and Space Science. 97, pp 57 – 61, (1983).

[4] Muthucumaraswamy.R and Vijayalakshmi.A "Effect of Heat and Mass Transfer on Flow past, an Oscillating vertical plate with variable temperature" Int.J.of. Appl Math and Mech 4(1) pp 59-65, (2008).

[5] Basant kumar Jha. Ravindra Prasad "Free convection and Mass Transfer Effects on the Flow past an Accelerated vertical plate with Heat Source". Mechanics Research Communication 17, pp 143-148, (1990).

[6] Muthucumaraswamy.R Sundar Raj .M and Subramanian USA "Unsteady flow past an Accelerated Infinite vertical plate with variable temperature and uniform Mass Diffusion". Int. J. of Appl. Math and Mech 5(6), pp 51-56, (2009).

[7] Chaudhary.R.C and Jain.A "Combined Heat and Mass Transfer Effect on MHD free convection Flow past an Oscillating plate embedded in porous medium". Rom.J. of phy 52,pp 505-524 , (2007).

[8] R.Muthucumaraswamy and G.Senthil kumar "Heat and mass transfer effects on moving vertical plate in the presence of Thermal Radiation". Theoretical Applied Mechanics vol:31,pp 35-46 (2004).

[9] Muthucumaraswamy.R.Sathappan K.E and Natarajan.R "Diffusion and Heat Transfer Effects on exponentially Accelerated vertical plate with variable temperature". Theoretical Applied Mechanics vol 14(1) pp 73-77, (2010).

[10] Bala Siddulu Malga and Naikoti kishan "Viscous dissipation Effects on unsteady free convection and Mass Transfer Flow past an Accelerated vertical porous plate with suction". Advances in Applied Science Research 2(6): pp 460-469, (2011).

[11] Sahin Ahmed, Abdul Batin " Convective laminar Radiating Flow over an Accelerated vertical plate Embedded in a porous medium with an External Magnetic Field" Int.J.of Eng & Tech vol:3 No:1 ,pp 57-63, Jan (2013)

[12] Saraswat Amit and Srivastava.R.K "Heat and mass transfer effects on flow past an Oscillating Infinite vertical plate with variable Temperature through porous Media " Res. J Recent.Sci vol 2 (ISC-2012) , pp 316-321, (2013).

[13] Jain.M.K, Iyengar.S.R.K Jain R.K "Computational method for partial Differential equation". Wiles Eastern Limited (1994).

[14] Muthucumaraswamy.R and Valliamma.V "MHD flow past an exponentially Accelerated vertical plate with variable temperature and Mass Diffusion in the presence of Chemical Reaction" Annals of Faculty Engineering Hunedoana-International Journal of Engineering 1, pp 151-154, (2012).

[15] D.Chenna Kesavaiah and P.V.Satyanarayana "Effects of Radiation Chemical Rreaction and Soret on unsteady Flow past an Accelerated Isothermal Infinite Vertical plate". Int. J. of. Tech and Res .Adv (5) pp 136-145, (2013).