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Chemical Reaction Effects on a Three Dimensional MHD Mass Transfer Flow past a Vertical Plate

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Abstract: *The objective of the present paper is to study the effects of chemical reaction on a three dimensional MHD mass transfer flow past a vertical plate in presence of heat source. The governing non dimensional equations relevant to the problem are solved by regular perturbation technique. Under certain assumptions, the solutions for velocity field, temperature distribution and species concentration are obtained. The expressions for skin-friction, Nusselt number and Sherwood number are performed. The influences of various parameters on the velocity field, temperature field, concentration field, skin-friction, Nusselt number and Sherwood number are studied graphically. The obtained results have shown that the chemical reaction effect has a great influence in the study of flow and heat transfer process in the presence of magnetic field of some types of fluids considered.*
Keywords: *MHD, chemical reaction, mass transfer, perturbation.*

I. Introduction:

Combined heat and mass transfer in fluid finds applications in a variety of engineering processes such as heat exchanger devices, petroleum reservoirs, chemical catalytic reactors and processes, geothermal and geophysical engineering. Double diffusive flow is driven by buoyancy due to temperature and concentration gradients.

The investigation of MHD convection problems have attracted the attention of a number of scholars because of its wide application in meteorology, solar physics, cosmic fluid dynamics, astrophysics, geophysics and in the motion of earth's cone. From technological point of view, MHD convection flow problems are also very significant in the fields of stellar and planetary magnetospheres, aeronautics, chemical engineering and electronics. The application of MHD principles in medicines and biology are of paramount interest owing to their significance in bio-medical engineering in general and in the treatment of various pathological states in particular. The bio-medical engineering includes cardiac MRI, ECG etc. The problems of above phenomena of MHD convection have been studied by many authors. Raptis and Soundalgekar [13], Ferraro and Plumpton [6], Cramer and Pai [5], Sanyal and Bhattacharya [14], and Soundalgekar et al. [18] are some of them. Ahmed and Sarma [1], singh et al. [17], and Choudhary and Chand [4] have investigated the effect of three dimensional flow caused by the periodic motion perpendicular to the main flow when the difference between the wall temperature and free stream temperature gives rise to buoyancy force in the direction of the free stream on heat transfer characteristics.

In many transport processes in nature and in industrial applications, the heat and mass transfer with variable viscosity is a consequence of buoyancy effects caused by the diffusion of heat and chemical species. The study of such processes is useful for improving a number of chemical technologies such as polymer production and food processing. In nature the presence of pure air or water is impossible, because some foreign mass may be presented either naturally or mixed with air or water. The study of heat and mass transfer with chemical reaction is of considerable importance in the chemical and hydrometallurgical industries. Chemical reaction can be codified as either homogeneous or heterogeneous processes. A homogeneous reaction is one that occurs uniformly through a given phase. In contrast, a heterogeneous reaction takes place in a restricted region or within the boundary of a phase. A reaction is said to be first order, if the rate of reaction is directly proportional to the concentration itself. Chemical reactions usually accompany a large amount of exothermic and endothermic reactions. These characteristics can be easily seen in a lot of industrial processes such as the polymer production, the manufacturing of ceramics or glassware, the food processing and so on. It has been realized that it is not always permissible to neglect the convection effects in porous constructed chemical reactors. Das et al. [19] considered the effects of first order chemical reaction on the flow past an impulsively started infinite vertical plate with constant heat flux and mass transfer. Muthucumarswamy and Ganesan[9] and Muthucumarswamy [10] studied first order homogeneous chemical reaction on the flow past infinite vertical plate. In view of the importance of the chemical reaction effects, several authors have carried out their research works to investigate the effects of it on some mass transfer flow problems. Some of them are – Poornima and Reddy [11], Sharma et al. [15], Shivaiah and Rao[16], Ibrahim et al.[7], Rajeshwari et al. [12], Kandasamy et al.[8] and Chamkha [3].

The object of the present work is to investigate the chemical reaction effects on a three dimensional MHD mass transfer flow past a vertical plate. This work is an extension of the problem studied by Ahmed and Goswami [2].

II. Mathematical analysis:

A steady, three dimensional free and forced convection flow of an incompressible and electrically conducting viscous fluid past a vertical porous plate in presence of chemical reaction is considered by making the following assumptions.

(i) All the fluid properties except the density in the buoyancy force term are constant.

(ii) A magnetic field of uniform strength B_0 is applied transversely to the direction of the main flow.

(iii) The magnetic Reynolds number is so small that the induced magnetic field can be neglected.

(iv) The viscous dissipation and magnetic dissipations of energy are negligible.

(v) $\bar{T}_\omega > \bar{T}_\infty$ and $\bar{C}_\omega > \bar{C}_\infty$

Let us consider a co-ordinate system $(\bar{x},\bar{y},\bar{z})$ with $\bar{X}$ - axis vertically upwards along the plate, $\bar{Y}$ - axis perpendicular to it directed into the fluid region and $\bar{Z}$ - axis along the width of the plate. Let $\vec{q} = \hat{i}\bar{u} + \hat{j}\bar{v} + \hat{k}\bar{w}$ be the fluid velocity at the point $(\bar{x},\bar{y},\bar{z})$ and $\vec{B} = B_0\hat{j}$ be the applied magnetic field. The suction velocity distribution is taken as follows:

$$\bar{v}_w(\bar{z}) = -V_0[1+\varepsilon\cos\frac{\pi\bar{z}}{L}]$$

which consists of a basic steady distribution - V_0 with superimposed weak distribution $-V_0\varepsilon\cos\frac{\pi\bar{z}}{L}$. Since the plate is infinite in length in $\bar{X}$ - direction, therefore all the quantities except possibly the pressure are assumed to be independent of $\bar{x}$.

With these assumptions and under usual boundary layer approximations, the governing equations of the problem are:

Equation of continuity:

$$\frac{\partial\bar{v}}{\partial\bar{y}} + \frac{\partial\bar{w}}{\partial\bar{z}} = 0 \tag{1}$$

Momentum equations:

$$\bar{v}\frac{\partial\bar{u}}{\partial\bar{y}} + \bar{w}\frac{\partial\bar{u}}{\partial\bar{z}} = g\beta\left(\bar{T}-\bar{T}_\infty\right) + g\bar{\beta}\left(\bar{C}-\bar{C}_\infty\right) + \upsilon\left(\frac{\partial^2\bar{u}}{\partial\bar{y}^2}+\frac{\partial^2\bar{u}}{\partial\bar{z}^2}\right) + \frac{\sigma B0^2}{\rho}\left(\bar{U}-\bar{u}\right) \tag{2}$$

$$\bar{v}\frac{\partial\bar{v}}{\partial\bar{y}} + \bar{w}\frac{\partial\bar{v}}{\partial\bar{z}} = -\frac{1}{\rho}\frac{\partial\bar{p}}{\partial\bar{y}} + \upsilon\left(\frac{\partial^2\bar{v}}{\partial\bar{y}^2}+\frac{\partial^2\bar{v}}{\partial\bar{z}^2}\right) \tag{3}$$

$$\bar{v}\frac{\partial\bar{w}}{\partial\bar{y}} + \bar{w}\frac{\partial\bar{w}}{\partial\bar{z}} = -\frac{1}{\rho}\frac{\partial\bar{p}}{\partial\bar{z}} + \upsilon\left(\frac{\partial^2\bar{w}}{\partial\bar{y}^2}+\frac{\partial^2\bar{w}}{\partial\bar{z}^2}\right) - \frac{\sigma B_0^2\bar{w}}{\rho} \tag{4}$$

Energy equation:

$$\bar{v}\frac{\partial\bar{T}}{\partial\bar{y}} + \bar{w}\frac{\partial\bar{T}}{\partial\bar{z}} = \frac{k}{\rho C_p}\left(\frac{\partial^2\bar{T}}{\partial\bar{y}^2}+\frac{\partial^2\bar{T}}{\partial\bar{z}^2}\right) + \frac{\bar{Q}}{\rho Cp}\left(\bar{T}_\infty-\bar{T}\right) \tag{5}$$

Species concentration equation:

$$\bar{v}\frac{\partial\bar{C}}{\partial\bar{y}} + \bar{w}\frac{\partial\bar{C}}{\partial\bar{z}} = D_m\left(\frac{\partial^2\bar{C}}{\partial\bar{y}^2}+\frac{\partial^2\bar{C}}{\partial\bar{z}^2}\right) + D_T\left(\frac{\partial^2\bar{T}}{\partial\bar{y}^2}+\frac{\partial^2\bar{T}}{\partial\bar{z}^2}\right) - \bar{K}\left(\bar{C}-\bar{C}_\infty\right) \tag{6}$$

The relevant boundary conditions are:

At $\bar{y}=0$: $\bar{u}=0, \bar{v}=\bar{v}_w, \bar{w}=0, \bar{T}=\bar{T}_w, \bar{C}=\bar{C}_w$ (7)

At $\bar{y}\to\infty$: $\bar{u}=\bar{U}, \bar{v}=-V_0, \bar{w}=0, \bar{T}=\bar{T}_\infty, \bar{C}=\bar{C}_\infty, \bar{p}=\bar{p}_\infty$ (8)

To normalize the flow model, we introduce the following non-dimensional quantities:

$$y=\frac{\bar{y}}{L},\ z=\frac{\bar{z}}{L},\ u=\frac{\bar{u}}{V_0},\ v=\frac{\bar{v}}{V_0},\ w=\frac{\bar{w}}{V_0},\ U=\frac{\bar{U}}{V_0},\ \theta=\frac{\bar{T}-\bar{T}_\infty}{\bar{T}_w-\bar{T}_\infty},\ \phi=\frac{\bar{C}-\bar{C}_\infty}{\bar{C}_w-\bar{C}_\infty},\ \ Sr=\frac{D_T(\bar{T}_w-\bar{T}_\infty)}{\upsilon(\bar{C}_w-\bar{C}_\infty)}$$

$$Gr=\frac{Lg\beta(\bar{T}_w-\bar{T}_\infty)}{V_0^2},\ Gm=\frac{Lg\bar{\beta}(\bar{C}_w-\bar{C}_\infty)}{V_0^2},\ \Pr=\frac{\mu C_p}{k},\ Sc=\frac{\upsilon}{D_m},\ M=\frac{\sigma B_0^2\upsilon}{\rho V_0^2},\ \mathrm{Re}=\frac{V_0L}{\upsilon}$$

$$p=\frac{\bar{p}}{\rho\left(\frac{\upsilon}{L}\right)^2},\ p_\infty=\frac{\bar{p}_\infty}{\rho\left(\frac{\upsilon}{L}\right)^2},\ K=\frac{\bar{K}\upsilon}{V_0^2},\ Q=\frac{\bar{Q}L\upsilon}{V_0k}$$

The non-dimensional forms of the equations (1) to (6) are:

$$\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{9}$$

$$v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=Gr\theta+Gm\phi+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2u}{\partial y^2}+\frac{\partial^2u}{\partial z^2}\right)+M\,\mathrm{Re}\left(U-u\right) \tag{10}$$

$$v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=-\frac{1}{\mathrm{Re}^2}\frac{\partial p}{\partial y}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2v}{\partial y^2}+\frac{\partial^2v}{\partial z^2}\right) \tag{11}$$

$$v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{1}{\mathrm{Re}^2}\frac{\partial p}{\partial z}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2w}{\partial y^2}+\frac{\partial^2w}{\partial z^2}\right)-M\,\mathrm{Re}\,w \tag{12}$$

$$v\frac{\partial \theta}{\partial y}+w\frac{\partial \theta}{\partial z}=\frac{1}{\mathrm{Re}\Pr}\left(\frac{\partial^2\theta}{\partial y^2}+\frac{\partial^2\theta}{\partial z^2}\right)-\frac{Q}{\Pr}\theta \tag{13}$$

$$v\frac{\partial \phi}{\partial y}+w\frac{\partial \phi}{\partial z}=\frac{1}{Sc\,\mathrm{Re}}\left(\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}\right)+\frac{Sr}{\mathrm{Re}}\left(\frac{\partial^2\theta}{\partial y^2}+\frac{\partial^2\theta}{\partial z^2}\right)-K\,\mathrm{Re}\,\phi \tag{14}$$

With relevant boundary conditions:

$$y=0:\quad u=0,\ v=-\left(1+\varepsilon\cos\pi z\right),\ w=0,\ \theta=1,\ \phi=1 \tag{15}$$

$$y\to\infty:\quad u=U,\ v=-1,\ w=0,\ \theta=0,\ \phi=0,\ p=p_\infty \tag{16}$$

Method of Solution:

Let us suppose the solutions of the equations from (9) to (14) be of the form:

$$u=u_0\left(y\right)+\varepsilon u_1\left(y,z\right)+O\left(\varepsilon^2\right)$$

$$v=v_0\left(y\right)+\varepsilon v_1\left(y,z\right)+O\left(\varepsilon^2\right)$$

$$w=w_0\left(y\right)+\varepsilon w_1\left(y,z\right)+O\left(\varepsilon^2\right)$$

$$p=p_0\left(y\right)+\varepsilon p_1\left(y,z\right)+O\left(\varepsilon^2\right)$$

$$\theta=\theta_0\left(y\right)+\varepsilon\theta_1\left(y,z\right)+O\left(\varepsilon^2\right)$$

$$\phi=\phi_0\left(y\right)+\varepsilon\phi_1\left(y,z\right)+O\left(\varepsilon^2\right)$$

with $p_0=p_\infty$, $w_0=0$

Substituting these in equations (9) to (14) and by equating the coefficients of the similar terms and neglecting ε^2, we get the following zeroth-order and first-order equations.

Zeroth-order equations:

$$\frac{dv_0}{dy}=0 \tag{17}$$

$$v_0 \frac{du_0}{dy} = Gr\theta_0 + Gm\phi_0 + \frac{1}{\mathrm{Re}}\frac{d^2u_0}{dy^2} + M\,\mathrm{Re}\left(U - u_0\right) \tag{18}$$

$$v_0 \frac{d\theta_0}{dy} = \frac{1}{\Pr \mathrm{Re}}\frac{d^2\theta_0}{dy^2} - \frac{Q}{\Pr}\theta_0 \tag{19}$$

$$v_0 \frac{d\phi_0}{dy} = \frac{1}{Sc\,\mathrm{Re}}\frac{d^2\phi_0}{dy^2} + \frac{Sr}{\mathrm{Re}}\frac{d^2\theta_0}{dy^2} - K\,\mathrm{Re}\,\phi_0 \tag{20}$$

First-order equations:

$$\frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0 \tag{21}$$

$$v_0 \frac{\partial u_1}{\partial y} + v_1 \frac{du_0}{dy} = Gr\theta_1 + Gm\phi_1 + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2}\right) - M\,\mathrm{Re}\,u_1 \tag{22}$$

$$v_0 \frac{\partial v_1}{\partial y} = -\frac{1}{\mathrm{Re}^2}\frac{\partial p_1}{\partial y} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v_1}{\partial y^2} + \frac{\partial^2 v_1}{\partial z^2}\right) \tag{23}$$

$$v_0 \frac{\partial w_1}{\partial y} = -\frac{1}{\mathrm{Re}^2}\frac{\partial p_1}{\partial z} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 w_1}{\partial y^2} + \frac{\partial^2 w_1}{\partial z^2}\right) - M\,\mathrm{Re}\,w_1 \tag{24}$$

$$v_0 \frac{\partial \theta_1}{\partial y} + v_1 \frac{d\theta_0}{dy} = \frac{1}{\Pr \mathrm{Re}}\left(\frac{\partial^2 \theta_1}{\partial y^2} + \frac{\partial^2 \theta_1}{\partial z^2}\right) - \frac{Q}{\Pr}\theta_1 \tag{25}$$

$$v_0 \frac{\partial \phi_1}{\partial y} + v_1 \frac{d\phi_0}{dy} = \frac{1}{Sc\,\mathrm{Re}}\left(\frac{\partial^2 \phi_1}{\partial y^2} + \frac{\partial^2 \phi_1}{\partial z^2}\right) + \frac{Sr}{\mathrm{Re}}\left(\frac{\partial^2 \theta_1}{\partial y^2} + \frac{\partial^2 \theta_1}{\partial z^2}\right) - K\,\mathrm{Re}\,\phi_1 \tag{26}$$

with the boundary conditions:

$$\left.\begin{array}{l} y=o: u_0=0, u_1=0, v_0=-1, v_1=-\cos\pi z, w_0=0, w_1=0, \theta_0=1, \theta_1=0, \phi_0=1, \phi_1=0 \\ y\to\infty: u_0=U, u_1=0, v_0=-1, v_1=0, w_0=0, w_1=0, \theta_0=0, \theta_1=0, \phi_0=0, \phi_1=0, p_1=0 \end{array}\right\} \tag{27}$$

The solutions of the equations (17) to (20) under the boundary condition (27) are:

$$v_0 = -1$$

$$\theta_0 = e^{-A_1 y}$$

$$\phi_0 = \left(1 - A_3\right)e^{-A_2 y} + A_3 e^{-A_1 y}$$

$$u_0 = A_5 e^{-A_1 y} + A_6 e^{-A_2 y} + A_7 e^{-A_4 y} + U$$

Cross flow solution: We shall first consider the equations (21), (23) and (24) for $v_1(y, z)$, $w_1(y, z)$ and $p_1(y, z)$ which are independent of the main flow component u_1, temperature field θ_1 and concentration field ϕ_1.

We assume v_1, w_1 and p_1 as:

$$v_1 = -\pi v_{11}(y)\cos\pi z$$

$$w_1 = v_{11}'(y)\sin\pi z$$

$$p_1 = \mathrm{Re}^2\, p_{11}(y)\cos\pi z$$

On substitution of the above, the equation (21) is satisfied and the equations (23) and (24) reduce to the ordinary differential equations as:

$$v_{11}'' + \mathrm{Re}\,v_{11}' - \pi^2 v_{11} = -\frac{\mathrm{Re}}{\pi}p_{11}' \tag{28}$$

$$v_{11}''' + \mathrm{Re}\,v_{11}'' - (\pi^2 + M\,\mathrm{Re}^2)v_{11}' = -\pi\,\mathrm{Re}\,p_{11} \tag{29}$$

The corresponding boundary conditions are:

$$y = 0 : v_{11} = \frac{1}{\pi}, v'_{11} = 0$$
$$y \to \infty : v_{11} = 0, v'_{11} = 0$$

Under these boundary conditions, the solutions of the equations (28) and (29) are

$$v_{11} = \frac{1}{\pi\left(A_{10} - A_9\right)}\left[A_{10}\, e^{-A_9 y} - A_9\, e^{-A_{10} y}\right]$$
$$p_{11} = \frac{A_9 A_{10}}{\mathrm{Re}\,\pi^2\left(A_{10} - A_9\right)}\left[A_{11} e^{-A_9 y} - A_{12} e^{-A_{10} y}\right]$$

Hence the solutions for the velocity component v_1, w_1 and pressure p_1 are as follows.

$$v_1 = \frac{1}{\left(A_9 - A_{10}\right)}\left[A_{10}\, e^{-A_9 y} - A_9\, e^{-A_{10} y}\right]\cos \pi z$$
$$w_1 = \frac{A_9 A_{10}}{\pi\left(A_9 - A_{10}\right)}\left[e^{-A_9 y} - e^{-A_{10} y}\right]\sin \pi z$$
$$p_1 = \frac{\mathrm{Re}\, A_9 A_{10}}{\pi^2\left(A_{10} - A_9\right)}\left[A_{11} e^{-A_9 y} - A_{12} e^{-A_{10} y}\right]\cos \pi z$$

Solution for first order flow, concentration and temperature field: we now consider the equations (22), (25) and (26).To reduce these partial differential equations into ordinary differential equations, we consider the following assumptions for u_1, θ_1 and ϕ_1.

$$u_1 = u_{11}\left(y\right)\cos \pi z\,,$$
$$\theta_1 = \theta_{11}\left(y\right)\cos \pi z\,, \quad \phi_1 = \phi_{11}\left(y\right)\cos \pi z$$

Substituting these expressions in equations (22),(25) and (26), we get the following ordinary differential equations.

$$u''_{11} + \mathrm{Re}\, u'_{11} - \left(\pi^2 + M\,\mathrm{Re}^2\right)u_{11} = -\pi\,\mathrm{Re}\, v_{11} u'_0 - Gr\,\mathrm{Re}\,\theta_{11} - Gm\,\mathrm{Re}\,\phi_{11} \tag{30}$$
$$\theta''_{11} + \mathrm{Pr}\,\mathrm{Re}\,\theta'_{11} - \left(\pi^2 + Q\,\mathrm{Re}\right)\theta_{11} = -\pi\,\mathrm{Pr}\,\mathrm{Re}\, v_{11}\theta'_0 \tag{31}$$
$$\phi''_{11} + Sc\,\mathrm{Re}\,\phi'_{11} - \left(\pi^2 + KSc\,\mathrm{Re}^2\right)\phi_{11} = -\pi Sc\,\mathrm{Re}\, v_{11}\phi'_0 - SrSc\left(\theta''_{11} - \pi^2\theta_{11}\right) \tag{32}$$

With boundary conditions:

$$y = 0:\; u_{11} = 0, \theta_{11} = 0, \phi_{11} = 0$$
$$y \to 0:\; u_{11} = 0, \theta_{11} = 0, \phi_{11} = 0$$

The solution of the equations (30), (31) and (32) subject to the above boundary conditions are as follows:

$$\theta_{11} = A_{16} e^{-A_{13} y} + A_{14} e^{-A_{22} y} + A_{15} e^{-A_{24} y}$$
$$\phi_{11} = A_{32} e^{-A_{26} y} + A_{27} e^{-A_{18} y} + A_{28} e^{-A_{20} y} + A_{29} e^{-A_{22} y} + A_{30} e^{-A_{24} y} + A_{31} e^{-A_{13} y}$$
$$u_{11} = L_0 e^{-A_{56} y} + L_1 e^{-A_{22} y} + L_2 e^{-A_{18} y} + L_3 e^{-A_{36} y} + L_4 e^{-A_{24} y} + L_5 e^{-A_{20} y} + L_6 e^{-A_{40} y} + L_7 e^{-A_{13} y} + L_8 e^{-A_{26} y}$$

Hence the solutions for the velocity component u_1, temperature θ_1 and the concentration ϕ_1 are as follows:

$$u_1 = [L_0 e^{-A_{56} y} + L_1 e^{-A_{22} y} + L_2 e^{-A_{18} y} + L_3 e^{-A_{36} y} + L_4 e^{-A_{24} y} + L_5 e^{-A_{20} y} + L_6 e^{-A_{40} y} + L_7 e^{-A_{13} y} + L_8 e^{-A_{26} y}]\cos \pi z$$
$$\theta_1 = [A_{16} e^{-A_{13} y} + A_{14} e^{-A_{22} y} + A_{15} e^{-A_{24} y}]\cos \pi z$$
$$\phi_1 = [A_{32} e^{-A_{26} y} + A_{27} e^{-A_{18} y} + A_{28} e^{-A_{20} y} + A_{29} e^{-A_{22} y} + A_{30} e^{-A_{24} y} + A_{31} e^{-A_{13} y}]\cos \pi z$$

Thus the complete solutions of the equations (9) to (14) are:

$$u = A_5 e^{-A_1 y} + A_6 e^{-A_2 y} + A_7 e^{-A_4 y} + U + \varepsilon[L_0 e^{-A_{56} y} + L_1 e^{-A_{22} y} + L_2 e^{-A_{18} y} + L_3 e^{-A_{36} y}$$
$$+ L_4 e^{-A_{24} y} + L_5 e^{-A_{20} y} + L_6 e^{-A_{40} y} + L_7 e^{-A_{13} y} + L_8 e^{-A_{26} y}]\cos \pi z$$

$$v = -1 + \varepsilon \frac{1}{(A_9 - A_{10})}\left[A_{10} e^{-A_9 y} - A_9 e^{-A_{10} y}\right]\cos \pi z$$

$$w = \varepsilon \frac{A_9 A_{10}}{\pi (A_9 - A_{10})}\left[e^{-A_9 y} - e^{-A_{10} y}\right]\sin \pi z$$

$$p = p_\infty + \varepsilon \frac{\operatorname{Re} A_9 A_{10}}{\pi^2 (A_{10} - A_9)}\left[A_{11} e^{-A_9 y} - A_{12} e^{-A_{10} y}\right]\cos \pi z$$

$$\theta = e^{-A_1 y} + \varepsilon[A_{16} e^{-A_{13} y} + A_{14} e^{-A_{22} y} + A_{15} e^{-A_{24} y}]\cos \pi z$$

$$\phi = (1 - A_3) e^{-A_2 y} + A_3 e^{-A_1 y} + \varepsilon[A_{32} e^{-A_{26} y} + A_{27} e^{-A_{18} y} + A_{28} e^{-A_{20} y} + A_{29} e^{-A_{22} y} + A_{30} e^{-A_{24} y} + A_{31} e^{-A_{13} y}]\cos \pi z$$

Skin-friction at the plate: The non-dimensional skin-friction at the plate in the direction of the free stream is given by

$$\tau = \frac{\mu \left[\frac{\partial \bar{u}}{\partial \bar{y}}\right]_{\bar{y}=0}}{\rho v_0^2} = -\frac{1}{\operatorname{Re}}\left(\frac{\partial u}{\partial y}\right)_{y=0}$$
$$= -\frac{1}{\operatorname{Re}}\left[u_0'(0) + \varepsilon u_{11}'(0)\cos \pi z\right]$$
$$= \tau_0 + \varepsilon Q_1 \cos \pi z$$

where $\tau_0 = -\frac{1}{\operatorname{Re}} u_0'(0)$

$$= \frac{1}{\operatorname{Re}}\left[A_1 A_5 + A_2 A_6 + A_4 A_7\right]$$

$$Q_1 = -\frac{1}{\operatorname{Re}} u_{11}'(0)$$
$$= \frac{1}{\operatorname{Re}}\left[L_0 A_{56} + L_1 A_{22} + L_2 A_{18} + L_3 A_{36} + L_4 A_{24} + L_5 A_{20} + L_6 A_{40} + L_7 A_{13} + L_8 A_{26}\right]$$

The coefficient of rate of heat transfer: The heat flux from the plate to the fluid in terms of Nusselt number Nu is given by

$$Nu = -\frac{k}{\rho v_0 C_p \left(\bar{T}_\omega - \bar{T}_\infty\right)}\left(\frac{\partial \bar{T}}{\partial \bar{y}}\right)_{\bar{y}=0} = -\frac{1}{\Pr \operatorname{Re}}\left(\frac{\partial \theta}{\partial y}\right)_{y=0}$$
$$= Nu_0 + \varepsilon Q_2 \cos \pi z$$

where $Nu_0 = -\frac{1}{\Pr \operatorname{Re}} \theta_0'(0)$

$$= \frac{A_1}{\Pr \operatorname{Re}}$$

$$Q_2 = -\frac{1}{\Pr \operatorname{Re}} \theta_{11}'(0)$$
$$= \frac{1}{\Pr \operatorname{Re}}\left[A_{16} A_{13} + A_{14} A_{22} + A_{15} A_{24}\right]$$

The coefficient of rate of mass transfer: the mass transfer at the wall $\bar{y}=0$ in terms of Sherwood number Sh is given by

$$Sh = \frac{1}{Sc\,\mathrm{Re}}\left(\frac{\partial \phi}{\partial y}\right)_{y=0}$$

$$= \frac{1}{Sc\,\mathrm{Re}}\left[\phi_0'(0) + \varepsilon \phi_{11}'(0)\cos \pi z\right]$$

$$= Sh_0 + \varepsilon Q_3 \cos \pi z$$

where $Sh_0 = \frac{1}{Sc\,\mathrm{Re}}\phi_0'(0) = \frac{1}{Sc\,\mathrm{Re}}\left[A_2\left(A_3 - 1\right) - A_1 A_3\right]$

$$Q_3 = \frac{1}{Sc\,\mathrm{Re}}\phi_{11}'(0)$$

$$= \frac{1}{Sc\,\mathrm{Re}}\left[-A_{26}A_{32} - A_{18}A_{27} - A_{20}A_{28} - A_{22}A_{29} - A_{24}A_{30} - A_{13}A_{31}\right]$$

III. Results And Discussion:

In order to get a physical insight of the problem, the numerical calculations are carried to illustrate the influence of various physical parameters viz, chemical reaction parameter K, heat source parameter Q, Soret number Sr, and magnetic parameter on the velocity, concentration, skin-friction and Sherwood number. The effect of heat source parameter Q on temperature profile is also presented. Throughout the calculations, Pr (Prandtl number) is considered to be equal 0.71 which corresponds to air. Since the water vapour is used as a diffusing chemical species of common interest in air therefore the value of Sc (Schmidt number) is taken to be 0.60 (water vapour). The value of the Grashof number Gr for heat transfer has chosen as 10 (externally cooled plate), The value of Grashof number Gm for mass transfer is considered to be 15, the free stream velocity U is selected to be 1, the value of ε (frequency of oscillation) is considered as 0.001 and the remaining parameters are chosen arbitrarily. The variation of velocity profiles u against y for different values of chemical reaction parameter K, heat source parameter Q, Soret number Sr, and Hartmann number M are depicted in figures 1-4 respectively. It is seen from these figures that the velocity quickly increases up to some thin layer of the liquid adjacent to the plate and after this liquid layer the fluid velocity decreases asymptotically towards 1 as $y \to \infty$.

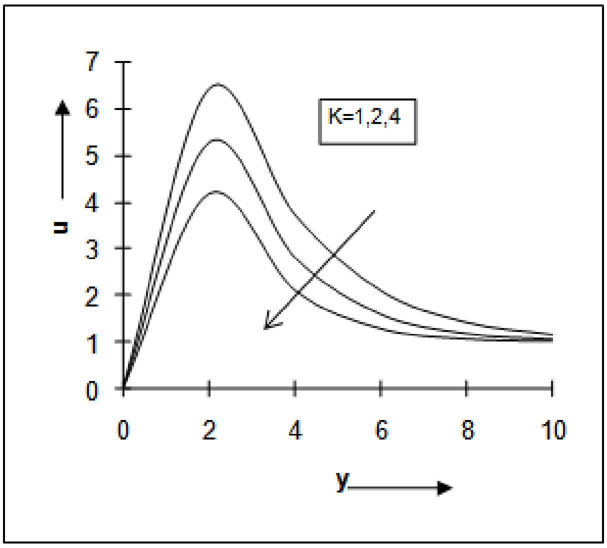

Fig.1: Velocity versus y for different K when Gr=10, M=2, U=1,Gm=15,Re=0.5,Pr=0.71,Q=1,Sc=0.6,Sr=0.5, ε=0.001

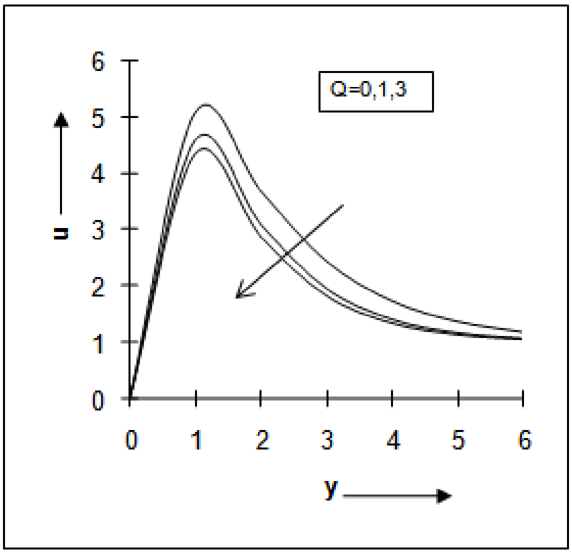

Fig.2: Velocity versus y for different Q when Gr=10, M=2, U=1,Gm=15,Re=1,Pr=0.71,K=0.5,Sc=0.6,Sr=1, ε=0.001

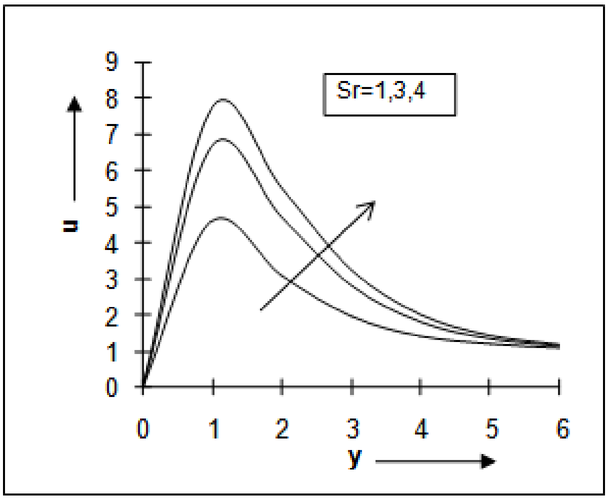

Fig.3: Velocity versus y for different Sr when Gr=10,M=2, Q=1,Gm=15,Re=1, Pr=0.71,Q=1,Sc=0.6, ε=0.001,K=0.5,U=1

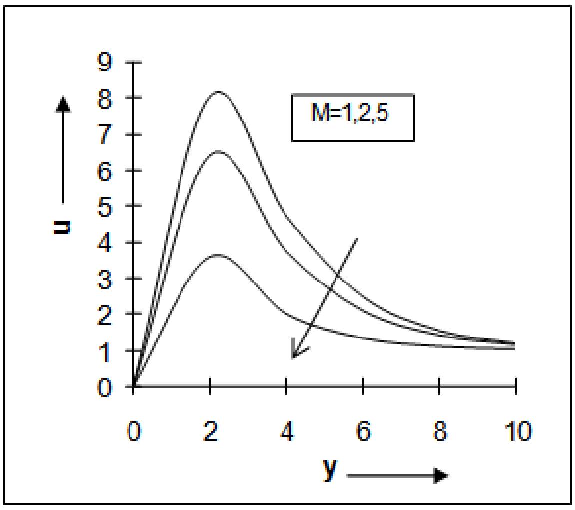

Fig.4: Velocity versus y for different M when Gr=10, Gm=15,Re=0.5,Pr=0.71, Sc=0.6, ε=0.001,K=1,Sr=0.5,U=1

It is observed from figures 1,2 and 4 that chemical reaction parameter K, heat source parameter Q and Hartmann number M lead the fluid motion to retard whereas the motion is accelerated due to the Soret number Sr as seen from figure 3. The Soret number Sr defines the effect of temperature gradients inducing significant mass diffusion effects. Hence mass diffusion leads to increase the fluid motion. The application of the transverse magnetic field will result in a resistive type force (Lorentz force) similar to drag force which leads to resist the fluid flows and thus reduces its velocity.

Figure 5 shows the effect of the heat source parameter Q on the temperature θ. It is observed from this figure that with an increase in the heat source parameter, the temperature decreases within the boundary layer.

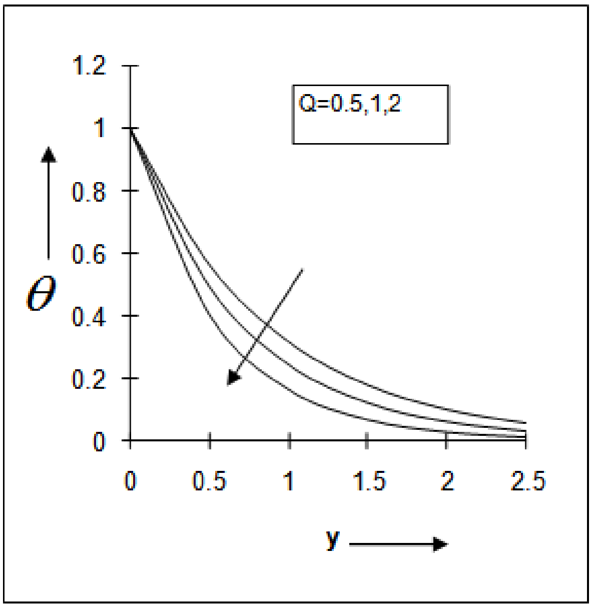

Fig.5: Temperature distribution against y for Gr=10,U=1 Gm=15,Re=1,M=2,Pr=0.71,Sc=0.6,Sr=1, ε=0.001,K=1

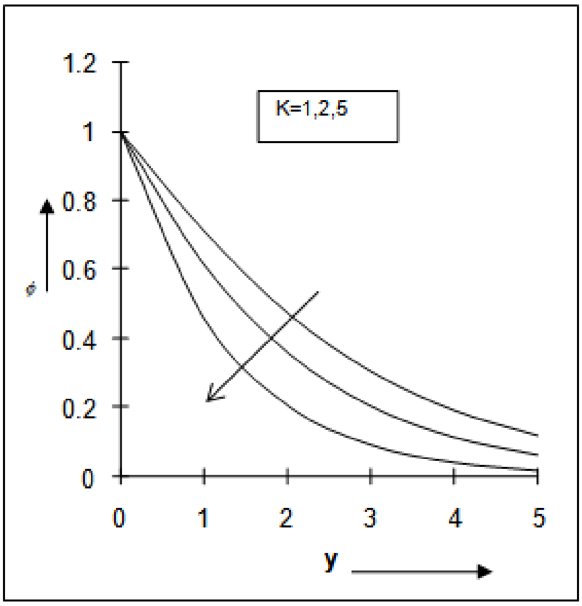

Fig.6: Species concentration versus y for different K whenGr=10 Gm=15,Re=0.5,M=2,Pr=0.71,Q=1,Sc=0.6,Sr=0.5, ε=0.001,U=1

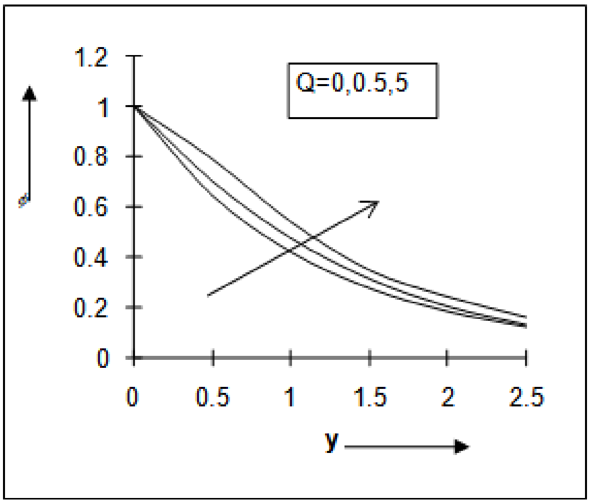

Fig.7: Species concentration versus y for different Q Gr=10,Gm=15,Re=1,M=2,Pr=0.71,K=1,Sc=0.6,Sr=1, ε=0.001

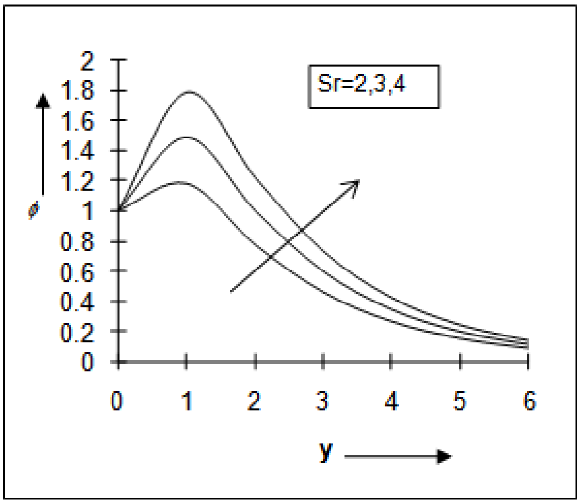

Fig.8: Species concentration versus y for different Sr when U=1,Gr=10,Gm=15,Re=0.5,M=1,Pr=0.71,K=1,Sc=0.6,Q=5, ε=0.001

The concentration profiles for different values of the chemical reaction parameter, heat source parameter and Soret number are presented in figures 6-8. Figure 6 exhibits that, as the chemical reaction parameter K increases, the concentration decreases. Figures 7 and 8 show that the concentration increases with the increase of heat source parameter Q and Soret number Sr.

Variation of Nusselt number Nu (rate of heat transfer) against Reynolds number Re for different values of the heat source parameter Q is presented in figure 9. It is inferred from this figure that Nu increases with the increasing values of Q whereas it decreases with increasing Re.

Figures 10, 11 and 12 demonstrate how the skin-friction τ is effected by the Reynolds number Re under the influence of chemical reaction parameter K, heat source parameter Q and Soret number Sr. It is noticed from these figures that an increase in the value of Re causes the magnitude of τ to decrease. It is clear from figures 10 and 11 that there is steady fall in the magnitude of τ due to chemical reaction parameter and the generating heat source whereas $|\tau|$ increases with Soret number.

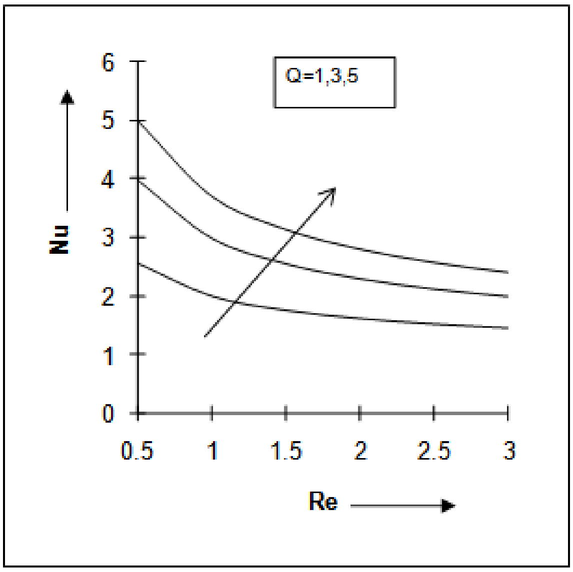

Fig.9: Nusselt number against Re for different Q whenU=1 Gr=10,Gm=15,M=2,Pr=0.71,Sc=0.6,Sr=0.5, ε=0.001

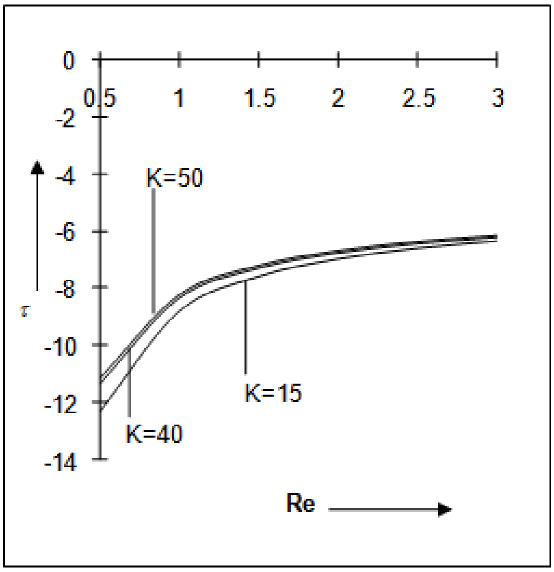

Fig.10: Skin friction against Re for different K when U=1 Gr=10,Gm=15,M=20,Pr=0.71,Q=3,Sc=0.6,Sr=0.5, ε =0.001

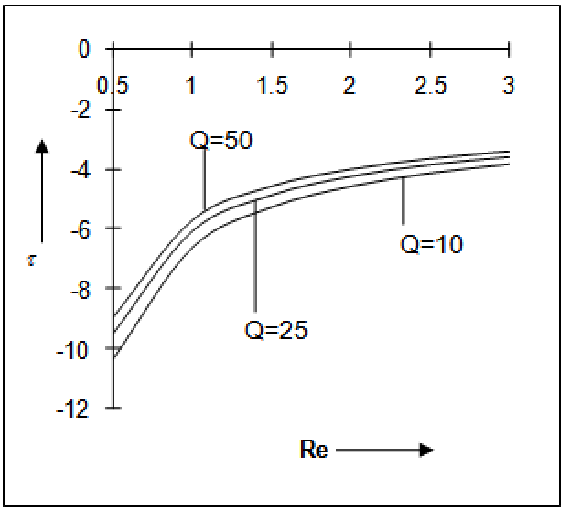

Fig.11: Skin friction against Re for different Q when U=1 Gr=10,Gm=15,M=2,Pr=0.71,Sc=0.6,Sr=0.5, ε =0.001,K=50

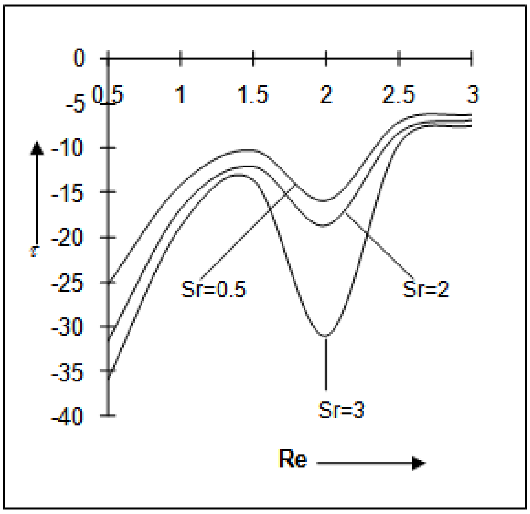

Fig.12: Skin friction against Re for different Sr when U=1 Gr=10,Gm=15,M=2,Pr=0.71,Sc=0.6,Q=1, ε =0.001,K=1

The variations of Sherwood number (rate of mass transfer) Sh against Reynolds number Re for different values of K,Q and Sr are displayed in figures 13, 14 and 15. It is observed from these figures that an increase in the value of Re causes Sh to decrease. Figures13 and 15 show that $|Sh|$ increases with the increasing values of K and Sr whereas figure 14 exhibits that $|Sh|$ decreases with the increasing values of Q.

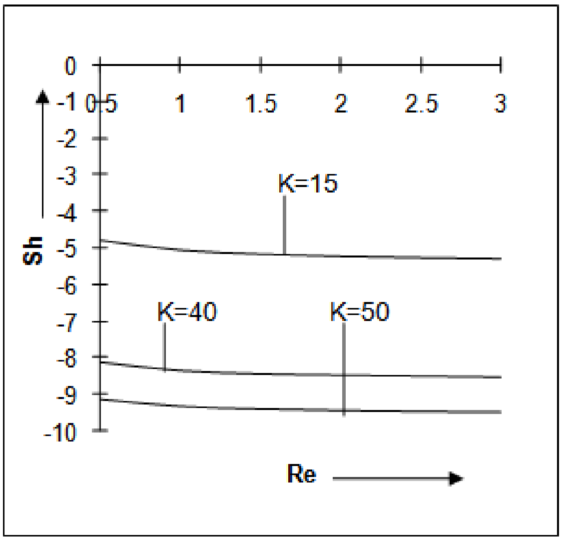

Fig.13: Sherwood number against Re for different K, U=1 Gr=10,Gm=15,M=20,Pr=0.71,Q=3,Sc=0.6,Sr=0.5 ε =0.001

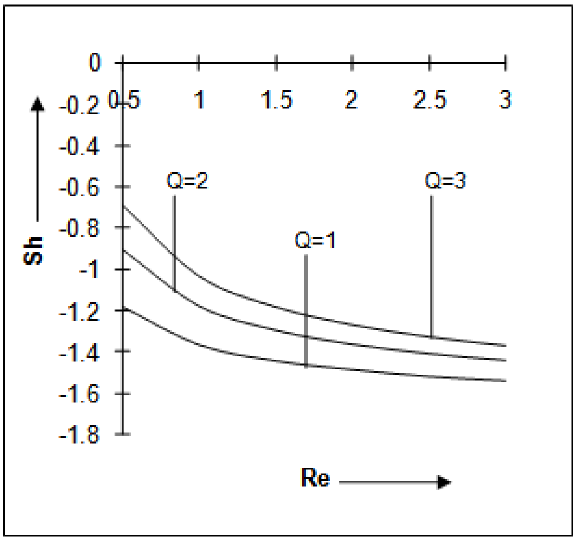

Fig.14: Sherwood number against Re for different Q, U=1 Gr=10,Gm=15,M=2,,Pr=0.71,Sc=0.6,Sr=0.5, ε =0.001,k=1

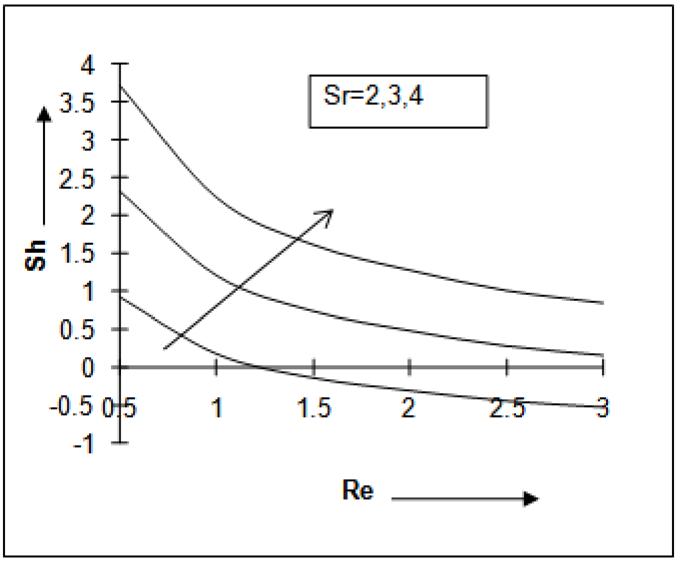

Fig.15: Sherwood number against Re for different Sr when Gr=10, Gm=15,M=2,Pr=0.71,Q=1,Sc=0.6, ε =0.001K=1,U=1

IV. Conclusion:

From the results and discussions of the present study, the following conclusions can be drawn:

- The velocity as well as concentration of the fluid decreases with an increase in the chemical reaction parameter.
- The heat source effect leads the fluid motion to retard whereas this motion is accelerated due to the thermal diffusion effect.
- An increase in value of magnetic parameter leads to fall in the velocity which is consistent with the law of physics.
- Magnitude of skin friction decreases due to the application of generating heat source as well as chemical reaction parameter.
- Magnitude of Sherwood number increases due to the chemical reaction parameter and it falls due to the generating heat source.
- It is interesting to note that the velocity increases near the plate and then decreases smoothly away from the plate in all the cases.

NOMENCLATURE:

$(\bar{x},\bar{y},\bar{z})$ is the coordinate system

$(\bar{u},\bar{v},\bar{w})$ are the components of the fluid velocity $\vec{q}$

(u,v,w) are the non dimensional components of the fluid velocity

V_0 is the mean suction velocity

$\bar{U}$ is the free stream velocity

U is the non dimensional free stream velocity

B_0 is the strength of applied magnetic force

$\bar{C}$ is the species concentration

$\bar{C}_\infty$ is the species concentration in the free stream

$\bar{C}_w$ is the species concentration at the plate

C_p is the specific heat at constant pressure

D_m is the co-efficient of chemical molecular diffusivity

D_T is the co-efficient of chemical thermal diffusivity

g is the acceleration due to gravity

Gr is the Grashof number for heat transfer

Gm is the Grashof number for mass transfer

k is the thermal conductivity
L is the wave length of the periodic suction
M is the Hartmann number
$\bar{K}$ is the chemical reaction parameter
K is the non dimensional chemical reaction parameter
$\bar{p}$ is the pressure
$\bar{p}_\infty$ is the pressure in the free stream
p is the non dimensional pressure
p_∞ is the non dimensional pressure in the free stream
$\bar{Q}$ is the first order heat source
Q is the non dimensional first order heat source
Re is the Reynolds number
Sr is the Soret number
Pr is the Prandtl number
Sc is the Schmidt number
$\bar{T}$ is the temperature in the boundary layer
$\bar{T}_w$ is the temperature at the plate
$\bar{T}_\infty$ is the fluid temperature in the free stream

Greek symbols:
β is the co-efficient of volume expansion for heat transfer
$\bar{\beta}$ is the co-efficient of volume expansion for mass transfer
υ is the kinematic viscosity
σ is the electrical conductivity
ρ is the density of the fluid
ε is a small reference parameter
θ is the non dimensional temperature
ϕ is the non dimensional concentration
μ is the coefficient of viscosity.

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