

## Strongly $g^\#$ -Continuous and Perfectly $g^\#$ -Continuous Maps in Ideal Topological Spaces

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**Abstract:** J. Antony Rex Rodrigo and P. Mariappan introduced the characterizations and properties of  $g^\#$ -closed sets in ideal topological space. In this paper, we introduced  $T_{I_{g^\#}}$ -space,  $T_{I_{g^\#}}^*$ -space, strongly  $I_{g^\#}$ -continuous maps, perfectly  $I_{g^\#}$ -continuous and  $I_{g^\#}$ -compactness.

**Keywords:**  $T_{I_{g^\#}}$ -space,  $T_{I_{g^\#}}^*$ -space,  $T_{I_{g^\#}}^*$ -space, Strongly  $I_{g^\#}$ -continuous maps, Perfectly  $I_{g^\#}$ -continuous,  $I_{g^\#}$ -compactness.

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### I. Introduction

Levine [8] introduced and investigated the concept of strong continuity in topological spaces. Sundaram [10] introduced strongly  $g$ -continuous maps and perfectly  $g$ -continuous maps in topological spaces. In [10], Sundaram introduced the concept of  $GO$ -compact space by using  $g$ -open covers. Antony Rex Rodrigo and Mariappan [3] introduced the characterizations and properties of  $g^\#$ -closed sets in ideal topological spaces. In this paper, we introduce the notion of  $T_{I_{g^\#}}$ -space,  $T_{I_{g^\#}}^*$ -space, strongly  $I_{g^\#}$ -continuous maps, perfectly  $I_{g^\#}$ -continuous and  $I_{g^\#}$ -compactness in ideal topological spaces and obtain some of its properties.

An ideal  $I$  on a topological space  $(X, \tau)$  is non-empty collection of subsets of  $X$  which satisfies (i)  $A \in I$  and  $B \subset A \Rightarrow B \in I$  and (ii)  $A \in I$  and  $B \in I \Rightarrow A \cup B \in I$ . Given a topological space  $(X, \tau)$  with an ideal  $I$  on  $X$  and if  $\wp(X)$  is the set of all subsets of  $X$ , a set operator  $(.)^* : \wp(X) \rightarrow \wp(X)$ , called a local function [4] of  $A$  with respect to  $\tau$  and  $I$  is defined as follows: for  $A \subseteq X$ ,  $A^*(I, \tau) = \{x \in X : U \cap A \notin I \text{ for every } U \in \tau(X)\}$  where  $\tau(X) = \{U \in \tau : x \in U\}$ . We will make use of the basic facts about the local functions [1, Theorem 2.3] without mentioning it explicitly. A Kuratowski closure operator  $cl^*(.)$  for a topology  $\tau^*(I, \tau)$ , called the  $*$ -topology, finer than  $\tau$  is defined by  $cl^*(A) = A \cup A^*(I, \tau)$  [12]. When there is no chance for confusion, we will simply write  $A^*$  for  $A^*(I, \tau)$  and  $\tau^*$  for  $\tau^*(I, \tau)$ . If  $I$  is an ideal on  $X$ , then  $(X, \tau, I)$  is called an ideal space.

### II. Preliminaries

**Definition 2.1:** A subset  $A$  of a topological space  $(X, \tau)$  is an  $\alpha$ -open set [8] if  $A \subseteq \text{int}(cl(\text{int}(A)))$ .

**Definition 2.2:** A subset  $A$  of a topological space  $(X, \tau)$  is called

- (i) Generalized closed (briefly  $g$ -closed) [11] if  $cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $X$ .
- (ii)  $\alpha$ -generalized closed (briefly  $\alpha g$ -closed) [2] if  $\alpha-cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $X$ .
- (iii)  $g^\#$ -closed [6] if  $cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $\alpha g$ -open in  $X$ .

**Definition 2.3:** A function  $f : (X, \tau) \rightarrow (Y, \sigma)$  is called

- (i) Strongly continuous [8] if  $f^{-1}(V)$  is both open and closed in  $X$  for each subset  $V$  in  $Y$ .
- (ii) Perfectly continuous [13] if  $f^{-1}(V)$  is both open and closed in  $X$  for each open set  $V$  in  $Y$ .

**Definition 2.4:** A topological space  $X$  is called  $T_{1/2}$ -space [7] if every  $g$ -closed set of  $X$  is closed in  $X$ .

**Definition 2.5:** A subset  $A$  of an ideal space  $(X, \tau, I)$  is said to be  $I_{g^\#}$ -closed [3] if  $A^* \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $\alpha g$ -open in  $X$ .

**Definition 2.6:** A function  $f : (X, \tau, I) \rightarrow (Y, \sigma)$  is called  $I_{g^\#}$ -continuous [3] if the inverse image of every closed set in  $Y$  is  $I_{g^\#}$ -closed in  $X$ .

**Definition 2.7:** A function  $f: (X, \tau, I) \rightarrow (Y, \sigma, J)$  is called  $I_{g^\#}$ -irresolute [3] if the inverse image of every  $I_{g^\#}$ -closed set in  $Y$  is  $I_{g^\#}$ -closed in  $X$ .

### III. Separation Axioms In Ideal Topological Space

**Definition 3.1:** An ideal topological space  $(X, \tau, I)$  is called a  $T_{I_{g^\#}}$ -space if every  $I_{g^\#}$ -closed set of  $X$  is closed in  $X$ .

**Definition 3.2:** An ideal Topological space  $(X, \tau, I)$  is called a  $T_{I_g}$ -space if every  $I_g$ -closed set of  $X$  is closed in  $X$ .

**Definition 3.3:** An ideal Topological space  $(X, \tau, I)$  is called a  $T_{I_{g^\#}}^*$ -space if every  $I_g$ -closed set of  $X$  is  $I_{g^\#}$ -closed in  $X$ .

**Theorem 3.4:** If  $X$  is  $T_{I_g}$  then it is  $T_{I_{g^\#}}$  but not conversely.

**Proof** Let  $X$  be a  $T_{I_g}$ -space and  $A$  be a  $I_{g^\#}$ -closed set in  $X$ . Since every  $I_{g^\#}$ -closed set is  $I_g$ -closed and  $X$  is  $T_{I_g}$ ,  $A$  is closed in  $X$ . Hence  $X$  is  $T_{I_{g^\#}}$ .

The converse need not be true as seen from the following example.

**Example 3.5:** Let  $X = \{a, b, c, d\}$ ,  $\tau = \{\emptyset, X, \{b\}, \{c\}, \{b, c\}, \{b, c, d\}\}$  and  $I = \{\emptyset, \{a\}\}$ . Then  $(X, \tau, I)$  is  $T_{I_{g^\#}}$ -space but not  $T_{I_g}$ -space. Since  $I_{g^\#}$ -closed sets of  $X$  are closed in  $X$  but the  $I_g$ -closed set  $\{d\}$  is not closed in  $X$ .

**Theorem 3.6:** If  $X$  is  $T_{I_{g^\#}}^*$  then it is  $I_{g^\#}$  but not conversely.

**Proof.** Let  $X$  be a  $T_{I_{g^\#}}^*$ -space and  $A$  be a  $I_g$ -closed set in  $X$ . Since  $X$  is  $T_{I_{g^\#}}^*$ -space and every closed set is  $I_{g^\#}$ -closed. Hence  $X$  is  $T_{I_{g^\#}}$ . The converse need not be true as seen from the following example.

**Example 3.7:** Let  $X = \{a, b, c\}$ ,  $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$  and  $I = \{\emptyset, \{a\}\}$ . Then  $(X, \tau, I)$  is  $T_{I_{g^\#}}^*$ -space but not  $T_{I_{g^\#}}$ -space. Since  $I_{g^\#}, I_g$ -closed sets of  $X$  are  $\emptyset, X, \{a\}, \{c\}, \{b, c\}, \{a, c\}$  and closed sets of  $X$  are  $\emptyset, X, \{c\}, \{b, c\}, \{a, c\}$ .

**Remark 3.8:**  $T_{1/2}$  and  $T_{I_{g^\#}}$  spaces are independent from the following example.

**Example 3.9:** This is obvious from remark 2.4[3].

**Theorem 3.10:** If a function  $f: (X, \tau, I) \rightarrow (Y, \sigma, J)$  is continuous and  $Y$  is a  $T_{I_{g^\#}}$ -space, then  $f$  is  $I_{g^\#}$ -irresolute.

**Proof.** Assume that  $f$  is continuous. Let  $G$  be any  $I_{g^\#}$ -closed set in  $Y$ . Since  $Y$  is a  $T_{I_{g^\#}}$ -space, then  $G$  is closed in  $Y$ . Hence  $f^{-1}(G)$  is closed in  $X$ . But every closed set is  $I_{g^\#}$ -closed. Therefore  $f$  is  $I_{g^\#}$ -irresolute.

**Theorem 3.11:** If a function  $f: (X, \tau, I) \rightarrow (Y, \sigma, J)$  is continuous and  $Y$  is a  $T_{I_{g^\#}}$ -space, then  $f$  is strongly  $I_{g^\#}$ -continuous.

**Proof.** Assume that  $f$  is continuous. Let  $G$  be any  $I_{g^\#}$ -closed set in  $Y$ . Since  $Y$  is a  $T_{I_{g^\#}}$ -space, then  $G$  is closed in  $Y$ . Hence  $f^{-1}(G)$  is closed in  $X$ . Therefore  $f$  is strongly  $I_{g^\#}$ -continuous.

#### IV. Strongly $g^\#$ -Continuous And Perfectly $g^\#$ -Continuous Maps In Ideal Topological Spaces

**Definition 4.1:** A function  $f : (X, \tau) \rightarrow (Y, \sigma, J)$  is said to be strongly  $I_{g^\#}$ -continuous if the inverse image of every  $I_{g^\#}$ -closed set in  $Y$  is closed in  $X$ .

**Remark 4.2:** When  $Y$  is  $T_{I_{g^\#}}$ , strongly  $I_{g^\#}$ -continuity coincides with continuity.

**Theorem 4.3:** If a map  $f : (X, \tau) \rightarrow (Y, \sigma, J)$  from a topological space into an ideal topological space is strongly  $I_{g^\#}$ -continuous then it is continuous but not conversely.

**Proof.** Assume that  $f$  is strongly  $I_{g^\#}$ -continuous. Let  $G$  be any open set in  $Y$ . Since every open set is  $I_{g^\#}$ -open,  $G$  is  $I_{g^\#}$ -open in  $Y$ . Since  $f$  is strongly  $I_{g^\#}$ -continuous,  $f^{-1}(G)$  is open in  $X$ . Therefore  $f$  is continuous.

The converse need not be true as seen from the following example.

**Example 4.4:** Let  $X = Y = \{a, b, c\}$  with the topologies  $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ ,  $\sigma = \{\emptyset, X, \{a\}, \{a, b\}\}$  and  $J = \{\emptyset, \{b\}\}$ . Define a map  $f : (X, \tau) \rightarrow (Y, \sigma, J)$  by  $f(a)=b$ ,  $f(b)=a$ ,  $f(c)=c$  then  $f$  is continuous. But  $f$  is not strongly  $I_{g^\#}$ -continuous. Since  $f^{-1}(\{b\})=\{a\}$  is not closed in  $X$ , where  $\{b\}$  is  $I_{g^\#}$ -closed in  $Y$ .

**Theorem 4.5:** A function  $f : (X, \tau) \rightarrow (Y, \sigma, J)$  from a topological space  $(X, \tau)$  into an ideal topological space  $(Y, \sigma, J)$  is strongly  $I_{g^\#}$ -continuous if and only if the inverse image of every  $I_{g^\#}$ -closed set in  $Y$  is closed in  $X$ .

**Proof.** Assume that  $f$  is strongly  $I_{g^\#}$ -continuous. Let  $F$  be any  $I_{g^\#}$ -closed set in  $Y$ . Then  $F^c$  is  $I_{g^\#}$ -open in  $Y$ . Since  $f$  is strongly  $I_{g^\#}$ -continuous,  $f^{-1}(F^c)$  is open in  $X$ . But  $f^{-1}(F^c) = X - f^{-1}(F)$ ,  $f^{-1}(F)$  is closed in  $X$ . Conversely assume that the inverse image of every  $I_{g^\#}$ -closed set in  $Y$  is closed in  $X$ . Let  $G$  be any  $I_{g^\#}$ -open in  $Y$ . Then  $G^c$  is  $I_{g^\#}$ -closed in  $Y$ . By assumption,  $f^{-1}(G^c)$  is closed set in  $X$ . But  $f^{-1}(G^c) = X - f^{-1}(G)$  and so  $f^{-1}(G)$  is open in  $X$ . Therefore  $f$  is strongly  $I_{g^\#}$ -continuous.

**Theorem 4.6:** If a function  $f : (X, \tau) \rightarrow (Y, \sigma, J)$  is strongly continuous, then it is strongly  $I_{g^\#}$ -continuous but not conversely.

**Proof.** Let  $G$  be any  $I_{g^\#}$ -open set in  $Y$ . Since  $f$  is strongly continuous,  $f^{-1}(G)$  is open in  $X$  (by Definition). Hence  $f$  is strongly  $I_{g^\#}$ -continuous.

The converse need not be true as seen from the following example.

**Example 4.7:** Let  $X = Y = \{a, b, c\}$  with the topologies  $\tau = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$ ,  $\sigma = \{\emptyset, X, \{a\}, \{a, b\}\}$  and  $J = \{\emptyset, \{b\}\}$ . Define a map  $f : (X, \tau) \rightarrow (Y, \sigma, J)$  by  $f(a)=a$ ,  $f(b)=c$ ,  $f(c)=b$ , then  $f$  is strongly continuous. But  $f$  is not strongly continuous. Since for the set  $\{a\}$  in  $Y$ ,  $f^{-1}(\{a\})=\{a\}$  is open but not closed in  $X$ .

**Theorem 4.8:** If a map  $f : (X, \tau) \rightarrow (Y, \sigma, J)$  is strongly  $I_{g^\#}$ -continuous and a map  $g : (Y, \sigma, J) \rightarrow (Z, \gamma)$  is  $I_{g^\#}$ -continuous, then the composition  $g \circ f : (X, \tau) \rightarrow (Z, \gamma)$  is continuous.

**Proof.** Let  $G$  be any open set in  $Z$ . Since  $g$  is  $I_{g^\#}$ -continuous,  $g^{-1}(G)$  is  $I_{g^\#}$ -open in  $Y$ . Since  $f$  is strongly  $I_{g^\#}$ -continuous,  $f^{-1}g^{-1}(G)$  is open in  $X$ . But  $f^{-1}g^{-1}(G) = (g \circ f)^{-1}(G)$ . Therefore  $g \circ f$  is continuous.

**Theorem 4.9:** If a map  $f : (X, \tau) \rightarrow (Y, \sigma, J)$  is strongly  $I_{g^\#}$ -continuous and a map  $g : (Y, \sigma, J) \rightarrow (Z, \gamma, K)$  is strongly  $I_{g^\#}$ -continuous, then the composition  $g \circ f : (X, \tau) \rightarrow (Z, \gamma, K)$  is strongly  $I_{g^\#}$ -continuous.

(i.e) Composition of two strongly  $I_{g^\#}$ -continuous functions is strongly  $I_{g^\#}$ -continuous.

**Proof.** Let  $G$  be any  $I_{g^\#}$ -open set in  $Z$ . Since  $g$  is strongly  $I_{g^\#}$ -continuous,  $g^{-1}(G)$  is open in  $Y$ . Since  $f$  is strongly  $I_{g^\#}$ -continuous and every open set is  $I_{g^\#}$ -open,  $f^{-1}g^{-1}(G)$  is open in  $X$ . But  $f^{-1}g^{-1}(G) = (g \circ f)^{-1}(G)$ . Therefore  $g \circ f$  is strongly  $I_{g^\#}$ -continuous.

**Theorem 4.11:** If a map  $f : (X, \tau, I) \rightarrow (Y, \sigma, J)$  is  $I_{g^\#}$ -continuous and a map  $g : (Y, \sigma, J) \rightarrow (Z, \gamma, K)$  is strongly  $I_{g^\#}$ -continuous, then the composition  $g \circ f : (X, \tau, I) \rightarrow (Z, \gamma, K)$  is  $I_{g^\#}$ -irresolute.

**Proof.** Let  $G$  be any  $I_{g^\#}$ -open set in  $Z$ . Since  $g$  is strongly  $I_{g^\#}$ -continuous,  $g^{-1}(G)$  is open in  $Y$ . Since  $f$  is  $I_{g^\#}$ -continuous,  $f^{-1}g^{-1}(G)$  is  $I_{g^\#}$ -open in  $X$ . But  $f^{-1}g^{-1}(G) = (g \circ f)^{-1}(G)$ . Therefore  $g \circ f$  is  $I_{g^\#}$ -irresolute.

**Theorem 4.12:** If a map  $f : (X, \tau) \rightarrow (Y, \sigma, J)$  is strongly  $I_{g^\#}$ -continuous and a map  $g : (Y, \sigma, J) \rightarrow (Z, \gamma, K)$  is  $I_{g^\#}$ -irresolute, then the composition  $g \circ f : (X, \tau, I) \rightarrow (Z, \gamma, K)$  is continuous.

**Proof.** Let  $G$  be any open set in  $Z$ . Since  $g$  is  $I_{g^\#}$ -irresolute and every open set is  $I_{g^\#}$ -open,  $g^{-1}(G)$  is  $I_{g^\#}$ -open in  $Y$ . Since  $f$  is strongly  $I_{g^\#}$ -continuous,  $f^{-1}g^{-1}(G)$  is open in  $X$ . But  $f^{-1}g^{-1}(G) = (g \circ f)^{-1}(G)$ . Therefore  $g \circ f$  is continuous.

**Definition 4.13:** A map  $f : (X, \tau) \rightarrow (Y, \sigma, J)$  is said to be perfectly  $I_{g^\#}$ -continuous if the inverse image of every  $I_{g^\#}$ -open set in  $(Y, \sigma, J)$  is both open and closed in  $(X, \tau)$ .

**Theorem 4.14:** A map  $f : (X, \tau) \rightarrow (Y, \sigma, J)$  from a topological space  $(X, \tau)$  into an ideal topological space  $(Y, \sigma, J)$  is perfectly  $I_{g^\#}$ -continuous then it is strongly  $I_{g^\#}$ -continuous but not conversely.

**Proof.** Assume that  $f$  is perfectly  $I_{g^\#}$ -continuous. Let  $G$  be any  $I_{g^\#}$ -open set in  $(Y, \sigma, J)$ . Since  $f$  is perfectly  $I_{g^\#}$ -continuous,  $f^{-1}(G)$  is open in  $(X, \tau)$ . Therefore  $f$  is strongly  $I_{g^\#}$ -continuous.

The converse of the above theorem need not be true as seen from the following example.

**Example 4.15:** Let  $X = Y = \{a, b, c\}$  with the topologies  $\tau = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$  and  $\sigma = \{\emptyset, X, \{a\}, \{a, b\}\}$  and  $J = \{\emptyset, \{b\}\}$ . Define a map  $f : (X, \tau) \rightarrow (Y, \sigma, J)$  as the identity map. Then  $f$  is strongly  $I_{g^\#}$ -continuous but not perfectly  $I_{g^\#}$ -continuous. For the  $I_{g^\#}$ -open set  $\{a\}$  of  $Y$ ,  $f^{-1}(\{a\}) = \{a\}$  which is open but not closed in  $X$ .

**Theorem 4.16:** A map  $f : (X, \tau) \rightarrow (Y, \sigma, J)$  from a topological space  $(X, \tau)$  into an ideal topological space  $(Y, \sigma, J)$  is perfectly  $I_{g^\#}$ -continuous iff  $f^{-1}(G)$  is both open and closed in  $(X, \tau)$  for every  $I_{g^\#}$ -open set in  $(Y, \sigma, J)$ .

**Proof.** Assume that  $f$  is perfectly  $I_{g^\#}$ -continuous. Let  $F$  be any  $I_{g^\#}$ -closed set in  $(Y, \sigma, J)$ . Since  $f$  is perfectly  $I_{g^\#}$ -continuous,  $f^{-1}(F^c)$  is both open and closed in  $(X, \tau)$ . But  $f^{-1}(F^c) = X - f^{-1}(F)$  and so  $f^{-1}(F)$  is both open and closed in  $(X, \tau)$ . Conversely assume that the inverse image of every  $I_{g^\#}$ -closed is both open and closed in  $(X, \tau)$ . Let  $G$  be any  $I_{g^\#}$ -open set in  $(Y, \sigma, J)$ . Then  $G^c$  is  $I_{g^\#}$ -closed set in  $(Y, \sigma, J)$ . By assumption  $f^{-1}(G^c)$  is both open and closed in  $(X, \tau)$ . But  $f^{-1}(G^c) = X - f^{-1}(G)$  and so  $f^{-1}(G)$  is both open and closed in  $(X, \tau)$ . Therefore  $f$  is perfectly  $I_{g^\#}$ -continuous.

**Remark 4.17:** From the above observations we have the following implications.

### V. $g^\#$ -Compactness in Ideal Topological Space

**Definition 5.1:** A collection  $\{A_i; i \in I\}$  of  $I_{g^\#}$ -open sets in an ideal topological space  $(X, \tau, I)$  is called a  $I_{g^\#}$ -open cover of a subset  $B$  in  $X$  if  $B \subseteq \bigcup_{i \in I} A_i$ .

**Definition 5.2:** An ideal topological space  $(X, \tau, I)$  is  $I_{g^\#}$ -compact if every  $I_{g^\#}$ -open cover of  $X$  has a finite subcover of  $X$ .

**Definition 5.3:** A subset  $B$  of an ideal topological space  $(X, \tau, I)$  is called  $I_{g^\#}$ -compact relative to  $X$ , if for every collection  $\{A_i; i \in I\}$  of  $I_{g^\#}$ -open subsets of  $X$  such that  $B \subseteq \bigcup_{i \in I} A_i$ , there exist a finite subset  $I_0$  of  $I$  such that  $B \subseteq \bigcup_{i \in I_0} A_i$ .

**Definition 5.4:** A subset  $B$  of an ideal topological space  $(X, \tau, I)$  is called  $I_{g^\#}$ -compact if  $B$  is  $I_{g^\#}$ -compact as the subspace of  $X$ .

**Theorem 5.5:** A  $I_{g^\#}$ -closed subset of  $I_{g^\#}$ -compact space is  $I_{g^\#}$ -compact relative to  $X$ .

**Proof.** Let  $A$  be a  $I_{g^\#}$ -closed subset of  $I_{g^\#}$ -compact space  $X$ . Then  $A^c$  is  $I_{g^\#}$ -open in  $X$ . Let  $S$  be a  $I_{g^\#}$ -open cover of  $A$  in  $X$ . Then  $S$  along with  $A^c$  form a  $I_{g^\#}$ -open cover of  $X$ . Since  $X$  is  $I_{g^\#}$ -compact, it has a finite subcover, say  $\{G_1, G_2, G_3, \dots, G_n\}$ . If this subcover contains  $A^c$ , we discard it. Otherwise leave the subcover as it is. Thus we have obtained a finite subcover of  $A$  and so  $A$  is  $I_{g^\#}$ -compact relative to  $X$ .

**Theorem 5.6:** A  $I_{g^\#}$ -continuous image of a  $I_{g^\#}$ -compact space is compact.

**Proof.** Let  $f: (X, \tau, I) \rightarrow (Y, \sigma)$  be a  $I_{g^\#}$ -continuous map from a  $I_{g^\#}$ -compact space  $X$  onto a topological space  $Y$ . Let  $\{A_i; i \in I\}$  be an open cover of  $Y$ . Then  $\{f^{-1}(A_i); i \in I\}$  is a  $I_{g^\#}$ -open cover of  $X$ . Since  $X$  is  $I_{g^\#}$ -compact, it has a finite subcover say  $\{f^{-1}(A_1), f^{-1}(A_2), \dots, f^{-1}(A_n)\}$ . Since  $f$  is onto,  $\{A_1, A_2, A_3, \dots, A_n\}$  is an open cover of  $Y$  and so  $Y$  is compact.

**Theorem 5.7:** If  $f: (X, \tau) \rightarrow (Y, \sigma, J)$  is strongly  $I_{g^\#}$ -continuous map from a compact space  $X$  onto an ideal topological space, then  $Y$  is  $I_{g^\#}$ -compact.

**Proof.** Let  $\{A_i; i \in I\}$  be an  $I_{g^\#}$ -open cover of  $Y$ . Then  $\{f^{-1}(A_i); i \in I\}$  is a open cover of  $X$ . Since  $f$  is strongly  $I_{g^\#}$ -continuous. Since  $X$  is compact, it has a finite sub cover say  $\{f^{-1}(A_1), f^{-1}(A_2), \dots, f^{-1}(A_n)\}$  and since  $f$  is onto,  $\{A_1, A_2, A_3, \dots, A_n\}$  is a finite subcover of  $Y$ . Therefore  $Y$  is  $I_{g^\#}$ -compact.

**Theorem 5.8:** If  $f: (X, \tau) \rightarrow (Y, \sigma, J)$  is perfectly  $I_{g^\#}$ -continuous map from a compact space  $X$  onto an ideal topological space  $Y$ , then  $Y$  is  $I_{g^\#}$ -compact

**Proof.** It follows from theorem 5.7.

**Theorem 5.9:** Let  $(X, \tau, I)$  be an ideal space. If  $A$  is an  $I_{g^\#}$ -closed subset of  $X$ , then  $A$  is  $I$ -compact. [5, Theorem 2.17]

**Corollary 5.11:** Let  $(X, \tau, I)$  be an ideal space. If  $A$  is an  $I_{g^\#}$ -closed subset of  $X$ , then  $A$  is  $I$ -compact.

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