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Application of Use Rate for Estimating Parameter and Finding the Approximate Failure Number using Warranty Claims in Linear Scale

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Abstract: *Recently product warranty and technological development are closely related in this world. Every product company makes a policy about warranty claims because of setting up an appropriate product price. But find out the approximate failure number is very difficult work due to the censoring information about usage or age or both. In this paper, increasing failure rate and linear model based on age and usage for finding the approximate failure numbers are applied. For this purpose, this paper contains the application of different use rate to make the incomplete data due to censored information as complete information's. Using two-dimensional warranty scheme, it is found that the approximate failure number and compared with age based forecasting through a brief simulation.*

Keywords: *Warranty Policy, Censoring, Increasing Failure Rate (IFR), Month-in-Service (MIS).*

I. Introduction

Offering product warranty to the manufacturer is a challenging work due to the claims resulting from product failures under warranty. Product failures depend mainly on product reliability. Reliability is quality over time (Condra, 1993).Warranty data provide a valuable source of information for assessing the reliability of an item in operation (called the ''field reliability'') and to make decisions. At present, manufacturing industries are facing the task of improving product quality while minimizing warranty costs, Facing due to rapidly changing technologies, global markets, development of high sophisticated products, and increasing customer expectations. Companies improve product development processes, advance product design and make modifications to their manufacturing and assembly systems by analyzing the warranty claims. As a result, products are becoming more complex and their performance capabilities are increasing with each new generation.

Usually, the degradation of a product depends on its age or usage or both. Namely, there are linear or multiple failure timescales. Each scale contributes partial failure information. As a result, consideration of multiple timescales will lead to better failure prediction and better maintenance decisions. Composite scale models are the models which deals multiple failure timescales (multidimensional). Since the information capacity carried by the composite variable is larger than that carried by any individual variable. Thus, the finding approximate failure number based on the composite scale model is more accurate than that based on the model of single variable (Jiang, R. and Jardine, A.K.S., 2006). Two typical composite models: (i) Linear models, and (ii) Multiplicative models. The composite scale is a weighted average of age and usage. As such, measures on this scale are not directly interpretable. They have no physical meaning, but capture the joint effects of age and usage. In spite of the difficulty in interpreting the results directly, the transformed data can be used in warranty analysis to evaluate various alternative warranty policies and the warranty parameters (Blischke, W. R., Karim, M. R., and Murthy, D. N.P., 2011). So far as we know; no research has been made to finding the approximate warranty claims using composite scale. In previous research, forecasting failure claims is available when the lifetime variable is measured in age only. In this paper, linear composite scales are used to perform warranty claims predictions under two-dimensional warranty scheme because the use of linear scale model is very easy and simple.

II. Product Warranty Policy and Competing Risk Models

Blischke and Murthy (1994) give a detailed discussion of the various types of warranty policies. The policy can broadly be grouped into two types: one and two-dimensional. When a warranted product fails within the warranty period, and the consumer makes legitimate claim to the manufacturer for repair or replacement of product, the claim is known as warranty claim. This paper considered warranty claims data with a two-dimensional scheme: age warranty and usage warranty. Different manufacturer offer different warranty schemes. Now-a-days warranty periods are being longer like as 1, 2, 3, 10 and so many years. The manufacturer companies are paying a lot of money for warranty costs, which are increasing.

When a warranty claim is made, failure related information is recorded in the warranty database. Two variables that are important in the field reliability studies are month-in-service (MIS) and usages at the time of warranty claim. The warranty claim databases can be represented as in **Table 1**.

Table 1: Warranty Database Information.

Sales Month	MIS, t	Sales Amount	MTF					Failure Number	Censored Number
			1	…	l	…	T		
1	T	N_T	r_T^1	…	r_T^l	…	r_T^T	r_T	c_T
…	…	…	…	…	…	…		…	
$T-t+1$	t	N_t	r_t^1	…	r_t^l			r_t	c_t
…	…	…	…	…				…	
T	1	N_1	r_1^1					r_1	c_1
Total		N						n	$N-n$

The notation in the **Table 1** is summarized as follows:

- t : months-in-service (MIS); $t = 1,2\ldots,T$
- T : maximum MIS where $T = \min(W, S, M)$
- M : observation period in calendar time
- N_t : number of product or component with t MIS
- r_t^l : number of failures claimed at l MTF for the product or component with t MIS
- c_t : number of non-failures with t MIS, $c_t = N_t - r_t$
- W : warranty period in MIS
- S : total months of sales
- n : total number of failures
- N : total number of product or component in the field
- r_t : number of failures with t MIS, $r_t = \sum_{l=1}^{t} r_t^l, t = 1, \ldots, T$

Let, X be a lifetime variable under study measured by actual usage, and let Y_t be a censoring random variable representing the total usage accumulated by an item at t MIS, which is independent of X. The following competing risks model is often used for censoring problems in which the observations$(U_{ti}, \delta_{ti}), i = 1, \ldots, N_t; t = 1, \ldots, T$, have been obtained:

$$U_{ti} = \min\{X_i, Y_{ti}\}, \text{ and}$$
$$\delta_{ti} = \begin{cases} 1 \text{ if } X_i \le Y_{ti} \\ 0 \text{ if } X_i > Y_{ti} \end{cases}$$

for the i-th usages $(i = 1, \ldots, N_t; t = 1, \ldots, T)$. Several studies have been done on the censoring problem (Alam, Suzuki and Yamamoto, 2009).

III. Linear Composite Model

Linear composite scale modeling is used to combine several scales or variables into a single scale or variable. When products are sold with two-dimensional warranties, the warranty claims data are two dimensional. Usually the first dimension refers to the age of the products or items at the time of failure and the second to usage (Alam and Suzuki, 2009). Now, considering, T be the age and U be the usage. The two dimensional warranty provides convergence over a region $R = [0, A) \times [0, B)$, for example, $A = 12$ months and $B = 100{,}000$ km. Thus the failures are covered under warranty only if $(T, U) \in R$. Composite scale model involves forming a new variable V that is the combination of usage U and age T. According to Jiang, R. and Jardine, A.K.S. (2006) the linear composite model

$$V = a*T + b*U \tag{1}$$

The parameters 'a' and 'b' can be estimated by minimizing the sample variance subject to the condition

$$\mu_U = \max(\mu_T, \mu_U)$$

Where,

μ_T = mean of the actual age T, and μ_U = mean of the actual age U.

IV. Approaches to Approximate Censored Usage

The two-dimensional warranty data are effectively reduced to one-dimensional data by treating usage as a function of age or age as a function of usage. Modeling of failures under warranty is then done using 1-D models by conditioning on the usage rate (Blischke, Karim and Murthy, 2011). It is assumed that usage rate is different for each different customer. Let (T_{ij}, U_{ij}) denote the age and usage at the time of the j-th warranty claim for customer i. The usage rate for customer i with a single failure is

$$z_i = \frac{U_{ij}}{T_{ij}}, j = 1, i = 1,2, \ldots \ldots \ldots, I_1 \tag{2}$$

The underlying model formulations for the several approaches we look at failures and censors. Subsequent failures data are available by their age but not their usage. So we involve the different approaches to modeling 2-dimensinal failures. Approach 1 and Approach 2 exist and discussed for one dimensional data (Blischke, Karim and Murthy, 2011). Now we use these approaches for two dimensional data. So that the approaches are given bellow:

Approach-1: In this approach we compute average sample usage rate from the warranty claims data, which can be treated as a censoring observation for each age (e.g., day, week, month etc.) by multiplying usage by sequential age (e.g., week, month etc.). Here usage rate is computed by dividing the accumulated usage by its corresponding age.

Approach-2: In this approach we compute median sample usage rate from the claims data, which can be treated as a censoring observation for each age by multiplying usage by sequential age.

V. Parameter Estimation Procedure

The analysis may be done at the component or product level. For simplicity, let N denote the total number of items in the data set, including original purchases, and let n be the total number of failures. Observed values are denoted x_i, andy_i. Renumber the data so that (x_i, y_i) correspond to failures for, $i = 1, 2 \dots n$, and to age and usage of non-failed items for, $i = n + 1 \dots N$. Age data for all items are easily obtained from the sales data (at an aggregated level, if necessary). Usage data, however, are rarely available for un-failed items.
Assuming that data have been aggregated at some level, with k = number of periods for which the following data are available for $j = 1, 2, \dots, k$.

N_j	=	sales in period j
A_j	=	number of valid claims on the N_j units sold in period j
M_j	=	number of items sold in period j for which no claims were made = $N_j - A_j$
T_j	=	age at censoring for items sold in period j
K	=	total number of valid claims = $\sum A_j$
U_j	=	usage at censoring for items sold in period j

If usage is not known, it may be estimated as indicated in the previous section that can be used forecasting. Let

$$R_j = (x, y): x \in [0, U_j), y \in [0, T_j) \tag{3}$$

Valid warranty claims resulting from item failures must be such that $(x_i, y_i) \in R_j$. R_j may be viewed as service ages and usages for un-failed items sold in period j. Data for analysis are failure data, namely those for which$(x_i, y_i) \in R_j$, and censored data, i.e., unobserved random variables (X_j, Y_j) which are not in the region R_j for any j.
Let T_j and U_j denote the values of censored age and usage for items sold in period j. The corresponding censored values of V_j, denoted $\widetilde{V}$, are calculated as

$$\widetilde{V} = a * T_j + b * U_j, j = 1,2, \dots \dots, k \tag{4}$$

Failure data on the v-scale are

$$v_i = a * x_j + b * y_j, i = 1,2, \dots \dots, n \tag{5}$$

The value of ‘a’ and ‘b’ are determined by a procedure, which discuss in **Section III**.

Now we select a distribution F and use the one-dimensional data V and $\widetilde{V}$ associated with the values of ‘a’ and ‘b’ to calculate MLEs of the parameters of F basedon the following likelihood function

$$L(\theta) = \prod_{i=1}^{n} f(v_i; \theta) \prod_{j=1}^{K} [1 - F(\widetilde{V}_j; \theta)]^{M_j} \tag{6}$$

In the G-K approach, the Weibull distribution is usually used for this purpose. We also use the Weibull distribution in our purpose. We consider the distribution function and density function of the chosen distribution are $F(v; \theta)$ and $f(v; \theta)$, respectively.

MLEs of the elements of the parameter θ are obtained by maximizing $\log L(\theta)$ given in (6) for the selected values of ‘a’ and ‘b’, using Newton Raphson iteration method for solution (Alam and Suzuki, 2009). The asymptotic normality of the MLEs may be used to obtain confidence intervals (CI) for the estimated parameters. This is done by forming the matrix as inverting the matrix, and substituting the estimated values of unknown parameters into the result. This provides estimated variances and covariance of the estimators and the confidence intervals (CI) are obtained by use of the standard normal distribution.

VI. Simulation Study

In this study we find the approximate failure number using the linear composite scale model in warranty claims data with different approach and compare the linear model with age based model. Mainly we perform a simulation study in this paper. The simulation processes were repeated 1000 times and we use average values from 1000 repetitions. To perform the simulation study data were generated assuming that the lifetime variable $X \sim \text{Weibull}(\beta_0, \eta_0)$ and the censoring variable $Y_t \sim \text{LN}(\mu + \log t, \sigma^2)$ for a set of true parameter values (Alam, Suzuki and Yamamoto, 2009). In this paper we consider about increasing failure rate (IFR) types of parameter because generally the failure rates of product are increasing day by day. The true parameter settings with different number of sales considered for the simulation studies are given in the **Table 2**.

Table 2: The parameter sets of the simulation study for IFR information.

Data Set	Parameters				No of Sales, N_t
	β_0	η_0	μ	σ	
Set 1	3.00	30000	6.50	0.70	1000
Set 2	3.00	30000	6.50	0.70	2000
Set 3	3.00	30000	6.50	0.70	3000
Set 4	3.00	30000	6.50	0.70	4000
Set 5	3.00	30000	6.50	0.70	5000

In this paper we want to find the approximate failure number of company product over the next year. The warranty period that the company offers is 12 months but 9 month failure data are available that are generated using the parameter **(Table 2)** and we want to find the approximate failure numbers in next 3 months that completes 12 months warranty. The products are sold N_t units per month and we also consider that the company will have sales of N_t units per month over the next year. We generate warranty claims (failures) data to month-in-service and we transform this data to month-to-failure and fit a distribution. We have chosen to fit a 2-parameter Weibull distribution using MLE as the parameter estimation method due to the large number of suspended data points in the data set. We find the approximate failure number using the concept of conditional reliability. The equation of the conditional probability of failure is:

$$Q(\tau|T) = 1 - R(\tau|T) = 1 - \frac{R(T+\tau)}{R(T)} \qquad (7)$$

Here $Q(\tau|T)$ is the unreliability of a unit for the next τ months given that it has already operated successfully for T months, $R(\tau|T)$ is the reliability of a unit for the next τ months given that it has already operated successfully for T months and $R(.)$ is the reliability function.

Let, out of Z units that are sold in Month-1(M-1), B units are still out in the field as of the end of Month 10 (M-10). That is calculated by subtracting the total number of returns of the M-1 shipment from the number of sales for that month. Then the forecasted number of failure (NF) can be calculated as, $NF = B.Q(\tau|T)$.

For the IFR type data **(Table 2)** and different approaches to approximate censored usage as discussed. The average MLE of the parameters of Weibull distribution under linear composite scale model is presented in the **Table 3**.

Table 3: The average MLE and standard error of the Weibull parameters with different number of sales.

No. of Sales	Approaches	MLE of the Parameters		Standard Error	
		$\hat{\eta}$	$\hat{\beta}$	$S.E.(\hat{\eta})$	$S.E.(\hat{\beta})$
1000	Approach 1	23.25159	3.165355	0.058951	0.006686
	Approach 2	22.23426	3.254923	0.054991	0.006951
2000	Approach 1	23.23606	3.152294	0.040979	0.004496
	Approach 2	22.23254	3.240190	0.038412	0.004687
3000	Approach 1	23.27064	3.148403	0.032503	0.003657
	Approach 2	22.25288	3.237286	0.030732	0.003834
4000	Approach 1	23.27977	3.147004	0.029680	0.003398
	Approach 2	22.26337	3.235623	0.028087	0.003556
5000	Approach 1	23.24560	3.150354	0.026247	0.002987
	Approach 2	22.23661	3.238542	0.024547	0.003111

Table 4: Approximate failure number of failures by MIS for different approaches with N_t = 1000.

Approaches	Number of failure in MIS												
	M-1	M-2	M-3	M-4	M-5	M-6	M-7	M-8	M-9	M-10	M-11	M-12	Total
Actual	**1**	**1**	**3**	**6**	**11**	**17**	**25**	**35**	**46**	**58**	**72**	**86**	**361**
Approach 1	**0**	**0**	**1**	**4**	**8**	**15**	**24**	**36**	**51**	**69**	**90**	**115**	**413**
Approach 2	0	0	1	4	8	15	25	38	54	73	96	121	435
Age	0	1	3	3	8	15	26	40	52	69	92	116	425

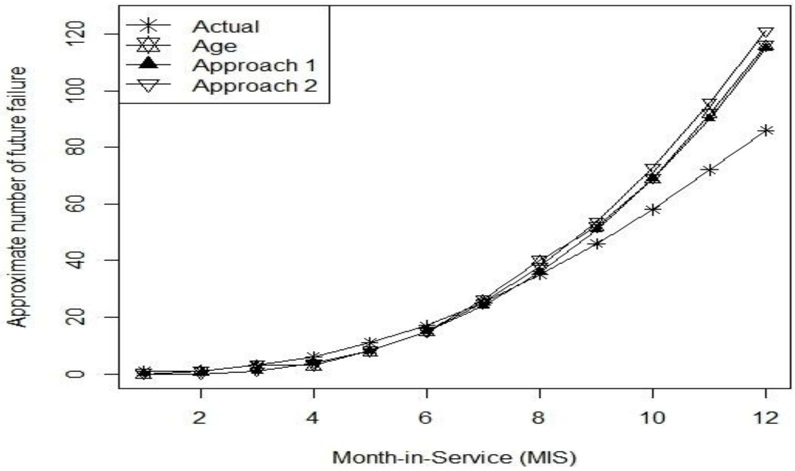

Figure 1: Comparison of different approaches in finding approximate future failure claims with $N_t = 1000$.

To make a simple comparison, the results are presented in **Table 4 and Figure 1**, when the number of sale is 1000 per month. From **Table 4 and Figure 1**, it is easily evident that the Approach 1 gives the better performance in finding the approximate future warranty claims.

Using 9 month data we find the approximate number of failures for the next 3 months for choosing the appropriate use rate (Approach). Since Approach 1 performs better, the expected failure number in next 12 months are also computed and is given in the **Table 5**.

Table 5: Approximate failure number in the next 12 months for Approach 1 with $N_t = 1000$.

Sales Month	MIS	Sales Amount	No of Failure	M-10	M-11	M-12	M-13	M-14	M-15	M-16	M-17	M-18	M-19	M-20	M-21
M-1	9	1000	46	19	23	27									
M-2	8	1000	35	15	18	22	27								
M-3	7	1000	25	11	15	18	22	27							
M-4	6	1000	17	8	11	15	18	22	27						
M-5	5	1000	11	6	8	11	15	18	22	27					
M-6	4	1000	6	4	6	8	11	15	18	22	27				
M-7	3	1000	3	2	4	6	8	11	15	18	22	27			
M-8	2	1000	1	1	2	4	6	8	11	15	18	22	27		
M-9	1	1000	1	0	1	2	4	6	8	11	15	18	22	27	
M-10				0	0	1	2	4	6	8	11	15	18	22	27
M-11					0	0	1	2	4	6	8	11	15	18	22
M-12						0	0	1	2	4	6	8	11	15	18
M-13							0	0	1	2	4	6	8	11	15
M-14								0	0	1	2	4	6	8	11
M-15									0	0	1	2	4	6	8
M-16										0	0	1	2	4	6
M-17											0	0	1	2	4
M-18												0	0	1	2
M-19													0	0	1
M-20														0	0
M-21															0
Total				**66**	**88**	**114**	**114**	**114**	**114**	**114**	**114**	**114**	**114**	**114**	**114**

Table 6 shows the expected number of failure for next 12 months along with its upper and lower 95% confidence limits.

Table 6: Confidence interval of Approach 1 with $N_t = 1000$.

Lower Value	64	86	112	112	112	112	112	112	112	112	112	112
Approach-1	66	88	114	114	114	114	114	114	114	114	114	114
Upper Value	68	91	118	118	118	118	118	118	118	118	118	118

To make the comparison, the approximate total failure numbers when product services in 12 months are presented in **Table 7**, with different number of sales (e.g. 1000,2000,3000 etc.) per month. From **Table 7** we observe that the approximate value of Approach 1 is closely related of Actual value than others, it is easily evident that the Approach 1 gives the better performance in finding the approximate future warranty claims.

Table 7: Total number of failures in 12 month in service with different number of sales.

Approaches	Total Number of Failures in 12 Month in Service				
	Monthly Sales 1000	Monthly Sales 2000	Monthly Sales 3000	Monthly Sales 4000	Monthly Sales 5000
Actual	361	717	1070	1425	1781
Approach 1	413	837	1253	1666	2086
Approach 2	435	883	1313	1748	2196
Age	425	856	1278	1704	2125

Figure 2 shows the plot of standard error of weibull parameter (lifetime parameter) with different number of sales. **Figure 2** indicate that the standard error reduces when we increase the monthly sales number.

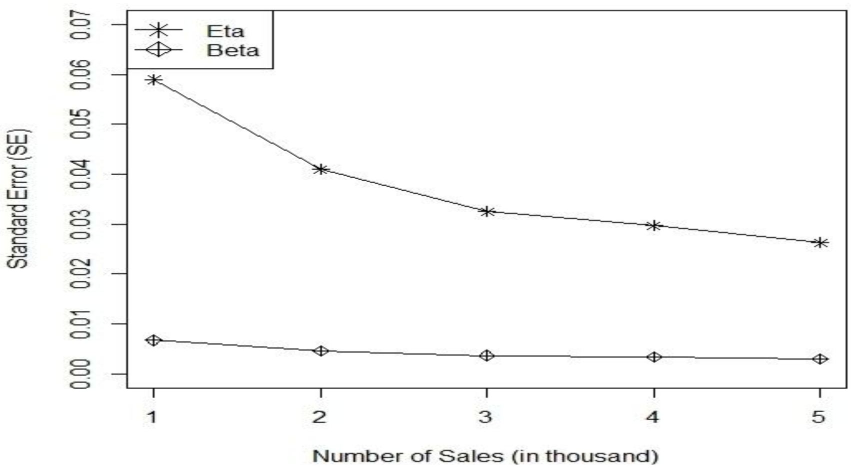

Figure 2: Plot of standard error in different number of sales.

VII. Conclusion

In this paper, the approximate number of future failure for linear composite scale model based on age and usage in the presence of censored observations has been examined. As warranty data is incomplete due to unavailability of censored usage, two approaches are used to approximate the censored usage. This information is used to make the incomplete data as complete. Weibull distribution is then fitted to the approximate complete data in linear composite scale and is used to find the approximate the failure claims.

The primary goal of this research was to investigate whether composite scale is capable or not to finding the approximate failure claims. If so, how it performs as compared to age based forecasting. For this purpose, a brief simulation was performed. The simulation results reveals that one can fairly find the approximate future failure claims in two-dimensional warranty scheme in composite model. Further we can use these approaches in real data analysis.

This paper considered only single failure mode and there is no outlier in lifetime data. In future we are interested to consider multiple failure modes in forecasting failure claims for warranted products. In such cases consideration of usage condition as a covariate would be interesting.

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