

An EOQ model for Price Discount Linked to Order Quantity under Fuzzy Environment in Quadratic Demand Pattern

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Abstract: In this paper a study has been carried out using crisp and fuzzy inventory model for the deteriorating items under trapezoidal fuzzy numbers when the supplier offered price discount to the retailer at the time of replenishment. In this model the deterioration rate is constant. Many researchers suggested, demand rate in the inventory models are constant, exponential (increase/decrease) and linearly increasing / decreasing demand patterns.. In practical situation, quadratic demand rate is more realistic.. In this paper, an inventory model is developed in crisp and fuzzy environment. This paper investigates the feasibility of regular order and special order offered by the supplier in which to maximize the total cost saving. Numerical example and sensitivity analysis carried out in fuzzy environment

Keywords: Fuzzy inventory ,Inventory , Deterioration, Price linked, Holding Cost

I. Introduction

Researchers developed inventory strategy by developing mathematical models to analyze profit or total cost depending upon various demand patterns like constant, exponential, linear and logarithmic by incorporating deterioration rate as constant or time dependent. In this paper the demand pattern deals with time dependent quadratic in nature and deterioration rate is constant. Both crisp and fuzzy inventory models are carried out when the supplier offered to retailer price discount when the retailer ordered more quantity at the time of replenishment. Zadeh (1965) proposed Fuzzy sets. In continuation Jain (1970) investigates decision making in fuzzy environment Zimmerman (1983) studied how to use fuzzy sets in Operations research. Datta and Pavan Kumar (2012) suggested fuzzy inventory model for the deteriorating items using trapezoidal fuzzy numbers. They also investigate using without shortages in inventory. Dutta and Pavan Kumar (2013) studied Fuzzy inventory model without shortages using fuzziness in the demand. Tersine (1982) studied the principles of inventory and materials management. Goyal (1990) suggested the EOQ model for on special discount periods under certainty for dynamic inventory problems. Martin (1994) proposed a note on EOQ model with temporary sale price. Aull-hyde⁹ suggested an inventory model which is under backlog also during restricted sale period. Abad (1997) proposed an optimal policy for a reseller when supplier offers temporary reduction in price. Wee and Yu (1997) suggested a deteriorated inventory model for temporary price discount. Baba and Mahmood (2006) suggested optimal ordering policies in response to a discount offer. Wee and Yu (1997) investigates deteriorating inventory model with a temporary price discount. Chang and Dye (2000) studied an EOQ model with deteriorating items in response to a temporary sale price. Bhavin (2005) investigates an EOQ model for time-dependent deterioration rate with a temporary price discount. Lal and Staelin (1984) proposed an optimal discount pricing policy in inventory. Wee (1999) proposed a backlogging deteriorating inventory model with quantity discount. Covert and Philip (1973) suggested an EOQ model incorporating Weibull distribution. An inventory model with deteriorating items, quantity discount, pricing and time dependent partial backlogging by Papachristos and Skouri¹⁷. Covert and Philip(1973) proposed an EOQ model items with weibull distribution. Philip (1974) studied an EOQ model for items with Weibull distribution. Shah (1977) studied an order level lot size model for deteriorating items

Assumptions and Notations:

- (i) $D(t) = (a_1 + b_1t + c_1t^2)$ $a_1 \geq 0, b_1 \neq 0, c_1 \neq 0$. Here a is the initial rate of demand, b is the rate with which the demand rate increases and c is the rate with which the change in the rate demand rate itself increases
- (ii) $I_1(t)$ is the inventory at any time 't'
- (iii) Replenishment rate is infinite
- (iv) Lead time is zero
- (v) θ is the deterioration rate which is constant
- (vi) C , is the cost per unit
- (vii) h is the holding cost

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- (viii) A is the ordering cost
- (ix) T* is the optimal length of the replenishment cycle
- (x) Q* is the Optimal ordering quantity
- (xi) I_{SD}(t) Inventory level during $0 \leq t \leq T_{sp}$

II. Mathematical Model

The inventory level depletes as the time passes due to selling rate and deterioration. The differential equation which describes the inventory level at time t can be written as

$$\frac{dI_1(t)}{dt} + \theta I_1(t) = -D(t), \quad 0 \leq t \leq T$$

Where $D(t) = (a_1 + b_1t + c_1t^2)$ (1)

The solution of equation (1) for the boundary condition I(T) = 0, is

$$I_1(t) = \left(\frac{-a_1}{\theta} + \frac{b_1}{\theta^2} - \frac{2c_1}{\theta^3}\right)(1 - e^{\theta(T-t)}) + \frac{b_1}{\theta}(-t + Te^{\theta(T-t)}) + \frac{c_1}{\theta}(-t^2 + T^2e^{\theta(T-t)}) + \frac{2c_1}{\theta^2}(t - Te^{\theta(T-t)})$$
(2)

$$\text{Carrying cost/holding cost per cycle} = h \int_0^T I_1(t) dt$$

Material cost per cycle (3)

(including Deterioration Loss) = $QC = I(0)C$ (4)

Total Cost(TC) = Carrying cost+ Material cost+ Ordering cost

$$= h \int_0^T I_1(t) dt + I(0)C + A$$
(5)

$$\text{Total cost}(TC) = \frac{h}{T} \left[\frac{-\left(a_1T + \frac{b_1T^2}{2} + \frac{c_1T^3}{3}\right)}{\theta} - \frac{a_1}{\theta^2} + \frac{b_1}{\theta^3} - \frac{2c_1}{\theta^4} + \left(\frac{a_1 + b_1T + c_1T^2}{\theta^2} - \frac{b_1 + 2c_1t}{\theta^3} + \frac{2c_1}{\theta^4}\right)e^{\theta T} \right]$$

$$+ \frac{C}{T} \left[\left(\frac{-a_1}{\theta} + \frac{b_1}{\theta^2} - \frac{2c_1}{\theta^3}\right) + \left(\frac{a_1 + b_1T + c_1T^2}{\theta} - \frac{b_1 + 2c_1t}{\theta^2} + \frac{2c_1}{\theta^3}\right)e^{\theta T} \right] + \frac{A}{T}$$
(6)

From the above (6) the unique value of T, optimal length of replenishment cycle time (say T*) can be obtained. Similarly the optimal order quantity Q* can be found out in from I(t). i.e I(0) = Q. The purpose of this paper is to study optimal order quantity by maximizing the total cost saving during the length of depletion time for the special order quantity.

Special order occurs(Retailers replenishment)

If the retailer order Q_{sp} units under special order policy, the inventory level at time 't' is

$$I_{sp}(t) = \left(\frac{-a_1}{\theta} + \frac{b_1}{\theta^2} - \frac{2c_1}{\theta^3}\right)(1 - e^{\theta(T-t)}) + \frac{b_1}{\theta}(-t + Te^{\theta(T-t)}) + \frac{c_1}{\theta}(-t^2 + T^2e^{\theta(T-t)}) + \frac{2c_1}{\theta^2}(t - Te^{\theta(T-t)})$$
, $0 \leq t \leq T_{sp}$

Similarly

$$Q_{sp} = \left(\frac{-a_1}{\theta} + \frac{b_1}{\theta^2} - \frac{2c_1}{\theta^3} \right) + \left(\frac{a_1 + b_1T + c_1T^2}{\theta} - \frac{b_{11} + 2c_1t}{\theta^2} + \frac{2c_1}{\theta^3} \right) e^{\theta T}$$
 Since the price discount rate being dependent on special order let price discount rate be δ_i in $(0, T_{sp})$ denoted by $TC_{sp}(T_{sp})$

$$\begin{aligned}
 TC_{sp}(T_{sp}) = & h(1 - \delta_i) \left[\frac{- \left(a_1 T_{sp} + \frac{b_1 T_{sp}^2}{2} + \frac{c_1 T_{sp}^3}{3} \right)}{\theta} - \frac{a_1}{\theta^2} + \frac{b_1}{\theta^3} - \frac{2c_1}{\theta^4} + \left(\frac{a_1 + b_1 T_{sp} + c_1 T_{sp}^2}{\theta^2} - \frac{b_1 + 2c_1 t}{\theta^3} + \frac{2c_1}{\theta^4} \right) e^{\theta T_{sp}} \right] \\
 & + C(1 - \delta_i) \left[\left(\frac{-a_1}{\theta} + \frac{b_1}{\theta^2} - \frac{2c_1}{\theta^3} \right) + \left(\frac{a_1 + b_1 T_{sp} + c_1 T_{sp}^2}{\theta} - \frac{b_{11} + 2c_1 t}{\theta^2} + \frac{2c_1}{\theta^3} \right) e^{\theta T_{sp}} \right] + A
 \end{aligned} \tag{7}$$

On the other hand, if the retailer adopts Q^* (regular order policy) in place of a large special order policy the TC (Total Cost) during $[0, T_{sp}]$ can be obtained by average cost approach. i.e. in the time interval $[0, T_{sp}]$ the total cost of regular order is $TC_{Ro}(T_{sp})$

$$\begin{aligned}
 TC_N(T_{sp}) = \frac{T_{sp}}{T^*} \left\{ \right. & h \left[\frac{- \left(a_1 T^* + \frac{b_1 T^{*2}}{2} + \frac{c_1 T^{*3}}{3} \right)}{\theta} - \frac{a_1}{\theta^2} + \frac{b_1}{\theta^3} - \frac{2c_1}{\theta^4} + \left(\frac{a_1 + b_1 T^* + c_1 T^{*2}}{\theta^2} - \frac{b_1 + 2c_1 t}{\theta^3} + \frac{2c_1}{\theta^4} \right) e^{\theta T^*} \right] \\
 & \left. + C \left[\left(\frac{-a_1}{\theta} + \frac{b_1}{\theta^2} - \frac{2c_1}{\theta^3} \right) + \left(\frac{a_1 + b_1 T^* + c_1 T^{*2}}{\theta} - \frac{b_{11} + 2c_1 t}{\theta^2} + \frac{2c_1}{\theta^3} \right) e^{\theta T^*} \right] + A \right\}
 \end{aligned} \tag{8}$$

Comparing (7) and (8) for the fixed price discount rate δ_i , the total cost saving can be formulated as follows.

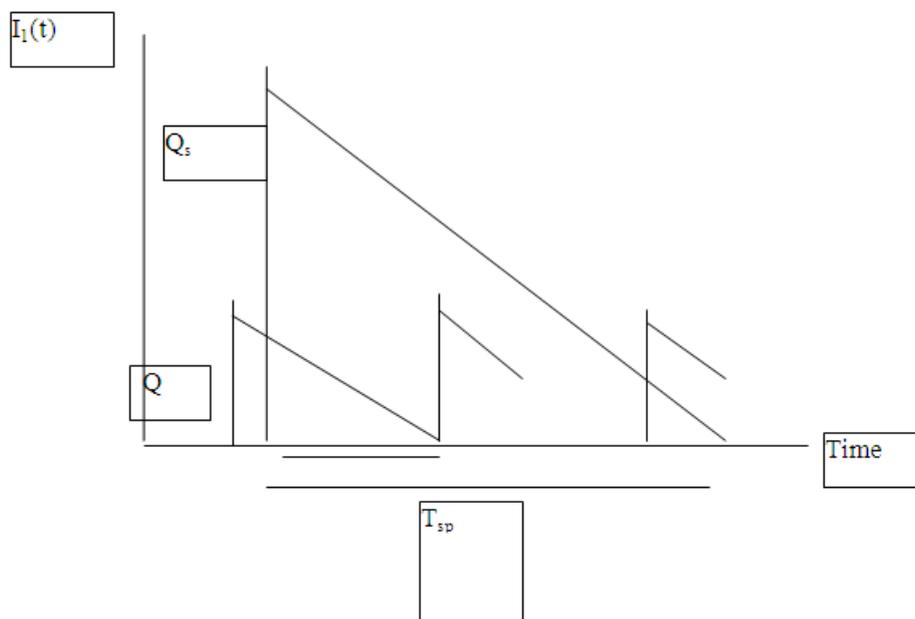


Fig (1): Regular order vs. special order policies when the special order time coincides with the retailer's

replenishment time.

$$G_s(T_{sp}) = \frac{T_{sp}}{T^*} \left\{ h \left[\frac{-\left(a_1 T^* + \frac{b_1 T^{*2}}{2} + \frac{c_1 T^{*3}}{3} \right)}{\theta} - \frac{a_1}{\theta^2} + \frac{b_1}{\theta^3} - \frac{2c_1}{\theta^4} + \left(\frac{a_1 + b_1 T^* + c_1 T^{*2}}{\theta^2} - \frac{b_1 + 2c_1 t}{\theta^3} + \frac{2c_1}{\theta^4} \right) e^{\theta T^*} \right] + C \left[\left(\frac{-a_1}{\theta} + \frac{b_1}{\theta^2} - \frac{2c_1}{\theta^3} \right) + \left(\frac{a_1 + b_1 T^* + c_1 T^{*2}}{\theta} - \frac{b_1 + 2c_1 t}{\theta^2} + \frac{2c_1}{\theta^3} \right) e^{\theta T^*} \right] + A \right\}$$

$$- h(1 - \delta_i) \left[\frac{-\left(a_1 T_{sp} + \frac{b_1 T_{sp}^2}{2} + \frac{c_1 T_{sp}^3}{3} \right)}{\theta} - \frac{a_1}{\theta^2} + \frac{b_1}{\theta^3} - \frac{2c_1}{\theta^4} + \left(\frac{a_1 + b_1 T_{sp} + c_1 T_{sp}^2}{\theta^2} - \frac{b_1 + 2c_1 t}{\theta^3} + \frac{2c_1}{\theta^4} \right) e^{\theta T_{sp}} \right]$$

$$- C(1 - \delta_i) \left[\left(\frac{-a_1}{\theta} + \frac{b_1}{\theta^2} - \frac{2c_1}{\theta^3} \right) + \left(\frac{a_1 + b_1 T_{sp} + c_1 T_{sp}^2}{\theta} - \frac{b_1 + 2c_1 t}{\theta^2} + \frac{2c_1}{\theta^3} \right) e^{\theta T_{sp}} \right] - A$$

$$\frac{d(G_s(T_s))}{dT_s} = 0, \quad \frac{d^2(G_s(T_s))}{dT_s^2} < 0$$

The necessary condition for a maximize $G_s(T_s)$,

III. Fuzzy Model and solution

Let us consider the model in fuzzy environment. Due to fuzziness, precisely defining all parameter is not easy. Hence Let $A = (A_{11}, A_{12}, A_{13}, A_{14})$, $C_{hc} = (\theta_1, \theta_2, \theta_3, \theta_4)$, $C_{DC} = (C_{DC1}, C_{DC2}, C_{DC3}, C_{DC4})$ be trapezoidal fuzzy numbers in LR form. Now, in the fuzzy sense the total cost of the system is given by

$$F_s(T_{sp}) = \frac{T_{sp}}{T^*} \left\{ h \left[\frac{-\left(a_1 T^* + \frac{b_1 T^{*2}}{2} + \frac{c_1 T^{*3}}{3} \right)}{\theta} - \frac{a_1}{\theta^2} + \frac{b_1}{\theta^3} - \frac{2c_1}{\theta^4} + \left(\frac{a_1 + b_1 T^* + c_1 T^{*2}}{\theta^2} - \frac{b_1 + 2c_1 t}{\theta^3} + \frac{2c_1}{\theta^4} \right) e^{\theta T^*} \right] + C \left[\left(\frac{-a_1}{\theta} + \frac{b_1}{\theta^2} - \frac{2c_1}{\theta^3} \right) + \left(\frac{a_1 + b_1 T^* + c_1 T^{*2}}{\theta} - \frac{b_1 + 2c_1 t}{\theta^2} + \frac{2c_1}{\theta^3} \right) e^{\theta T^*} \right] + A \right\}$$

$$- h(1 - \delta_i) \left[\frac{-\left(a_1 T_{sp} + \frac{b_1 T_{sp}^2}{2} + \frac{c_1 T_{sp}^3}{3} \right)}{\theta} - \frac{a_1}{\theta^2} + \frac{b_1}{\theta^3} - \frac{2c_1}{\theta^4} + \left(\frac{a_1 + b_1 T_{sp} + c_1 T_{sp}^2}{\theta^2} - \frac{b_1 + 2c_1 t}{\theta^3} + \frac{2c_1}{\theta^4} \right) e^{\theta T_{sp}} \right]$$

$$- C(1 - \delta_i) \left[\left(\frac{-a_1}{\theta} + \frac{b_1}{\theta^2} - \frac{2c_1}{\theta^3} \right) + \left(\frac{a_1 + b_1 T_{sp} + c_1 T_{sp}^2}{\theta} - \frac{b_1 + 2c_1 t}{\theta^2} + \frac{2c_1}{\theta^3} \right) e^{\theta T_{sp}} \right] - A$$

In which where

$$\begin{aligned}
 W = \frac{T_{sp}}{T^*} & \left\{ \begin{aligned} & h \left[\frac{-\left(a_1 T^* + \frac{b_1 T^{*2}}{2} + \frac{c_1 T^{*3}}{3}\right)}{\theta 1} - \frac{a_1}{\theta 1^2} + \frac{b_1}{\theta 1^3} - \frac{2c_1}{\theta 1^4} + \left(\frac{a_1 + b_1 T^* + c_1 T^{*2}}{\theta 1^2} - \frac{b_1 + 2c_1 t}{\theta 1^3} + \frac{2c_1}{\theta 1^4}\right) e^{\theta 1 T^*} \right] \\ & + C \left[\left(\frac{-a_1}{\theta 1} + \frac{b_1}{\theta 1^2} - \frac{2c_1}{\theta 1^3}\right) + \left(\frac{a_1 + b_1 T^* + c_1 T^{*2}}{\theta 1} - \frac{b_{11} + 2c_1 t}{\theta 1^2} + \frac{2c_1}{\theta 1^3}\right) e^{\theta T^*} \right] + A_{11} \end{aligned} \right\} \\
 & - h(1 - \delta_{11}) \left[\frac{-\left(a_1 T_{sp} + \frac{b_1 T_{sp}^2}{2} + \frac{c_1 T_{sp}^3}{3}\right)}{\theta 1} - \frac{a_1}{\theta 1^2} + \frac{b_1}{\theta 1^3} - \frac{2c_1}{\theta 1^4} + \left(\frac{a_1 + b_1 T_{sp} + c_1 T_{sp}^2}{\theta 1^2} - \frac{b_1 + 2c_1 t}{\theta 1^3} + \frac{2c_1}{\theta 1^4}\right) e^{\theta 1 T_{sp}} \right] \\
 & - C(1 - \delta_{12}) \left[\left(\frac{-a_1}{\theta 1} + \frac{b_1}{\theta 1^2} - \frac{2c_1}{\theta 1^3}\right) + \left(\frac{a_1 + b_1 T_{sp} + c_1 T_{sp}^2}{\theta 1} - \frac{b_{11} + 2c_1 t}{\theta 1^2} + \frac{2c_1}{\theta 1^3}\right) e^{\theta 1 T_{sp}} \right] - A_{12} \\
 \\
 X = \frac{T_{sp}}{T^*} & \left\{ \begin{aligned} & h \left[\frac{-\left(a_1 T^* + \frac{b_1 T^{*2}}{2} + \frac{c_1 T^{*3}}{3}\right)}{\theta 2} - \frac{a_1}{\theta 2^2} + \frac{b_1}{\theta 2^3} - \frac{2c_1}{\theta 2^4} + \left(\frac{a_1 + b_1 T^* + c_1 T^{*2}}{\theta 2^2} - \frac{b_1 + 2c_1 t}{\theta 2^3} + \frac{2c_1}{\theta 2^4}\right) e^{\theta 2 T^*} \right] \\ & + C \left[\left(\frac{-a_1}{\theta 2} + \frac{b_1}{\theta 2^2} - \frac{2c_1}{\theta 2^3}\right) + \left(\frac{a_1 + b_1 T^* + c_1 T^{*2}}{\theta 2} - \frac{b_{11} + 2c_1 t}{\theta 2^2} + \frac{2c_1}{\theta 2^3}\right) e^{\theta 2 T^*} \right] + A_{21} \end{aligned} \right\} \\
 & - h(1 - \delta_{21}) \left[\frac{-\left(a_1 T_{sp} + \frac{b_1 T_{sp}^2}{2} + \frac{c_1 T_{sp}^3}{3}\right)}{\theta 2} - \frac{a_1}{\theta 2^2} + \frac{b_1}{\theta 2^3} - \frac{2c_1}{\theta 2^4} + \left(\frac{a_1 + b_1 T_{sp} + c_1 T_{sp}^2}{\theta 2^2} - \frac{b_1 + 2c_1 t}{\theta 2^3} + \frac{2c_1}{\theta 2^4}\right) e^{\theta 2 T_{sp}} \right] \\
 & - C(1 - \delta_{22}) \left[\left(\frac{-a_1}{\theta 2} + \frac{b_1}{\theta 2^2} - \frac{2c_1}{\theta 2^3}\right) + \left(\frac{a_1 + b_1 T_{sp} + c_1 T_{sp}^2}{\theta 2} - \frac{b_{11} + 2c_1 t}{\theta 2^2} + \frac{2c_1}{\theta 2^3}\right) e^{\theta 2 T_{sp}} \right] - A_{22} \\
 \\
 Y = \frac{T_{sp}}{T^*} & \left\{ \begin{aligned} & h \left[\frac{-\left(a_1 T^* + \frac{b_1 T^{*2}}{2} + \frac{c_1 T^{*3}}{3}\right)}{\theta 3} - \frac{a_1}{\theta 3^2} + \frac{b_1}{\theta 3^3} - \frac{2c_1}{\theta 3^4} + \left(\frac{a_1 + b_1 T^* + c_1 T^{*2}}{\theta 3^2} - \frac{b_1 + 2c_1 t}{\theta 3^3} + \frac{2c_1}{\theta 3^4}\right) e^{\theta 3 T^*} \right] \\ & + C \left[\left(\frac{-a_1}{\theta 3} + \frac{b_1}{\theta 3^2} - \frac{2c_1}{\theta 3^3}\right) + \left(\frac{a_1 + b_1 T^* + c_1 T^{*2}}{\theta 3} - \frac{b_{11} + 2c_1 t}{\theta 3^2} + \frac{2c_1}{\theta 3^3}\right) e^{\theta 3 T^*} \right] + A_{31} \end{aligned} \right\} \\
 & - h(1 - \delta_{31}) \left[\frac{-\left(a_1 T_{sp} + \frac{b_1 T_{sp}^2}{2} + \frac{c_1 T_{sp}^3}{3}\right)}{\theta 3} - \frac{a_1}{\theta 3^2} + \frac{b_1}{\theta 3^3} - \frac{2c_1}{\theta 3^4} + \left(\frac{a_1 + b_1 T_{sp} + c_1 T_{sp}^2}{\theta 3^2} - \frac{b_1 + 2c_1 t}{\theta 3^3} + \frac{2c_1}{\theta 3^4}\right) e^{\theta 3 T_{sp}} \right] \\
 & - C(1 - \delta_{32}) \left[\left(\frac{-a_1}{\theta 3} + \frac{b_1}{\theta 3^2} - \frac{2c_1}{\theta 3^3}\right) + \left(\frac{a_1 + b_1 T_{sp} + c_1 T_{sp}^2}{\theta 3} - \frac{b_{11} + 2c_1 t}{\theta 3^2} + \frac{2c_1}{\theta 3^3}\right) e^{\theta 3 T_{sp}} \right] - A_{32}
 \end{aligned}$$

$$Z = \frac{T_{sp}}{T^*} \left[h \left[\left(-\frac{a_1 T^* + \frac{b_1 T^{*2}}{2} + \frac{c_1 T^{*3}}{3}}{\theta 4} - \frac{a_1}{\theta 4^2} + \frac{b_1}{\theta 4^3} - \frac{2c_1}{\theta 4^4} + \left(\frac{a_1 + b_1 T^* + c_1 T^{*2}}{\theta 4^2} - \frac{b_1 + 2c_1 t}{\theta 4^3} + \frac{2c_1}{\theta 4^4} \right) e^{\theta 4 T^*} \right) \right] + C \left[\left(\frac{-a_1}{\theta 4} + \frac{b_1}{\theta 4^2} - \frac{2c_1}{\theta 4^3} \right) + \left(\frac{a_1 + b_1 T^* + c_1 T^{*2}}{\theta 4} - \frac{b_1 + 2c_1 t}{\theta 4^2} + \frac{2c_1}{\theta 4^3} \right) e^{\theta 4 T^*} \right] + A_{41} \right] - h(1 - \delta_{41}) \left[\left(-\frac{a_1 T_{sp} + \frac{b_1 T_{sp}^2}{2} + \frac{c_1 T_{sp}^3}{3}}{\theta 4} - \frac{a_1}{\theta 4^2} + \frac{b_1}{\theta 4^3} - \frac{2c_1}{\theta 4^4} + \left(\frac{a_1 + b_1 T_{sp} + c_1 T_{sp}^2}{\theta 4^2} - \frac{b_1 + 2c_1 t}{\theta 4^3} + \frac{2c_1}{\theta 4^4} \right) e^{\theta 4 T_{sp}} \right) \right] - C(1 - \delta_{42}) \left[\left(\frac{-a_1}{\theta 4} + \frac{b_1}{\theta 4^2} - \frac{2c_1}{\theta 4^3} \right) + \left(\frac{a_1 + b_1 T_{sp} + c_1 T_{sp}^2}{\theta 4} - \frac{b_1 + 2c_1 t}{\theta 4^2} + \frac{2c_1}{\theta 4^3} \right) e^{\theta 4 T_{sp}} \right] - A_{42}$$

The α -cuts, $C_L(\alpha)$ and $C_R(\alpha)$ of the Trapezoidal fuzzy number $F_s(T_{sp})$ are given by $C_L(\alpha) = W+(X-W) \alpha$ and $C_R(\alpha) = Z-(Z-Y) \alpha$ (9)

By using signed distance method, the defuzzified value of fuzzy number $F_z(T)$, is given by

$$F_s(T_{sp})_{SD} = \frac{1}{2} \int_0^1 [C_L(\alpha) + C_R(\alpha)] d\alpha$$

$$\frac{\partial(F_s(T_{sp}))_{SD}}{\partial T_{sp}} = 0 \quad \text{provided} \quad \frac{\partial^2 F_s(T_{sp})_{SD}}{\partial T_{sp}^2} < 0$$

The necessary condition for minimizing the total cost is

The optimal value of T_{sp}^* and the total cost $(F_s(T_{sp}))_{SD}$ is obtained using mathematical software MATHCAD

3.1. Numerical example.

To illustrate the effectiveness of the model, we consider the following values for the parameters $a = 100, b = 70$ and $c = 3, A = 300, h = 1, C = 10, \Theta = 0.01, T^* = 0.804, Q = 103.993$ (T find T^*)

Crisp Model:

When $a = 100, b = 70$ and $c = 3, A = 300, h = 1, C = 10, \Theta = 0.01, T^* = 0.804, Q^* = 135.522$

Fuzzy Model:

When $a = 100, b = 70$ and $c = 3, A = 300, h = 1, C = 10, \Theta = 0.01, \delta = 0.1, T_{sp}^* = 1.003, G_s = 124.835, Q_{sp}^* = 136.929$ ($T^* < T_{sp}^*$)

Case I:

When all A, δ, Θ are fuzzy trapezoidal numbers, solution of fuzzy model is $T_{sp}^* = 1.003, G_s(T_{sp}) = 126.658, Q_{sp}^* = 136.929$

Case II:

When all δ, Θ are fuzzy trapezoidal numbers then solution of fuzzy model is $T_{sp}^* = 1.003, G_s(T_{sp}) = 126.658, Q_{sp}^* = 136.929$

Case III:

When Θ is fuzzy trapezoidal numbers then solution of fuzzy model is $T_{sp}^* = 1.001, G_s(T_{sp}) = 125.76, Q_{sp}^* = 136.581$

When none of these parameters are fuzzy trapezoidal number, then

$T_s^* = 1.001, G_s(T_{sp}) = 125.76, Q_{sp}^* = 136.581$

3.2 Sensitivity Analysis:

Sensitivity analysis for the parameter A

Defuzzify value of A	Fuzzify value of A	T_{sp}^*	$Gs(T_{sp})^*$	Q_{sp}^*
200	(50,150,250,350)	0.827	33.538	107.492
250	(100,200,300,400)	0.932	115.637	124.771
300	(150,250,350,450)	1.001	125.762	136.581
350	(200,300,400,500)	1.069	140.128	148.577

Sensitivity analysis for the parameter Θ

Defuzzify value of Θ	Fuzzify value of Θ	T_{sp}^*	$Gs(T_{sp})^*$	Q_{sp}^*
.004	(.001,.003,.005,.007)	1.008	126.498	137.8
.006	(.003,.005,.007,.009)	1.006	126.248	137.451
.008	(.005,.007,.009,.011)	1.003	126.003	136.929
.010	(.007,.009,.011,.013)	1.001	125.762	136.58

Sensitivity analysis for the parameter δ

Defuzzify value of δ	Fuzzify value of δ	T_{sp}^*	$Gs(T_{sp})^*$	Q_{sp}^*
.01	(.04,.08,.012,.016)	1.003	126.658	136.929
.02	(.05,.15,.25,.35)	1.238	295.007	179.742
.03	(0, .2, .4, .6)	1.533	523.333	240.009

IV. Conclusion

This paper investigates an EOQ model with time dependent quadratic demand pattern approach is derived. Here the deterioration rate is constant. This paper an\contains the analysis of temporary price discount offered by a supplier on a retailer replenishment policy for deteriorating items. Numerical example and sensitivity analysis also carried out.

Scope For Further Research

This paper can be extended by incorporating with shortages. Instead of special order vs regular order policy , this paper can be modified , when special order time occurs during the retailer's sales period.

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