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Orthogonal Semiderivations on Semiprime Semirings

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Abstract: *Motivated by some results on orthogonal derivations of semiprime rings, in* [2], *orthogonal reverse derivations on semiprime gamma rings, in* [9] *and introduction of semiderivations in* [7], *we, authors have defined the notion of orthogonal semiderivations on semiprime semirings. We also investigated some necessary and sufficient conditions for two semiderivations to be orthogonal.*
Key words: *Semirings, prime semirings, Semiprime semirings, Derivation, orthogonal derivation, Reverse derivation, Orthogonal reverse derivation, Semiderivation, Orthogonal semiderivation.*

I. Introduction

The notion of semiring was introduced by H.S.Vandiver [1] as a generalization of ring in the year 1934. Jonathan.S.Golan [13] introduced the definition of derivations in semirings in 1969. J. Bergen [7] introduced the notion of semiderivations in prime rings in the year 1983. Bresar.M and J.Vukman [14] introduced the notion of reverse derivations in rings in 1989. M.Bresar and J.Vukman [2] obtained some results concerning orthogonal derivations in semiprime rings which are related to the result that is well known to a theorem of Posner [3] for the product of two derivations in prime rings. J.C.Chang [4] studied on semi derivations of prime rings. He obtained some results on derivations of prime rings into semiderivations. C.L.Chuang [5] studied on the structure of semiderivations in prime rings. He obtained some remarkable results in connection with the semiderivatons. A.Firat [6] generalized some results of prime rings with derivations to the prime rings with semiderivations. Shakir Ali and Mohammad Salahuddin Khan [8] obtained some results on orthogonal (σ,τ)- derivations on semiprime gamma rings. Kalyan Kumar Dey, Akhil Chandra Paul, Isamiddin S. Rakhimov [9] studied on semiprime gamma rings with orthogonal reverse derivations. Motivated by all the above notions, in this paper we extend the results to semiprime semirings. The notion of orthogonality of two semiderivations is given and conditions of two semiderivations to be orthogonal are provided. We also obtain some characterizations of semiprime semirings with orthogonal semiderivation. Throughout this paper S will represent a 2-torsion free semiprime semiring.

II. Preliminaries

Definition 2.1
A **semiring** S is a nonempty set S equipped with two binary operations $+$ and $\bullet$ such that
1. $(S,+)$ is a commutative monoid with identity element 0
2. $(S,\bullet)$ is a monoid with identity element 1
3. Multiplication is distributive with respect to addition both from left and from right.

Definition 2.2
A semiring S is said to be **prime** if $xSy = 0$ implies $x = 0 \; or \; y = 0 \; for all \; x, y \in S.$

Definition 2.3
A semiring S is said to be **semiprime** if $xSx = 0$ implies $x = 0 \; for all \; x \in S.$

Definition 2.4
A semiring S is said to be **2- torsion free** if $2x = 0$ implies $x = 0 \; for all \; x \in S.$

Definition 2.5
An additive mapping $d : S \to S$ is called a **derivation** if $d(xy) = d(x)y + xd(y)$ holds $for all \; x, y \in S$.

Definition 2.6
Let S be a semiring. The derivations d and g of S into S is said to be **orthogonal** if $d(x)sg(y)=0=g(y)sd(x)$ *for all* $x,y\in S$

Definition 2.7
An additive mapping $d:S\to S$ is called a **reverse derivation** if $d(xy)=d(y)x+yd(x)$ holds *forall* $x,y\in S$.

Definition 2.8
Let S be a semiring. The reverse derivations d and g of S into S is said to be **orthogonal** if $d(x)sg(y)=0=g(y)sd(x)$ *for all* $x,y\in S$

Definition 2.9
An additive mapping $f:S\to S$ is called a **semiderivation** associated with a function $g:S\to S$ if for all $x,y\in S$ $(i)\, f(xy)=f(x)g(y)+xf(y)=f(x)y+g(x)f(y)$, $(ii)\, f(g(x))=g(f(x))$.
If $g=I$, i.e., an identity mapping of S then all semiderivations associated with g are merely ordinary derivations. If g is any endomorphism of S, then semiderivations are of the form $f(x)=x-g(x)$.

III. Orthogonal Semiderivations

Definition 3.1
Let f_1 *and* f_2 be two semiderivations of a semiprime semiring S associated with functions $g_1:S\to S$ *and* $g_2:S\to S$ respectively. Then f_1 *and* f_2 are said to be orthogonal if $f_1(x)sf_2(y)=0=f_2(y)sf_1(x)$ for all $x,y\in S$.

Example 3.2
Let S_1 be a semiprime semiring. Let $S=S_1\oplus S_1$. Define $f_1:S\to S$ such that $f_1(a,b)=(a,0)$ and $g_1:S\to S$ such that $g_1(a,b)=(0,ab)$ for all $a,b\in S_1$. Also define $f_2:S\to S$ such that $f_2(a,b)=(0,b)$ and $g_2:S\to S$ such that $g_2(a,b)=(ab,0)$ for all $a,b\in S_1$. Define addition and multiplication on S by $(a_1,b_1)+(a_2,b_2)=(a_1+a_2,b_1+b_2)$ and $(a_1,b_1)\bullet(a_2,b_2)=(a_1\bullet a_2,b_1\bullet b_2)$. Then it can easily be seen that f_1 *and* f_2 are semiderivations of S (with associated mappings g_1 *and* g_2 respectively) which are not derivations. Also it is clear that f_1 *and* f_2 are orthogonal semiderivations of S.

Example 3.3
Let S_1 be a semiprime semiring. Let $S=S_1\oplus S_1$. Let $\alpha_1:S_1\to S_1$ be an additive map, $\alpha_2:S_1\to S_1$ be a left and right S_1 module which is not a derivation. Define $f_1:S\to S$ such that $f_1(x_1,x_2)=(0,\alpha_2(x_2))$ and $g_1:S\to S$ such that $g_1(x_1,x_2)=(\alpha_1(x_1),0)$ for all $x_1,x_2\in S_1$. Also define $f_2:S\to S$ such that $f_2(x_1,x_2)=(\alpha_2(x_1),0)$ and $g_2:S\to S$ such that $g_2(x_1,x_2)=(0,\alpha_1(x_2))$ for all $x_1,x_2\in S_1$. Define addition and multiplication on S by $(x_1,x_2)+(y_1,y_2)=(x_1+y_1,x_2+y_2)$ and $(x_1,x_2)\bullet(y_1,y_2)=(x_1\bullet y_1,x_2\bullet y_2)$. Then it can easily be seen that f_1 *and* f_2 are semiderivations of S (with associated mappings g_1 *and* g_2 respectively) which are not derivations. Also it is clear that f_1 *and* f_2 are orthogonal semiderivations of S.

Note: The following example shows semiderivations which are not orthogonal.

Example 3.4
Let S be a semiprime semiring. Let $M_2(S)=\left\{\begin{bmatrix} a & b \\ 0 & c\end{bmatrix}: a,b,c\in S\right\}$. Define

$$f_1:M_2(S)\to M_2(S)\, by\; f_1\begin{bmatrix} a & b \\ 0 & c\end{bmatrix}=\begin{bmatrix} a & 0 \\ 0 & 0\end{bmatrix}, g_1:M_2(S)\to M_2(S)\, by\; g_1\begin{bmatrix} a & b \\ 0 & c\end{bmatrix}=\begin{bmatrix} 0 & 0 \\ 0 & c\end{bmatrix},$$

$$f_2:M_2(S)\to M_2(S)\, by\; f_2\begin{bmatrix} a & b \\ 0 & c\end{bmatrix}=\begin{bmatrix} 0 & b \\ 0 & 0\end{bmatrix}, g_2:M_2(S)\to M_2(S)\, by\; g_2\begin{bmatrix} a & b \\ 0 & c\end{bmatrix}=\begin{bmatrix} a & 0 \\ 0 & c\end{bmatrix}$$

It is clear that $f_1 and\ f_2$ are semiderivations which are not orthogonal.

IV. Results

Lemma 4.1

Let S be a semiprime semiring, and $a \in S$ .If S admits a semiderivation f such that $af(x)=0 or\ f(x)a=0\ for\ all\ x \in S\ then\ a=0 or\ f=0.$

Proof:

By hypothesis $af(x)=0\ \ for\ all\ x \in S$

Replacing x by xy $\quad for all\ x, y \in S$

$af(xy)=0\ \ for all\ x, y \in S$

$af(x)g(y)+axf(y)=0\ for all\ x, y \in S$

$axf(y)=0\ for all\ x, y \in S$

$asf(y)=0\ for all\ y \in S$

Since S is prime $a=0\ or\ f(y)=0\ for\ all\ y \in S$. Hence $a=0\ or\ f=0$.

Similarly we can prove for $f(x)a=0$

Lemma 4.2

Let S be a 2- torsion free semiprime semiring, and $a, b \in S$. Then the following are equivalent.

$(i) asb=0$

$(ii) bsa=0$

$(iii) asb+bsa=0$

If one of these conditions are fulfilled then $ab=ba=0.$

Proof:

Refer the proof of lemma 3.3 in [13]

Lemma 4.3

Let S be a 2- torsion free semiprime semiring. Suppose that additive mappings $f\ and\ g$ of S into S satisfying $f(x)sg(x)=0$ for all $x \in S$ then $f(x)sg(y)=0$ for all $x, y \in S.$

Proof:

Refer the proof of lemma 3.4 in [13]

Theorem 4.4

Let S be a 2- torsion free semiprime semiring and let $f_1\ and\ f_2$ be semiderivations of S into S associated with functions $g_1 : S \rightarrow S\ and\ g_2 : S \rightarrow S$ respectively. Then $f_1\ and\ f_2$ are orthogonal if and only if $f_1(x)f_2(y)+f_2(x)f_1(y)=0$ for all $x, y \in S.$

Proof:

Suppose that $f_1 and\ f_2$ are orthogonal. Then $f_1(x)sf_2(y)=0=f_2(y)sf_1(x)\ for\ all\ x, y \in S$

Since $f_1(x)sf_2(y)=0\ for\ all\ x, y \in S$ by lemma 4.2 we have

$f_1(x)f_2(y)=0=f_2(x)f_1(y)\ for\ all\ x, y \in S$

Hence $f_1(x)f_2(y)+f_2(x)f_1(y)=0$ for all $x, y \in S.$

Conversely assume that $f_1(x)f_2(y)+f_2(x)f_1(y)=0$ for all $x, y \in S.$ (1)

Replace y by yx

$$0=f_1(x)f_2(yx)+f_2(x)f_1(yx)\ for all\ x, y \in S$$
$$=f_1(x)(f_2(y)g_2(x)+yf_2(x))+f_2(x)(f_1(y)g_1(x)+yf_1(x))\ for all\ x, y \in S$$
$$=f_1(x)f_2(y)g_2(x)+f_1(x)yf_2(x)+f_2(x)f_1(y)g_1(x)+f_2(x)yf_1(x)\ for all\ x, y \in S$$

Since $g_1\ and\ g_2$ are surjective we have

$$0 = f_1(x)f_2(y)x + f_1(x)yf_2(x) + f_2(x)f_1(y)x + f_2(x)yf_1(x)\ for all\ x, y \in S$$
$$= 2f_1(x)yf_2(x)\ for all\ x, y \in S\ by (1)$$
$$= 2f_1(x)sf_2(x)\ for all\ x \in S$$

Since S is 2- torsion free $f_1(x)sf_2(x) = 0$ *for all* $x \in S.$

By lemma 4.3 $f_1(x)sf_2(y) = 0$ *for all* $x, y \in S.$

Hence by lemma 4.2 we have $f_1(x)sf_2(y) = 0 = f_2(y)sf_1(x)$ *for all* $x, y \in S.$

Thus $f_1 and\ f_2$ are orthogonal.

Theorem 4.5

Let S be a 2- torsion free semiprime semiring and let $f_1\ and\ f_2$ be semiderivations of S into S associated with functions $g_1 : S \to S\ and\ g_2 : S \to S$ respectively. Then $f_1\ and\ f_2$ are orthogonal if and only if $f_1 f_2 = 0 = f_2 f_1.$

Proof:

Suppose that $f_1 and\ f_2$ are orthogonal. Then $f_1(x)sf_2(y) = 0 = f_2(y)sf_1(x)$ *for all* $x, y \in S$

Then by lemma 4.2 $f_1(x)f_2(y) = 0$ *for all* $x, y \in S.$ (1)

$$ie, 0 = f_1(x)f_2(y)\ for all\ x, y \in S$$
$$= f_1(f_1(x)f_2(y))\ for all\ x, y \in S$$
$$= f_1(f_1(x))g_1(f_2(y)) + f_1(x)f_1(f_2(y))\ for all\ x, y \in S$$
$$= f_1(f_1(x))(f_2(y)) + f_1(x)f_1(f_2(y))\ for all\ x, y \in S,\ \text{since}\ g_1\ is\ surjective$$
$$= f_1(x)f_1(f_2(y))\ for all\ x, y \in S,\ by (1)$$

Premultiplying by $f_1 f_2(y)$

$$0 = f_1 f_2(y)f_1(x)f_1(f_2(y))\ for all\ x, y \in S$$
$$= f_1 f_2(y)sf_1(f_2(y))\ for all\ y \in S$$

By the semiprimeness of S $f_1 f_2(y) = 0$ *for all* $y \in S$

Hence $f_1 f_2 = 0.$ Similarly we can prove $f_2 f_1 = 0.$

Conversely assume $f_1 f_2 = 0 = f_2 f_1.$

$$0 = f_1 f_2(xy)\ for all\ x, y \in S$$
$$= f_1(f_2(x)g_2(y) + xf_2(y))\ for all\ x, y \in S$$
$$= f_1 f_2(x)g_1 g_2(y) + f_2(x)f_1 g_2(y) + f_1(x)g_1 f_2(y) + xf_1 f_2(y)\ for all\ x, y \in S$$
$$= f_2(x)f_1 g_2(y) + f_1(x)g_1 f_2(y)\ for all\ x, y \in S\ \text{since}\ f_1 f_2 = 0$$
$$= f_2(x)f_1(y) + f_1(x)f_2(y)\ for all\ x, y \in S,\ \text{since}\ g_1\ and\ g_2\ are\ surjective$$

Now by Theorem 4.4 $f_1\ and\ f_2$ are orthogonal.

Theorem 4.6

Let S be a 2- torsion free semiprime semiring and let $f_1\ and\ f_2$ be semiderivations of S into S associated with functions $g_1 : S \to S\ and\ g_2 : S \to S$ respectively. Then $f_1\ and\ f_2$ are orthogonal if and only if $f_1 f_2 + f_2 f_1 = 0$.

Proof:

Suppose that $f_1 and\ f_2$ are orthogonal. Then $f_1(x)sf_2(y) = 0 = f_2(y)sf_1(x)$ *for all* $x, y \in S$

Since $f_1 and\ f_2$ are orthogonal by Theorem 4.5 $f_1 f_2 = 0 = f_2 f_1.$

Hence $f_1 f_2 + f_2 f_1 = 0.$

Conversely assume $f_1 f_2 + f_2 f_1 = 0$

$$0=(f_1 f_2+f_2 f_1)(xy)\ for all\ x,y\in S$$
$$=f_1 f_2(xy)+f_2 f_1(xy)\ for all\ x,y\in S$$
$$=f_1(f_2(x)g_2(y)+xf_2(y))+f_2(f_1(x)g_1(y)+xf_1(y))\ for all\ x,y\in S$$
$$=f_1(f_2(x))g_1(g_2(y))+f_2(x)f_1(g_2(y))+f_1(x)g_1(f_2(y))+xf_1(f_2(y))$$
$$+f_2(f_1(x))g_2(g_1(y))+f_1(x)f_2(g_1(y))+f_2(x)g_2(f_1(y))+xf_2(f_1(y))\ for all\ x,y\in S$$
$$=f_1(f_2(x))y+f_2(x)f_1(y)+f_1(x)f_2(y)+xf_1(f_2(y))$$
$$+f_2(f_1(x))y+f_1(x)f_2(y)+f_2(x)f_1(y)+xf_2(f_1(y))\ for all\ x,y\in S, \text{since } g_1\ and\ g_2\ are surjective.$$
$$=2(f_1(x)f_2(y)+f_2(x)f_1(y))\ for all\ x,y\in S$$

Since is S is 2-torsion free $f_1(x)f_2(y)+f_2(x)f_1(y)=0\ for\ all\ x,y\in S$

Now by Theorem 4.4 $f_1 and\ f_2$ are orthogonal.

Theorem 4.7

Let S be a 2- torsion free semiprime semiring and let $f_1\ and\ f_2$ be semiderivations of S into S associated with functions $g_1:S\to S\ and\ g_2:S\to S$ respectively. Then $f_1\ and\ f_2$ are orthogonal if and only if $f_1 f_2\ or\ f_2 f_1$ is a semiderivation associated with the function $g_1 g_2:S\to S.$

Proof:

Suppose that $f_1 and\ f_2$ are orthogonal.

Then by Theorem 4.4 $f_1(x)f_2(y)+f_2(x)f_1(y)=0$ for all $x,y\in S.$ (1)

Also by Theorem 4.5 $f_1 f_2=0.$

$$ie, 0=f_1 f_2(xy)\ for all\ x,y\in S$$
$$=f_1(f_2(x)g_2(y)+xf_2(y))\ for all\ x,y\in S$$
$$=f_1(f_2(x))g_1(g_2(y))+f_2(x)f_1(g_2(y))+f_1(x)g_1(f_2(y))+xf_1(f_2(y))\ for all\ x,y\in S$$
$$=f_1 f_2(x)g_1g_2(y)+f_2(x)f_1(y)+f_1(x)f_2(y)+xf_1 f_2(y)\ for all\ x,y\in S\ \text{since } g_1\ and\ g_2\ are surjective$$
$$=f_1 f_2(x)g_1g_2(y)+xf_1 f_2(y)\ for all\ x,y\in S\ by\ (1)$$

Also $g_1g_2(f_1 f_2(x))=f_1 f_2(x)=f_1 f_2(g_1g_2(x)).$

Hence $f_1 f_2$ is a semiderivation associated with the function $g_1 g_2:S\to S.$

Similarly starting with $f_2 f_1=0$ we can prove $f_2 f_1$ is a semiderivation associated with the function $g_1 g_2:S\to S.$

Conversely assume that $f_1 f_2$ is a semiderivation associated with the function $g_1 g_2:S\to S.$

Then $f_1 f_2(xy)=f_1 f_2(x)g_1g_2(y)+xf_1 f_2(y)\ for\ all\ x,y\in S$ (2)

Also

$$f_1 f_2(xy)=f_1(f_2(x)g_2(y)+xf_2(y))\ for all\ x,y\in S$$
$$=f_1(f_2(x))g_1(g_2(y))+f_2(x)f_1(g_2(y))+f_1(x)g_1(f_2(y))+xf_1(f_2(y))\ for all\ x,y\in S$$
$$=f_1 f_2(x)g_1g_2(y)+f_2(x)f_1(y)+f_1(x)f_2(y)+xf_1 f_2(y)\ for all\ x,y\in S\ \text{since } g_1\ and\ g_2\ are surjective \quad (3)$$

Comparing (2) and (3) $f_1(x)f_2(y)+f_2(x)f_1(y)=0$ for all $x,y\in S.$

Then by Theorem 4.4 $f_1 and\ f_2$ are orthogonal.

Similarly we can prove if $f_2 f_1$ is a semiderivation associated with the function $g_1 g_2:S\to S$ then $f_1\ and\ f_2$ are orthogonal.

Corollary 4.8

Let S be a 2- torsion free semiprime semiring and let $f_1\ and\ f_2$ be semiderivations of S into S associated with functions $g_1:S\to S\ and\ g_2:S\to S$ respectively. If $f_1^2=f_2^2$ then $f_1+f_2\ and\ f_1-f_2$ are orthogonal.

Proof:

By hypothesis $f_1^{\,2} = f_2^{\,2}$. Hence $f_1^{\,2} - f_2^{\,2} = 0.$

Thus we have $(f_1 + f_2)(f_1 - f_2) = 0 = (f_1 - f_2)(f_1 + f_2)$

Hence by Theorem 4.5 we have $f_1 + f_2$ *and* $f_1 - f_2$ are orthogonal.

Theorem 4.9

Let S be a 2- torsion free semiprime semiring and let f_1 *and* f_2 be semiderivations of S into S associated with functions $g_1 : S \to S$ *and* $g_2 : S \to S$ respectively. Then f_1 *and* f_2 are orthogonal if and only if there exists $a, b \in S$ such that $f_1\ f_2(x) = ax + xb \text{ for all } x \in S.$

Proof:

Suppose that f_1 *and* f_2 are orthogonal.

Then by Theorem 4.5 $f_1 f_2 = 0.$

$ie,\ f_1 f_2(x) = 0.$

Choosing $a = b = 0$ we get $f_1 f_2(x) = ax + xb \text{ for all } x \in S$

Conversely assume that $f_1 f_2(x) = ax + xb \text{ for all } x \in S$

Replace x by xy

$f_1 f_2(xy) = axy + xyb \text{ for all } x, y \in S$

$f_1(f_2(x)g_2(y) + xf_2(y)) = axy + xyb \text{ for all } x, y \in S$

$f_1(f_2(x))g_1(g_2(y)) + f_2(x)f_1(g_2(y)) + f_1(x)g_1(f_2(y)) + xf_1(f_2(y)) = axy + xyb \text{ for all } x, y \in S$

$f_1 f_2(x)y + f_2(x)f_1(y) + f_1(x)f_2(y) + xf_1 f_2(y) = axy + xyb \text{ for all } x, y \in S \text{ since } g_1 \text{ and } g_2 \text{ are surjective}$

$(ax + xb)y + f_2(x)f_1(y) + f_1(x)f_2(y) + x(ay + yb) = axy + xyb \text{ for all } x, y \in S$

$xby + xay + f_2(x)f_1(y) + f_1(x)f_2(y) = 0 \text{ for all } x, y \in S$ (1)

Replace y by yx

$xbyx + xayx + f_2(x)f_1(yx) + f_1(x)f_2(yx) = 0 \text{ for all } x, y \in S$

$xbyx + xayx + f_2(x)f_1(y)g_1(x) + f_2(x)yf_1(x) + f_1(x)f_2(y)g_2(x) + f_1(x)yf_2(x) = 0 \text{ for all } x, y \in S$

$(xby + xay + f_2(x)f_1(y) + f_1(x)f_2(y))x + 2f_1(x)yf_2(x) = 0 \text{ for all } x, y \in S$

$2f_1(x)yf_2(x) = 0 \text{ for all } x, y \in S \text{ by } (1)$

$2f_1(x)sf_2(x) = 0 \text{ for all } x \in S$

Since S is 2-torsion free $f_1(x)sf_2(x) = 0 \text{ for all } x \in S$

Then by lemma 4.3 $f_1(x)sf_2(y) = 0 \text{ for all } x, y \in S$

$ie,\ f_1(x)sf_2(y) = 0 = f_2(y)sf_1(x) \text{ for all } x, y \in S$

Hence f_1 *and* f_2 are orthogonal.

Theorem 4.10

Let S be a 2- torsion free semiprime semiring and let f_1 *and* f_2 be semiderivations of S into S associated with functions $g_1 : S \to S$ *and* $g_2 : S \to S$ respectively such that $f_1 f_2$ is also a semiderivation associated with the function $g_1 g_2 : S \to S.$ Then either f_1 is zero or f_2 is zero

Proof:

By hypothesis $f_1 f_2$ is a semiderivation.

Then

$f_1 f_2(xy) = f_1 f_2(x)g_1 g_2(y) + xf_1 f_2(y) \text{ for all } x, y \in S$ (1)

Also

$f_1 f_2(xy) = f_1(f_2(x)g_2(y) + xf_2(y)) \text{ for all } x, y \in S$

$= f_1(f_2(x))g_1(g_2(y)) + f_2(x)f_1(g_2(y)) + f_1(x)g_1(f_2(y)) + xf_1(f_2(y)) \text{ for all } x, y \in S$

$= f_1 f_2(x)g_1 g_2(y) + f_2(x)f_1(y) + f_1(x)f_2(y) + xf_1 f_2(y) \text{ for all } x, y \in S \text{ since } g_1 \text{ and } g_2 \text{ are surjective}$ (2)

Comparing (1) and (2) $f_1(x)f_2(y)+f_2(x)f_1(y)=0$ for all $x, y \in S.$ (3)

Replace x by $xf_1(z)$

$f_1(xf_1(z))f_2(y)+f_2(xf_1(z))f_1(y)=0$ for all $x, y \in S.$

$ie, 0 = f_1(x)g_1(f_1(z))f_2(y)+xf_1(f_1(z))f_2(y)+f_2(x)g_2(f_1(z))f_1(y)+xf_2(f_1(z))f_1(y)\ forall\ x, y \in S$

$= f_1(x)g_1(f_1(z))f_2(y)+f_2(x)g_2(f_1(z))f_1(y)\ forall\ x, y \in S\ by$ (3)

$= f_1(x)f_1(z)f_2(y)+f_2(x)f_1(z)f_1(y)\ forall\ x, y \in S,\ \sin ce\ g_1\ and\ g_2\ are\ surjective$ (4)

In (3) replace x by z

$f_1(z)f_2(y)+f_2(z)f_1(y)=0\ forall\ y, z \in S$

$f_1(z)f_2(y)=-f_2(z)f_1(y)\ forall\ y, z \in S$

Hence (4) becomes

$-f_1(x)f_2(z)f_1(y)+f_2(x)f_1(z)f_1(y)=0\ forall\ x, y, z \in S$

$(f_2(x)f_1(z)-f_1(x)f_2(z))f_1(y)=0\ forall\ x, y, z \in S$

Now by lemma 4.1 either $f_2(x)f_1(z)-f_1(x)f_2(z)=0$ or $f_1(y)=0$

Let $f_2(x)f_1(z)-f_1(x)f_2(z)=0$ (5)

In (3) replace y by z

$f_1(x)f_2(z)+f_2(x)f_1(z)=0\ for\ all\ x, z \in S$ (6)

Adding (5) and (6)

$2f_2(x)f_1(z)=0\ for\ all\ x, z \in S$

Since S is 2-torsion free $f_2(x)f_1(z)=0\ for\ all\ x, z \in S$

Again by lemma 4.1 $f_2(x)=0\ or\ f_1(z)=0\ for\ all\ x, z \in S$

$ie,\ f_1=0\ or\ f_2=0.$

Corollary 4.11

Let S be a 2- torsion free semiprime semiring and let $f_1\ and\ f_2$ be semiderivations of S into S associated with functions $g_1 : S \to S\ and\ g_2 : S \to S$ respectively such that $f_1^{\ 2}$ is also a semiderivation associated with the function $g_1^{\ 2} : S \to S.$ Then f_1 is zero.

Proof:

In theorem 4.10 replacing f_2 by f_1 we get the result $f_1 = 0.$

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