IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 11, Issue 5 Ver. II (Sep. - Oct. 2015), PP 14-17
www.iosrjournals.org

Sugeno Integral Based On Some Inequalities

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Abstract: *In this paper we have to prove a general version of the chebyshev inequality for the Sugeno Integral type of intuitionistic fuzzy valued function with respect to intuitionistic fuzzy valued fuzzy measure and also present a Carlson type inequality for the generalized Sugeno Integral.*
Keywords: I*ntuitionistic fuzzy value, fuzzy measure, fuzzy Integral, Integral in equality, Sugeno Integral, Capacity, Carlson inequality, Shilkret Integral.*

I. Introduction

The theory of Fuzzy measure and fuzzy integrals was introduced by Sugeno(12) and intensively studied. Recently several classical inequalities were generalized to fuzzy integral. The series of paper on this topic was recently closed by Mesiar and ouyang(9) with a general version of chebyshev inequality for the sugeno integral on an abstract fuzzy measure space based on a product like operation the general result in (9) to obtain a general chebyshev type inequality for intuitionistic fuzzy valued fuzzy integrals of measurable intuitionistic fuzzy valued functions with respect to intuitionistic fuzzy valued fuzzy measures.

The fuzzy integral counterparts of several classical inequalities including Chebyshev's, Jensen's, Minkowski's and Holder inequalities are given by Flores Franulic and Roman Flores(7) Agahi et al., (6) L Wu et al(8) and others. The modified version of the Carlson inequality can be found in (11) and (3).

Preliminaries

We recall some basic definitions and previous results which will be used in the sequel.

Definition 1.1

Let$(X, \mathcal{A}, \mu)$ be a fuzzy measure space and $A \in \mathcal{A}$. The sugeno integral of $f \in \mathcal{A}_{+}^{\mu}(X)$ on A with respect to fuzzy measure μ is defined by

$$(S) \int_A f \, d\mu = \bigvee_{\alpha \geq 0} (\alpha \wedge \mu (A \cap F_\alpha))$$

where $F_\alpha = \{x \in X : f(x) \geq \alpha\}$, $\alpha \geq 0$

Definition 1.2

A mapping $: \mathcal{A} \to \mathcal{L}$ is called an intuitionistic fuzzy valued fuzzy measure or intuitionistic fuzzy measure in the following properties are satisfied:

(i) $\mathcal{v}(\emptyset) = (0, 1)$ and $\mathcal{v}(X) = (1,0)$

(ii) $A \subseteq B$ implies $\mathcal{v}(A) \subseteq_{\mathcal{L}} (B)$;

(iii) $A_1 \subseteq A_2 \subseteq . , . . .$ implies $\mathcal{v}\left(\bigcup_{n=1}^{\infty} A_n\right) = \lim_{n\to\infty} {}_{\mathcal{L}}\, \mathcal{v}(A_n)$;

(iv) $A_1 \supseteq A_2 \supseteq . . .$ implies $\mathcal{v}\left(\bigcap_{n=1}^{\infty} A_n\right) = \lim_{n\to\infty} {}_{\mathcal{L}}\, \mathcal{v}(A_n)$

The triple $(X, \mathcal{A}, \mathcal{v})$ is called an intuitionistic fuzzy measure space.

The Carlson inequality for the Lebesgue integral is of the form

$$\int_0^\infty f(x)\, dx \leq \sqrt{\pi} \left[\int_0^\infty f^2(x)\, dx\right]^{1/4} \left[\int_0^\infty x^2 f^2(x)\, dx\right]^{1/4} \quad (1)$$

Where f is any non-negative, measurable function such that the integrals on the right-hand side converge. The equality in (1) is attained iff $f(x) = \frac{\alpha}{\beta + x^2}$ for some constants $\alpha \geq 0$, $\beta > 0$.

The purpose of this paper is to study the Chebysheve inequality and Carlson inequality for the generalized Sugeno integrals. In Sec 2, we provide chebysheve's inequalities for the generalized Sugeno integral with respect to intuitionistic fuzzy measures. In Section 3 we present Carlson's inequality for the generalized Sugeno integrals.

II. Chebyshev Type Inequalities For Sugeno Integral With Respect To Intuitionistic Fuzzy Measures

The following results help us to reach the Chebyshev type inequalities for Sugeno integrals with respect to intuitionistic fuzzy measures.

Theorem: 2.1 (1,2)

Let : A → $\mathcal{L}$, $v = (v_1, v_2)$ be an intuitionistic fuzzy measure and f ∊ $A^{v}(X)$,
f (x) = (g (x) , h (x)).
Then

$$(S) \int_X^{\mathcal{L}} f \, dv = \left((S) \int_X g \, dv_1 , \; 1 - (S) \int_X h^c \, dv_2^c \right)$$

where $h^c(x) = 1 - h(x)$, x ∊ X, $v_2^c(A) = 1 - v_2(A)$, A∊$\mathcal{A}$

THEOREM 2.2

Let f,g ∊ $A_+^{\mu}(X)$ and μ be an arbitrary fuzzy measure such that both (S) $\int_X f \, d\mu$ and (S)$\int_X g \, d\mu$ are finite and let $* : [0,\infty)^2 \to [0,\infty)$ be continuous and non decreasing in both arguments and bounded from above by minimum.
If f, g are comontone, then the inequality

$$(S) \int_X f * g , d\mu \geq \left((S) \int_X f \, d\mu\right) * \left((S) \int_X g \, d\mu\right)$$

holds.

DEFINITION: 2.1

Two intuitionistic fuzzy valued functions f_1, f_2 : X → $\mathcal{L}$ is called comonotone if, for all x, y ∊ X,
$f_1(x) \leq_{\mathcal{L}} f_1(y)$ and $f_2(x) \leq_{\mathcal{L}} f_2(y)$
$f_1(x) \geq_{\mathcal{L}} f_1(y)$ and $f_2(x) \geq_{\mathcal{L}} f_2(y)$

EXAMPLE: 2.1

If v is an intuitionistic fuzzy measure and $f_1, f_2 \in \mathcal{A}^{v}(X)$ are comonotone then

$$(S) \int_X^{\mathcal{L}} f_1 \cdot f_2 \, dv \geq_{\mathcal{L}} (S)\int_X^{\mathcal{L}} f_1 \, dv \cdot (S) \int_X^{\mathcal{L}} f_2 \, dv$$

Where · is generated by triangular norm T(x,y) = xy, x,y ∊ [0,1]. If with the notations X = [0,1] , v is the Lebesgue measure on ℝ, $v_2 = 1- v_1$, g_1 and g_2 are both continuous strictly increasing or continuous strictly decreasing functions $h_1 = 1 - g_1$; $h_2 = 1 - g_2$ then the inequality becomes the Chebyshev inequality in the fuzzy case.
(See (5))

EXAMPLE: 2.2

If v is an intuitionistic fuzzy measure and $f_1; f_2 \in \mathcal{A}^{v}(X)$ are comonotone then

$$(S) \int_X^{\mathcal{L}} f_1 \wedge f_2 \, dv \geq_{\mathcal{L}} (S)\int_X^{\mathcal{L}} f_1 \, dv \wedge (S) \int_X^{\mathcal{L}} f_2 \, dv$$

where ∧ is generated as above by triangular norm T (x,y) = min(x,y), x,y ∊ [0,1]. Together with the monotonicity of Sugeno integral with respect to intuitionistic fuzzy measures. (See (2) Thm 3.45)
we obtain

$$(S) \int_X^{\mathcal{L}} (f_1 \wedge f_2) \, dv = (S) \int_X^{\mathcal{L}} f_1 \, dv \wedge (S) \int_X^{\mathcal{L}} f_2 \, dv$$

that is the property of comonotone minimitive property is valid for Sugeno integral with respect to intuitionistic fuzzy measures.

III. Carlson's Type Inequalities For Sugeno Integral

Let (X,$\mathcal{F}$) be a measurable space and μ: $\mathcal{F}$ → Y be a monotone measure, i.e., μ(∅) = 0, μ(X) > 0 and μ(A) ≤ μ(B) whenever A ⊂ B. Throughout this paper Y = [0, 1] or Y = [0,∞]. Suppose ∘: Y ×Y → Y is a non-decreasing operator, i.e.) a ∘ c > b ∘ d for a ≥ b and c ≥ d. An example of non-decreasing operators is a t-seminorm, also called a semicopula (4), (13).

For a measurable function h: X → Y, we define the generalized Sugeno integral of h on a set A ∈ $\mathcal{F}$ with respect to μ and a non-decreasing operator ∘: Y ×Y → Y as

$$\int_A h \circ \mu = \sup_{\alpha \in y} \{\alpha \circ \mu(A \cap \{h > \alpha\})\} \qquad (2)$$

where {h ≥ a} stands for {x ∈ X: h(x) ≥ a}. For ∘ = M, we get the Sugeno integral. If ∘ = ∏ then (2) is called shilkret Integral. We denoted integral the sugeno and the Shilkret Integral as (S) $\int f\ d\mu$ and (N) $\int f\ d\mu$ respectively.

THEOREM: 3.1

Suppose p, q ≥ 1 and r, s > 0. Then for arbitrary pairs of positively dependent functions f_{1A}, g_{1B} and f_{1A}, h_{1B}, the following inequality

$$\left(\left(\int_A f \circ \mu\right) \lozenge \left(\int_B g \circ \mu\right)\right)^r * \left(\left(\int_A f \circ \mu\right) \lozenge \left(\int_B h \circ \mu\right)\right)^s$$
$$\leq \left(\int_{A\cap B} (f \,\square\, g)^p \circ \mu\right)^{r/p} * \left(\int_{A\cap B} (f \square h)^q \circ \mu\right)^{s/q} \qquad (4)$$

is satisfied if for all a, b, c, d ∈ Y and s > 1,

$a^s \circ b \geq (a \circ b)^s$, $(a \,\square\, b) \circ (c \,\triangle\, d) \geq (a \circ c) \lozenge (b \circ d)$. (5)

Proof

Observe that all integrals in (4) are elements of Y . From the known inequality

$\int_A f^s \circ u \geq \left(\int_A f \circ \mu\right)^s$

It follows that

$$\int_{A\cap B} (f \,\square g) \circ \mu \leq \left(\int_{A\cap B} (f \square g)^{\,p} \circ \mu\right)^{1/p} \qquad (6)$$

$$\int_{A\cap B} (f \,\square\, h) \circ \mu \leq \left(\int_{A\cap B} (f \square h)^{\,q} \circ \mu\right)^{1/q} \qquad (7)$$

The operator * is non-decreasing so by (5) and (6)

$$\left(\int_{A\cap B} (f \square g) \circ\ \mu\right)^r * \left(\int_{A\cap B} (f \square h) \circ\ \mu\right)^s$$
$$\leq \left(\int_{A\cap B} (f \,\square\, g)^p \circ \mu\right)^{r/p} * \left(\int_{A\cap B} (f \square h)^q \circ \mu\right)^{s/q} \qquad (8)$$

From this inequality

$$\int_{A\cap B} (f_1 \,\square\, f_2) \circ \mu \geq \left(\int_A f_1 \circ\ \mu\right) \lozenge \left(\int_B f_2 \circ\ \mu\right)$$

We get

$$\int_{A\cap B} (f \,\square\, \psi) \circ \mu \geq \left(\int_A f\ \circ\ \mu\right) \lozenge \left(\int_B \psi\ \circ\ \mu\right)$$

for ψ = g,h (9)

From Theorem 3.1, one can obtain many other Carlson type inequalities since the conditions (5) are fulfilled by many systems of operators.

1. ∆ = ∧ and □ = ◊ = ∘, where ∘ is any t-norm satisfying the condition $(a^s \circ b) \geq (a \circ b)s$ for s ≥ 1 since a ∘ b ≤ a ∧ b and any t-norm is an associative and commutative operator;
2. ∆= □ = ∘ = ◊ = · on Y = [0, 1];
3. ∆= □ = ◊ = · and ∘ = ∧ with Y = [0, 1];
4. ∆= □ = ◊, ∘ = ∧ and Y = [0, 1];
5. ∆= □ = ◊ = ∘, where ∘ is any t-norm satisfying the condition $(a^s \circ b) > (a \circ b)^s$ for s ≥ 1,
   e.g. the Dombi t-norm a ∘ b = ab/(a + b − ab);
6. □ = ◊ , ∆ is any operator, a ∘ b = a for all a, b ∈ Y and Y = [0, 1] or Y = [0,∞];

Example: 3.1

The following inequality for the Shilkret integral of a non-decreasing function f is valid:

$(N) \int_A f\, d\mu \leq \frac{1}{\sqrt{k}} \left((N) \int_A f^2 d\mu\right)^{1/4} \left((N) \int_A x^2 f^2 d\mu\right)^{1/4}$

where K = μ(A) · $\left((N) \int_A x\, d\mu\right)$ to see this

put ∆ = ∧ or ∆ = · , g = 1, h = x,

◊ = * = □ = ∘ = ·, p = q = 2, r = s = 1

and A = B in Theorem 3.1.

Example 3.2

Let (X,$\mathcal{F}$, P) be a probability space. Put Y = [0, 1], r = s = 1, g =1, A = B = X, f = (U) and h = 1− (U), where U has the uniform distribution on [0, 1] and ϕ, ψ : [0, 1] → [0, 1] are decreasing functions. The functions f & h are not comonotone but

$P(f \geq a, h \geq b) = (\psi^{-1}(1 - b) - \phi^{-1}(a))_{+} = P(f \geq a) \circ_L P(h \geq b)$

so f and h are positively dependent with respect to P and $\circ_L$. The conditions are satisfied for $\triangle = \square = \lozenge = \circ_L$ and $* = \circ = \cdot$ (10 for 40) thus the corresponding Carlson inequality takes the form

$(N(f) \circ_L 1) \cdot (N(f) \circ_L N(h)) \leq (N(f^p))^{1/p} \, (N(f \circ_L h)^q)^{1/q}$

where N(f) = (N) $\int$ f dP.

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