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On Generalized (σ, σ)- n-Derivations in Prime Near–Rings

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Abstract: *In this paper, we investigate prime near – rings with generalized (σ, σ)- n-derivations satisfying certain differential identities . Consequently, some well known results have been generalized.*

Keywords: *prime near-ring, (σ,τ)- n-derivations, generalized (σ,τ)- n-derivations, generalized (σ,σ)-n-derivations*

I. Introduction

A right near – ring (resp. left near ring) is a set N together with two binary operations (+) and (.) such that (i) (N,+) is a group (not necessarily abelian). (ii) (N, .) is a semi group. (iii) For all a,b,c ϵ N ; we have (a + b).c = a.c + b.c (resp. a.(b + c) = a.b + b.c) . Trough this paper , N will be a zero symmetric left near – ring (i.e., a left near-ring N satisfying the property 0.x = 0 for all x ∈ N). we will denote the product of any two elements x and y in N , i.e.; x.y by xy. The symbol Z will denote the multiplicative centre of N, that is Z = {x ϵ N \ xy = yx for all y ϵ N}. For any x, y ∈ N the symbol [x, y] = xy - yx and (x , y) = x + y – x - y stand for multiplicative commutator and additive commutator of x and y respectively. Let σ and τ be two endomorphisms of N. For any x, y ∈ N, set the symbol $[x, y]_{\sigma,\tau}$ will denote xσ(y) - τ(y)x, while the symbol $(x \text{ o } y)_{\sigma,\tau}$ will denote xσ(y) + τ(y)x. N is called a prime near-ring if xNy = {0} implies that either x = 0 or y = 0. For terminologies concerning near-rings ,we refer to Pilz [1].

An additive mapping d :N ⟶N is called a derivation if d(xy) = d(x)y + xd(y), (or equivalently d(xy) = xd(y) + d(x)y for all x, y ϵ N, as noted in [2, Proposition 1]. The concept of derivation has been generalized in several ways by various authors. The notion of (σ,τ) derivation has been already introduced and studied by Ashraf [3]. An additive mapping d: N ⟶ N is said to be a (σ,τ) derivation if d(xy) = σ(x)d(y) + d(x)τ(y), (or equivalently d(xy) = d(x)τ(y) + σ(x)d(y) for all x, y ϵ N, as noted in [3, Lemma 2.1].

The notions of symmetric bi-(σ,τ) derivation and permuting tri-(σ,τ) derivation have already been introduced and studied in near-rings by Ceven [4] and Öztürk [5], respectively. Motivated by the concept of tri-derivation in rings, Park [6] introduced the notion of permuting n-derivation in rings. Further, the authors introduced and studied the notion of permuting n-derivation in near-rings (for reference see [7]). Inspired by these concepts, Ashraf [8] introduced (σ,τ)-n-derivation in near-rings and studied its various properties. In [9] Ashraf introduced the notion of generalized n-derivation in near-ring N and investigate several identities involving generalized n-derivations of a prime near-ring N which force N to be a commutative ring. In the present paper, motivated by these concepts, we define generalized (σ,τ)-n-derivation in near-rings and study commutativity of prime near-rings admitting suitably constrained additive mappings, as generalized n-derivation, generalized (σ, σ)-n-derivations.

Let n be a fixed positive integer. An n-additive (i.e.; additive in each argument) mapping $d:\underbrace{N\times N\times\ldots\times N}_{n-\text{times}}\longrightarrow N$ is called (σ,τ)-n-derivation of N if there exist functions σ, τ: N ⟶ N such that the equations

$$d(x_1x_1',x_2,\ldots,x_n)=d(x_1,x_2,\ldots,x_n)\sigma(x_1')+\tau(x_1)d(x_1',x_2,\ldots,x_n)$$
$$d(x_1,x_2x_2',\ldots,x_n)=d(x_1,x_2,\ldots,x_n)\sigma(x_2')+\tau(x_2)d(x_1,x_2',\ldots,x_n)$$
$$\vdots$$
$$d(x_1,x_2,\ldots,x_nx_n')=d(x_1,x_2,\ldots,x_n)\sigma(x_n')+\tau(x_n)d(x_1,x_2,\ldots,x_n')$$

hold for all $x_1,x_1',x_2,x_2',\ldots,x_n,x_n'\in N$

An n-additive mapping $f:\underbrace{N\times N\times\ldots\times N}_{n-\text{times}}\longrightarrow N$ is called a generalized (σ,τ)-n-derivation associated with (σ,τ)-n-derivation d if there exist functions σ, τ: N ⟶ N such that the equations

$$f(x_1x_1',x_2,\ldots,x_n)=d(x_1,x_2,\ldots,x_n)\sigma(x_1')+\tau(x_1)f(x_1',x_2,\ldots,x_n)$$
$$f(x_1,x_2x_2',\ldots,x_n)=d(x_1,x_2,\ldots,x_n)\sigma(x_2')+\tau(x_2)f(x_1,x_2',\ldots,x_n)$$
$$\vdots$$
$$f(x_1,x_2,\ldots,x_nx_n')=d(x_1,x_2,\ldots,x_n)\sigma(x_n')+\tau(x_n)f(x_1,x_2,\ldots,x_n')$$

hold for all $x_1,x_1',x_2,x_2',\ldots,x_n,x_n'\in N$.

For an example of a generalized (σ,τ)-n-derivation, Let S be a 2-torsion free zero-symmetric left near-ring. Let us define :

$N = \left\{\begin{pmatrix} x & y \\ 0 & 0 \end{pmatrix} \mid x, y, 0 \in S\right\}$. It can easily shown that N is a non commutative zero symmetric left near-ring with regard to matrix addition and matrix multiplication. Define d, f: $\underbrace{N \times N \times \ldots \times N}_{n-\text{times}} \longrightarrow N$ such that

$$d\left(\begin{pmatrix} x_1 & y_1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} x_2 & y_2 \\ 0 & 0 \end{pmatrix}, \ldots, \begin{pmatrix} x_n & y_n \\ 0 & 0 \end{pmatrix}\right) = \begin{pmatrix} 0 & x_1 x_2 \ldots x_n \\ 0 & 0 \end{pmatrix}$$

$$f\left(\begin{pmatrix} x_1 & y_1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} x_2 & y_2 \\ 0 & 0 \end{pmatrix}, \ldots, \begin{pmatrix} x_n & y_n \\ 0 & 0 \end{pmatrix}\right) = \begin{pmatrix} 0 & y_1 y_2 \ldots y_n \\ 0 & 0 \end{pmatrix}$$

Now we define $\sigma, \tau : N \rightarrow N$ by

$$\sigma\begin{pmatrix} x & y \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} -x & -y \\ 0 & y \end{pmatrix}, \tau\begin{pmatrix} x & y \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} x & -y \\ 0 & 0 \end{pmatrix}$$

It can be easily verified that f is a generalized (σ,τ)-n-derivation associated with (σ,τ)-n-derivation d. If f = d then generalized (σ,τ)-n-derivation f is just (σ,τ)-n-derivation. If σ = τ = 1, the identity map on N, then generalized (σ,τ)-n-derivation f is simply a generalized n-derivation. If σ = τ = 1 and d = f, then generalized (σ,τ)-n-derivation f is an n-derivation. Hence the class of generalized (σ,τ)-n-derivations includes those of n-derivations, generalized n-derivations and (σ,τ)-n-derivation. In this paper σ and τ will represent automorphisms of N.

II. Preliminary results.

We begin with the following lemmas which are essential for developing the proofs of our main results.

Lemma 2.1[8] Let N be a near-ring . Then d is a (σ,τ)-n-derivation of N if and only if

$$d(x_1 x_1', x_2, \ldots, x_n) = \tau(x_1)d(x_1', x_2, \ldots, x_n) + d(x_1, x_2, \ldots, x_n)\sigma(x_1')$$

$$d(x_1, x_2 x_2', \ldots, x_n) = \tau(x_2)d(x_1, x_2', \ldots, x_n) + d(x_1, x_2, \ldots, x_n)\sigma(x_2')$$

$$\vdots$$

$$d(x_1, x_2, \ldots, x_n x_n') = \tau(x_n)d(x_1, x_2, \ldots, x_n') + d(x_1, x_2, \ldots, x_n)\sigma(x_n')$$

hold for all $x_1, x_1', x_2, x_2', \ldots, x_n, x_n' \in N$.

Lemma 2.2 [8] Let N be a near-ring and d be a (σ,τ)-n-derivation of N. Then

$$(d(x_1, x_2, \ldots, x_n)\sigma(x_1') + \tau(x_1)d(x_1', x_2, \ldots, x_n))y = d(x_1, x_2, \ldots, x_n)\sigma(x_1')y + \tau(x_1)d(x_1', x_2, \ldots, x_n)y$$

$$(d(x_1, x_2, \ldots, x_n)\sigma(x_2') + \tau(x_2)d(x_1, x_2', \ldots, x_n))y = d(x_1, x_2, \ldots, x_n)\sigma(x_2')y + \tau(x_2)d(x_1, x_2', \ldots, x_n)y$$

$$\vdots$$

$$(d(x_1, x_2, \ldots, x_n)\sigma(x_n') + \tau(x_n)d(x_1, x_2, \ldots, x_n'))y = d(x_1, x_2, \ldots, x_n)\sigma(x_n')y + \tau(x_n)d(x_1, x_2, \ldots, x_n')y$$

hold for all $x_1, x_1', x_2, x_2', \ldots, x_n, x_n', y \in N$.

Lemma 2.3[8] Let N be a near-ring and d be a (σ,τ)-n-derivation of N . Then

$$(\tau(x_1)d(x_1', x_2, \ldots, x_n) + d(x_1, x_2, \ldots, x_n)\sigma(x_1'))y = \tau(x_1)d(x_1', x_2, \ldots, x_n)y + d(x_1, x_2, \ldots, x_n)\sigma(x_1')y$$

$$(\tau(x_2)d(x_1, x_2', \ldots, x_n) + d(x_1, x_2, \ldots, x_n)\sigma(x_2'))y = \tau(x_2)d(x_1, x_2', \ldots, x_n)y + d(x_1, x_2, \ldots, x_n)\sigma(x_2')y$$

$$\vdots$$

$$(\tau(x_n)d(x_1, x_2, \ldots, x_n') + d(x_1, x_2, \ldots, x_n)\sigma(x_n'))y = \tau(x_n)d(x_1, x_2, \ldots, x_n')y + d(x_1, x_2, \ldots, x_n)\sigma(x_n')y$$

hold for all $x_1, x_1', x_2, x_2', \ldots, x_n, x_n', y \in N$.

Lemma 2.4 [8] Let N be a prime near-ring and d a nonzero (σ,τ)-n-derivation d of N. If $d(N,N,\ldots,N) \subseteq Z$, then N is a commutative ring.

Lemma 2.5 Let d be a (σ, σ)-n-derivation of a near-ring N. Then $d(Z, N, \ldots, N) \subseteq Z$.

Proof. If $z \in Z$ then

$d(zx_1, x_2, \ldots, x_n) = d(x_1 z, x_2, \ldots, x_n)$ for all $x_1, x_2, \ldots, x_n \in N$.

Therefore, using defining property of d and Lemma 2.1 in previous equation, we get

$d(z, x_2, \ldots, x_n)\sigma(x_1) + \sigma(z)d(x_1, x_2, \ldots, x_n) = \sigma(x_1)d(z, x_2, \ldots, x_n) + d(x_1, x_2, \ldots, x_n)\sigma(z)$

for all $x_1, x_2, \ldots, x_n \in N$. Since $z \in Z$ and σ is an automorphism, we get

$d(z, x_2, \ldots, x_n)\sigma(x_1) = \sigma(x_1)d(z, x_2, \ldots, x_n)$ for all $x_1, x_2, \ldots, x_n \in N$. Thus we conclude that $d(Z, N, \ldots, N) \subseteq Z$.

Let N be a prime near-ring and d a nonzero (σ,τ)-n-derivation d of N. If $d(N,N,\ldots,N) \subseteq Z$, then N is a commutative ring.

Lemma 2.6 Let N be a near-ring. Then f is a generalized (σ,τ)-n-derivation of N if and only if

$$f(x_1 x_1', x_2, \ldots, x_n) = \tau(x_1)f(x_1', x_2, \ldots, x_n) + d(x_1, x_2, \ldots, x_n)\sigma(x_1')$$

$f(x_1, x_2x_2', \dots, x_n) = \tau(x_2)f(x_1, x_2', \dots, x_n) + d(x_1, x_2, \dots, x_n)\sigma(x_2')$

$\vdots$

$f(x_1, x_2, \dots, x_nx_n') = \tau(x_n)f(x_1, x_2, \dots, x_n') + d(x_1, x_2, \dots, x_n)\sigma(x_n')$

for all $x_1, x_1', x_2, x_2', \dots, x_n, x_n' \in N$.

Proof. By hypothesis, we get for all $x_1, x_1', x_2, \dots, x_n \in N$.

$f(x_1(x_1' + x_1'), x_2, \dots, x_n)$

$= d(x_1, x_2, \dots, x_n)\sigma(x_1' + x_1') + \tau(x_1)f(x_1' + x_1', x_2, \dots, x_n)$

$= d(x_1, x_2, \dots, x_n)\sigma(x_1') + d(x_1, x_2, \dots, x_n)\sigma(x_1') + \tau(x_1)f(x_1', x_2, \dots, x_n)$

$+ \tau(x_1)f(x_1', x_2, \dots, x_n)$ (1)

and

$f(x_1(x_1' + x_1'), x_2, \dots, x_n)$

$= f(x_1x_1' + x_1x_1', x_2, \dots, x_n)$

$= f(x_1x_1', x_2, \dots, x_n) + f(x_1x_1', x_2, \dots, x_n)$

$= d(x_1, x_2, \dots, x_n)\sigma(x_1') + \tau(x_1)f(x_1', x_2, \dots, x_n)$

$+ d(x_1, x_2, \dots, x_n)\sigma(x_1') + \tau(x_1)f(x_1', x_2, \dots, x_n)$ (2)

Comparing the two equations (1) and (2), we conclude that

$d(x_1, x_2, \dots, x_n)\sigma(x_1') + \tau(x_1)f(x_1', x_2, \dots, x_n) =$

$\tau(x_1)f(x_1', x_2, \dots, x_n) + d(x_1, x_2, \dots, x_n)\sigma(x_1')$ for all $x_1, x_1', x_2, \dots, x_n \in N$.

Similarly we can prove the remaining (n−1) relations. Converse can be proved in a similar manner.

III. Main results

Theorem 3.1 Let N be a prime near-ring, let f be a generalized (σ, σ)-n-derivation associated with a nonzero(σ,σ)-n-derivation d. If $f([x, y], x_2, \dots, x_n) = \sigma([x, y])$ for all $x, y, x_2, \dots, x_n \in N$. Then N is a commutative ring.

Proof. By our hypothesis, we have

$f([x,y], x_2, \dots, x_n) = \sigma([x,y])$ for all $x, y, x_2, \dots, x_n \in N$. (3)

Replace y by xy in (3) to get

$f([x, xy], x_2, \dots, x_n) = \sigma([x, xy])$ for all $x, y, x_2, \dots, x_n \in N$.

Which implies that

$f(x[x,y], x_2, \dots, x_n) = \sigma(x[x,y])$ for all $x, y, x_2, \dots, x_n \in N$.

Therefore

$d(x, x_2, \dots, x_n)\sigma([x, y]) + \sigma(x)f([x, y], x_2, \dots, x_n) = \sigma(x)\sigma([x, y])$ for all $x, y, x_2, \dots, x_n \in N$.

Using (3) in previous equation we get

$d(x, x_2, \dots, x_n)\sigma([x, y]) = 0$ for all $x, y, x_2, \dots, x_n \in N$, or equivalently,

$d(x, x_2, \dots, x_n)\sigma(x)\sigma(y) = d(x, x_2, \dots, x_n)\sigma(y)\sigma(x)$ for all $x, y, x_2, \dots, x_n \in N$. (4)

Replacing y by yz in (4) and using it again, we get

$d(x, x_2, \dots, x_n)\sigma(y)[\sigma(x), \sigma(z)] = 0$ for all $x, y, z, x_2, \dots, x_n \in N$.

Since σ is an automorphism of N, we get

$d(x, x_2, \dots, x_n)N[\sigma(x), \sigma(z)] = \{0\}$ for all $x, z, x_2, \dots, x_n \in N$. (5)

Primeness of N yields that for each fixed $x \in N$ either $d(x, x_2, \dots, x_n) = 0$ for all $x_2, \dots, x_n \in N$ or $x \in Z$. If $x \in Z$, by Lemma 2.5 we conclude that $d(x, x_2, \dots, x_n) \in Z$ for all $x_2, \dots, x_n \in N$. Therefore, in both cases we have $d(x, x_2, \dots, x_n) \in Z$ for all $x_2, \dots, x_n \in N$ and hence $d(N, N, \dots, N) \subseteq Z$. Thus by Lemma 2.4, we find that N is commutative ring.

Similar results hold in case $f([x, y], x_2, \dots, x_n) = -\sigma([x, y])$ for all $x, y, x_2, \dots, x_n \in N$.

Corollary 3.2 [14, Theorem 3.3] Let N be a prime near-ring, let f be a left generalized n-derivations with associated nonzero n-derivations d, If $f([x, y], x_2, \dots, x_n) = \pm [x, y]$ for all $x, y, x_2, \dots, x_n \in N$. Then N is a commutative ring.

Corollary 3.3 Let N be a prime near-ring, let d be a nonzero (σ, σ)-n-derivation d, If $d([x, y], x_2, \dots, x_n) = \pm \sigma([x, y])$ for all $x, y, x_2, \dots, x_n \in N$. Then N is a commutative ring.

Theorem 3.4 Let N be a prime near-ring, let f be a generalized (σ, σ)-n-derivation associated with a nonzero (σ, σ)-n-derivation d. If $f([x, y], x_2, \dots, x_n) = [\sigma(x), y]_{\sigma,\sigma}$ for all $x, y, x_2, \dots, x_n \in N$. Then N is a commutative ring.

Proof. By hypothesis, we have

$f([x, y], x_2, \dots, x_n) = [\sigma(x), y]_{\sigma,\sigma}$ for all $x, y, x_2, \dots, x_n \in N$. (6)

Replace y by xy in (6) to get

$f([x, xy], x_2, \dots, x_n) = [\sigma(x), xy]_{\sigma,\sigma}$ for all $x, y, x_2, \dots, x_n \in N$.

which implies that

$d(x, x_2, \dots, x_n)\sigma([x, y]) + \sigma(x)f([x, y], x_2, \dots, x_n) = \sigma(x)[\sigma(x), y]_{\sigma,\sigma}$

for all $x, y, x_2,\ldots,x_n \in N$.

Using hypothesis in previous equation we get

$d(x, x_2, \ldots, x_n)\sigma([x , y]) = 0$ for all $x, y, x_2,\ldots,x_n \in N$, or equivalently,

$d(x, x_2, \ldots, x_n)\sigma(x)\sigma(y) = d(x, x_2, \ldots, x_n)\sigma(y)\sigma(x)$ for all $x, y, x_2,\ldots,x_n \in N$. which is identical with the equation (4) in Theorem 3.1. Now arguing in the same way in the Theorem 3.1 we conclude that N is a commutative ring. Similar results hold in case $f([x, y], x_2, \ldots, x_n) = - [\sigma(x) , y]_{\sigma, \sigma}$ for all $x, y, x_2,\ldots,x_n \in N$.

Corollary 3.5 Let N be a prime near-ring, let d be a nonzero (σ, σ)-n-derivation. If $d([x, y], x_2, \ldots, x_n) = \pm [\sigma(x), y]_{\sigma, \sigma}$ for all $x, y, x_2,\ldots,x_n \in N$. Then N is a commutative ring.

Theorem 3.6 Let N be a prime near-ring, let f be a generalized (σ,σ)-n-derivation associated with a nonzero (σ,σ)-n-derivation d, If $f([x, y], x_2, \ldots, x_n) = (\sigma(x) \circ y)_{\sigma, \sigma}$ for all $x, y, x_2,\ldots,x_n \in N$. Then N is a commutative ring.

Proof. By hypothesis, we have

$f([x, y], x_2, \ldots, x_n) = (\sigma(x) \circ y)_{\sigma, \sigma}$ for all $x, y, x_2,\ldots,x_n \in N$. (7)

Replace y by xy in (7) to get

$f([x, xy], x_2, \ldots, x_n) = (\sigma(x) \circ xy)_{\sigma, \sigma}$ for all $x, y, x_2,\ldots,x_n \in N$.

which implies that

$d(x, x_2, \ldots, x_n)\sigma([x, y]) + \sigma(x)f([x, y], x_2, \ldots, x_n) = \sigma(x)(\sigma(x) \circ y)_{\sigma, \sigma}$
for all $x, y, x_2,\ldots,x_n \in N$.

Using hypothesis in previous equation we get

$d(x, x_2, \ldots, x_n)\sigma([x, y]) = 0$ for all $x, y, x_2,\ldots,x_n \in N$, or equivalently,

$d(x, x_2, \ldots, x_n)\sigma(x)\sigma(y) = d(x, x_2, \ldots, x_n)\sigma(y)\sigma(x)$ for all $x, y, x_2,\ldots,x_n \in N$. which is identical with the equation (4) in Theorem 3.1 Now arguing in the same way in the Theorem 3.1 we conclude that N is a commutative ring. Similar results hold in case $f([x, y], x_2, \ldots, x_n) = - (\sigma(x) \circ y)_{\sigma, \sigma}$ for all $x, y, x_2,\ldots,x_n \in N$.

Corollary 3.7 Let N be a prime near-ring, let f be a left generalized n-derivation associated with a nonzero n-derivation d. If $f([x, y], x_2, \ldots, x_n) = \pm (x\circ y)$ for all $x, y, x_2,\ldots,x_n \in N$. Then N is commutative ring.

Corollary 3.8 Let N be a prime near-ring, let d be a nonzero (σ, σ)-n-derivation. If $d([x, y], x_2, \ldots, x_n) = \pm (\sigma(x) \circ y)_{\sigma, \sigma}$ for all $x, y, x_2,\ldots,x_n \in N$. Then N is commutative ring.

Theorem 3.9 Let N be a prime near-ring, let f be a generalized (σ, σ)-n-derivation associated with a nonzero (σ, σ)-n-derivation d. If $f(x\circ y, x_2, \ldots, x_n) = (\sigma(x) \circ y)_{\sigma, \sigma}$ for all $x, y, x_2,\ldots,x_n \in N$. Then N is a commutative ring.

Proof. By hypothesis, we have

$f(x\circ y, x_2, \ldots, x_n) = (\sigma(x) \circ y)_{\sigma, \sigma}$ for all $x, y, x_2,\ldots,x_n \in N$. (8)

Replace y by xy in (8) to get

$f(x(x\circ y), x_2, \ldots, x_n) = (\sigma(x) \circ xy)_{\sigma, \sigma}$ for all $x, y, x_2,\ldots,x_n \in N$.

Which implies that

$d(x, x_2, \ldots, x_n)\sigma(x\circ y) + \sigma(x)f(x\circ y, x_2, \ldots, x_n) = \sigma(x)(\sigma(x) \circ y)_{\sigma, \sigma}$ for all $x, y, x_2,\ldots,x_n \in N$.

Using hypothesis in previous equation we get

$d(x, x_2, \ldots, x_n)\sigma(x\circ y) = 0$ for all $x, y, x_2,\ldots,x_n \in N$, or equivalently,

$d(x, x_2, \ldots, x_n)\sigma(x)\sigma(y) + d(x, x_2, \ldots, x_n)\sigma(y)\sigma(x) = 0$ for all $x, y, x_2,\ldots,x_n \in N$. (9)

Replacing y by yz in (9) and using it again, we get

$d(x, x_2, \ldots, x_n)\sigma(x)\sigma(y)\sigma(z) + d(x, x_2, \ldots, x_n)\sigma(y)\sigma(z)\sigma(x) = 0$ for all $x, y,z, x_2,\ldots,x_n \in N$.

Now substituting the values from (9) in the preceding equation we get

$\{- d(x, x_2, \ldots, x_n)\sigma(y)\sigma(x)\}\sigma(z) + d(x, x_2, \ldots, x_n)\sigma(y)\sigma(z)\sigma(x) = 0$ for all $x, y,z, x_2,\ldots,x_n \in N$.

So we get

$d(x, x_2, \ldots, x_n)\sigma(y)\sigma(- x)\sigma(z) + d(x, x_2, \ldots, x_n)\sigma(y)\sigma(z)\sigma(x) = 0$ for all $x, y,z, x_2,\ldots,x_n \in N$.

Replacing x by –x in the preceding equation we get

$d(-x, x_2, \ldots, x_n)\sigma(y)\sigma(x)\sigma(z) + d(-x, x_2, \ldots, x_n)\sigma(y)\sigma(z)\sigma(-x) = 0$ for all $x, y,z, x_2,\ldots,x_n \in N$. Thus we get

$d(-x, x_2, \ldots, x_n)\sigma(y)(\sigma(x)\sigma(z) - \sigma(z)\sigma(x)) = 0$ for all $x, y,z, x_2,\ldots,x_n \in N$. Since σ is an automorphism we conclude that

$d(-x, x_2, \ldots, x_n)N(\sigma(x)\sigma(z) - \sigma(z)\sigma(x)) = \{0\}$ for all $x, y,z, x_2,\ldots,x_n \in N$. For each fixed $x \in N$ primeness of N yields either $d(-x, x_2, \ldots, x_n) = 0$ for all $x_2,\ldots,x_n \in N$ or $x \in Z$. If $d(-x, x_2, \ldots, x_n) = 0$ for all $x_2,\ldots,x_n \in N$ then $d(x, x_2, \ldots, x_n) = 0$ for all $x_2,\ldots,x_n \in N$. Thus we conclude that for each fixed $x \in N$ either $d(x, x_2, \ldots, x_n) = 0$ for all $x_2,\ldots,x_n \in N$ or $x \in Z$. If $x \in Z$, by Lemma 2.5 we conclude that $d(x, x_2, \ldots, x_n) \in Z$ for all $x_2,\ldots,x_n \in N$. Therefore, in both cases we have $d(x, x_2, \ldots, x_n) \in Z$ for all $x_2,\ldots,x_n \in N$ and hence $d(N, N, \ldots, N) \subseteq Z$. Thus by Lemma 2.4, we find that N is a commutative ring.

Similar results hold in case $f(x\circ y, x_2, \ldots, x_n) = - (\sigma(x) \circ y)_{\sigma, \sigma}$ for all $x, y, x_2,\ldots,x_n \in N$.

Corollary 3.10 [14, Theorem 3.5] Let N be a prime near-ring, let f be a left generalized n-derivation associated with a nonzero n-derivation d, If $f(x\circ y, x_2, \ldots, x_n) = \pm (x\circ y)$ for all $x, y, x_2,\ldots,x_n \in N$. Then N is a commutative ring.

Corollary 3.11 Let N be a prime near-ring, let d be a nonzero (σ, σ)-n-derivation. If $d(x\circ y, x_2, \ldots, x_n) = \pm (\sigma(x) \circ y)_{\sigma, \sigma}$ for all $x, y, x_2,\ldots,x_n \in N$. Then N is a commutative ring.

Theorem 3.12 Let N be a prime near-ring, let f be a generalized (σ, σ)-n-derivation associated with a nonzero (σ, σ)-n-derivation d. If $f(x∘y, x_2, . . ., x_n) = [σ(x), y]_{σ, σ}$ for all $x, y, x_2,…,x_n ϵ N$. Then N is a commutative ring.

Proof. By hypothesis, we have

$f(x∘y, x_2, . . ., x_n) = [σ(x), y]_{σ, σ}$ for all $x, y, x_2,…,x_n ϵ N$. (10)

Replace y by xy in (10) to get

$f(x(x∘y), x_2, . . ., x_n) = [σ(x), xy]_{σ, σ}$ for all $x, y, x_2,…,x_n ϵ N$.

Which implies that

$d(x, x_2, . . ., x_n)σ(x∘y) + σ(x)f(x∘y, x_2, . . ., x_n) = σ(x)[σ(x), y]_{σ, σ}$ for all $x, y, x_2,…,x_n ϵ N$.

Using (10) in previous equation we get

$d(x, x_2, . . ., x_n)σ(x∘y) = 0$ for all $x, y, x_2,…,x_n ϵ N$, or equivalently,

$d(x, x_2, . . ., x_n)σ(x)σ(y) + d(x, x_2, . . ., x_n)σ(y)σ(x) = 0$ for all $x, y, x_2,…,x_n ϵ N$. which is identical with the relation (9) in Theorem 3.9. Now arguing in the same way in the Theorem 3.9, we conclude that N is a commutative ring.

Similar results hold in case $f(x∘y, x_2, . . ., x_n) = - [σ (x), y]_{σ, σ}$ for all $x, y, x_2,…,x_n ϵ N$.

Corollary 3.13 [14, Theorem 3.7] Let N be a prime near-ring, let f be a left generalized n-derivation associated with a nonzero n-derivation d. If $f(x∘y, x_2, . . ., x_n) = ± [x, y]$ for all $x, y, x_2,…,x_n ϵ N$. Then N is a commutative ring.

Corollary 3.14 Let N be a prime near-ring, let d be a nonzero (σ, σ)-n-derivation. If $d(x∘y, x_2, . . ., x_n) = [σ(x), y]_{σ, σ}$ for all $x, y, x_2,…,x_n ϵ N$. Then N is a commutative ring.

Theorem 3.15 Let N be a prime near-ring, let f be a generalized (σ, σ)-n-derivation associated with a nonzero (σ, σ)-n-derivation d, If $f([x, y], x_2, . . ., x_n) = σ(- xy + yx)$ for all $x, y, x_2,…,x_n ϵ N$. Then N is a commutative ring.

Proof. By hypothesis, we have

$f([x, y], x_2, . . ., x_n) = σ(- xy + yx)$ for all $x, y, x_2,…,x_n ϵ N$. (11)

Replace y by xy in (11) to get

$f([x, xy], x_2, . . ., x_n) = σ(- xxy + xyx)$ for all $x, y, x_2,…,x_n ϵ N$, which implies that

$f(x[x , y], x_2, . . ., x_n) = σ(x)σ(-xy + yx)$ for all $x, y, x_2,…,x_n ϵ N$.

$d(x, x_2, . . ., x_n)σ([x , y]) + σ(x)f([x , y], x_2, . . ., x_n) = σ(x)σ(-xy + yx)$ for all $x, y, x_2,…,x_n ϵ N$.

Using (11) in previous equation we get

$d(x, x_2, . . ., x_n)σ([x , y]) = 0$ for all $x, y, x_2,…,x_n ϵ N$, or equivalently,

$d(x, x_2, . . ., x_n)σ(x)σ(y) = d(x, x_2, . . ., x_n)σ(y)σ(x)$ for all $x, y, x_2,…,x_n ϵ N$. Now using again the same arguments as used after equation (4) in the last paragraph of the proof of Theorem 3.1, We conclude that N is a commutative ring.

Corollary 3.16 Let N be a prime near-ring, let f be a left generalized n-derivation associated with a nonzero n-derivation d , If $f([x , y], x_2, . . ., x_n) = - xy + yx$ for all $x, y, x_2,…,x_n ϵ N$. Then N is a commutative ring.

Corollary 3.17 Let N be a prime near-ring, let d be a nonzero (σ, σ)-n-derivation. If $d([x , y], x_2, . . ., x_n) = σ(-xy + yx)$ for all $x, y, x_2,…,x_n ϵ N$. Then N is a commutative ring.

The following example demonstrates that N to be prime is essential in the hypothesis of the previous theorems

Example 3.18 Let S be a 2-torsion free zero-symmetric left near-ring. Let us define :
$N = \left\{ \begin{pmatrix} 0 & x & y \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, x, y, 0 \in S \right\}$ is zero symmetric near-ring with regard to matrix addition and matrix multiplication .

Define f, d : $\underbrace{N \times N \times \ldots \times N}_{n-\text{times}} \longrightarrow N$ such that

$$f\left(\begin{pmatrix} 0 & x_1 & y_1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & x_2 & y_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \ldots, \begin{pmatrix} 0 & x_n & y_n \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right) = \begin{pmatrix} 0 & 0 & x_1 x_2 \ldots x_n \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$d\left(\begin{pmatrix} 0 & x_1 & y_1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & x_2 & y_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \ldots, \begin{pmatrix} 0 & x_n & y_n \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right) = \begin{pmatrix} 0 & 0 & y_1 y_2 \ldots y_n \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Now we define $\sigma : N \rightarrow N$ by $\sigma \begin{pmatrix} 0 & x & y \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & y & x \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

It can be easily seen that σ is an automorphisms of near-rings N which is not prime, having f is a nonzero generalized (σ,σ)-n-derivation associated with the (σ, σ)-n-derivation d. Further it can be easily also shown that

(i) $f([x, y], x_2, \ldots, x_n) = \sigma([x , y])$ for all $x, y, x_2,\ldots,x_n \in N$.
(ii) $f([x, y], x_2, \ldots, x_n) = \sigma(- xy + yx)$ for all $x, y, x_2,\ldots,x_n \in N$.
(iii) $f([x, y], x_2, \ldots, x_n) = [\sigma(x), y]_{\sigma, \sigma}$ for all $x, y, x_2,\ldots,x_n \in N$.
(iv) $f([x, y], x_2, \ldots, x_n) = (\sigma(x) \circ y)_{\sigma, \sigma}$ for all $x, y, x_2,\ldots,x_n \in N$.
(v) If $f(x\circ y, x_2, \ldots, x_n) = (\sigma(x) \circ y)_{\sigma, \sigma}$ for all $x, y, x_2,\ldots,x_n \in N$.
(vi) $f(x\circ y, x_2, \ldots, x_n) = [\sigma(x), y]_{\sigma, \sigma}$ for all $x, y, x_2,\ldots,x_n \in N$.
(vii) $f([x, y], x_2, \ldots, x_n) = \sigma(- xy + yx)$ for all $x, y, x_2,\ldots,x_n \in N$.
However N is not a ring.

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