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Structural Design Optimization using Generalized Fuzzy number

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Abstract : *This paper presents solution technique of geometric programming with fuzzy parameters to solve structural model. Here we are considered all fuzzy parameters as a generalized fuzzy number i.e. generalized triangular fuzzy number and generalized trapezoidal fuzzy number. Here material density of the bar, permissible stress of each bar and applied load are fuzzy numbers. We use geometric programming technique to solve structural problem. The structural problem whose aim is to minimize the weight of truss system subjected to the maximum permissible stress of each member. Decision maker can take the right decisions from the set of optimal solutions. Numerical examples are displayed to illustrate the model utilizing generalized fuzzy numbers.*
Keywords - *Structural optimization, Generalized fuzzy number, Geometric programming*

I. Introduction

Structural Optimization provides a means to help the structural engineer to achieve such an aim to find the best way to minimize the weight of structural design. This minimum weight design is subjected to various constraints on performance measures, such as stresses and displacements. Optimum shape design of structures is one of the challenging research areas of the structural optimization field. That is why the application of different optimization technique to structural problems has attracted the interest of many researchers. For example, artificial bee colony algorithm (Sonmez, M., [12]), particle swarm optimization (Luh et al., [11]), genetic algorithm (Dede et al., [10]), ant colony optimization (Kaveh et al., [9]) etc.

In general, structural optimization problem is solved with the assumption that the applied load, permissible stress of each members and thickness of the truss are specified in an exact mode. In real life, due to hesitation in judgments, lack of confirmation of otherwise. Sometimes it is not possible to get significant exact data for the structural system. This type of imprecise data is always well represented by fuzzy number, so fuzzy structural optimization model is needed in real life problem. Also making a decision, decision-makers have to review the alternatives with fuzzy numbers. It can be seen that fuzzy numbers have a very important role to describe fuzzy parameters in several fuzzy structural optimization model from the different view-points of decision makers. Zadeh [2] first introduced the concept of fuzzy set theory. Then Zimmermann [3] applied the fuzzy set theory concept with some suitable membership functions to solve linear programming problem with several objective functions. Some researchers applied the fuzzy set theory to Structural model. For example Wang et al. [1] first applied α -cut method to structural designs where the non-linear problems were solved with various design levels α , and then a sequence of solutions were obtained by setting different level-cut value of α . Rao [6] applied the same α -cut method to design a four–bar mechanism for function generating problem .Structural optimization with fuzzy parameters was developed by Yeh et al. [5]. In 1989, Xu [4] used two-phase method for fuzzy optimization of structures. In 2004, Shih et al.[7] used level-cut approach of the first and second kind for structural design optimization problems with fuzzy resources .Shih et al.[8] developed an alternative α -level cuts methods for optimum structural design with fuzzy resources in 2003.Dey and Roy [22] introduced fuzzy multi-objective mathematical programming technique based on generalized fuzzy set and they applied it in multi-objective structural models.

The non-linear optimization problems have been solved by various non-linear optimization techniques. Geometric Programming (GP) [14,16] is an effective method among those to solve a particular type of non-linear programming problem. Duffin, Peterson and Zener [16] laid the foundation stone to solve wide range of engineering problems by developing basic theories of geometric programming and its application in their text book. Chiang [26] used geometric programming in Communication Systems. One of the remarkable properties of Geometric programming is that a problem with highly nonlinear constraints can be stated equivalently with a dual program. If a primal problem is in posynomial form then a global minimizing solution of the problem can be obtained by solving its corresponding dual maximization problem because the dual constraints are linear, and linearly constrained programs are generally easier to solve than ones with nonlinear constraints. Cao [20] discussed fuzzy geometric programming (FGP) with zero degree of difficult. In 1987, Cao [24] first introduced FGP. There is a good book dealing with FGP by Cao [23]. Islam and Roy [27] used FGP to solve a fuzzy EOQ model with flexibility and reliability consideration and demand dependent unit production cost a space constraint. FGP method is rarely used to solve the structural optimization problem. But still there are enormous scopes to develop a fuzzy structural optimization model through fuzzy geometric programming (FGP). The parameter used in the GP problem may not be fixed. It is more fruitful to use fuzzy parameter instead of crisp parameter. Yang et al. [13] discussd about the basic and its applications of fuzzy geometric programming. Ojha et al.[18] used binary number for splitting the cost coefficients, constraints coefficient and exponents and then solved it by GP technique. A solution method of posynomial geometric programming with interval exponents and coefficients was developed by Liu (Liu, S.T., [19]). Nasseri et al.[21] solved two bar truss nonlinear problem by using geometric programming technique into the form of two-level mathematical programming.

In the present paper, we have considered the coefficients of the problem are generalized fuzzy number and solve it by fuzzy geometric programming technique.

The rest of this paper is organized in the following way. In section II, we discuss about structural optimization model. In section III and IV , we discuss about mathematical analysis and fuzzy mathematics prerequisites . In section V, different methods for defuzzification of fuzzy number are discussed. In section VI, we discuss about geometric programming technique with fuzzy coefficient. In section VII and VIII, crisp and fuzzy model of two bar truss are discussed and finally we apply geometric programming technique to solve two bar truss structural model respectively. In section IX, we discuss about an illustrative example. Finally we draw conclusions from the results in section X.

II. Structural Optimization Problem

In sizing optimization problems the aim is to minimize a single objective function, usually the weight of the structure, under certain behavioral constraints on stress and displacements. The design variables are most frequently chosen to be dimensions of the cross-sectional areas of the members of the structure. Due to fabrication limitations the design variables are not continuous but discrete since cross-sections belong to a certain set. A discrete structural optimization problem can be formulated in the following form

$$\begin{aligned} &Minimize \;\; f(A) \\ &subject\ to \;\; g_i(A) \le 0, \quad i = 1,2,\ldots,m. \\ &\qquad A_j \in R^d, \; j = 1,2,\ldots.,n. \end{aligned} \tag{1}$$

where $f(A)$ represents objective function, $g(A)$ is the behavioral constraint, m and n are the number of constraints and design variables, respectively. A given set of discrete values is expressed by R^d and design variables A_j can take values only from this set.

In this paper, objective function is taken as

$$f(A) = \sum_{i=1}^{m} \rho_i A_i l_i \tag{2}$$

and constraints are chosen to be stress of structures

$$g_i(A) = \frac{\sigma_i}{\sigma_i^0} - 1 \le 0 \;, \quad i = 1,2,\ldots,m \tag{3}$$

where ρ_i and l_i are weight of unit volume and length of i^{th} element, respectively, m is the number of the structural elements, σ_i and σ_i^0 are the i^{th} stress and allowable stress, respectively.

III. Mathematical Analysis

3.1 Geometric Programming

Geometric program (GP) can be considered to be an innovative modus operandi to solve a nonlinear problem in comparison with other nonlinear technique. It was originally developed to design engineering problems. It has become a very popular technique since its inception in solving nonlinear problems. The advantages of this method is that ,this technique provides us with a systematic approach for solving a class of nonlinear optimization problems by finding the optimal value of the objective function and then the optimal values of the design variables are derived. Also this method often reduces a complex nonlinear optimization problem to a set of simultaneous equations and this approach is more amenable to the digital computers.

GP is an optimization problem of the form:

$$Minimize \;\; g_0(x) \tag{4}$$

subject to

$$\begin{aligned} &g_j(x) \le 1, \quad j = 1,2,\ldots,m \\ &x_i > 0 \qquad i = 1,2,\ldots..,n \end{aligned}$$

where $g_j(x)$ $(j = 0,1,2,\ldots,m)$ are posynomial or signomial functions, x is decision variable vector of n components x_i $(i = 1,2,\ldots,n)$.

3.2. Geometric Programming Problem

$$Minimize\ g_0(x) \tag{5}$$

subject to $g_j(x) \le \delta_j b_j,\ (j = 1,2,\ldots\ldots,m)$

$x_i > 0,\ (i = 1,2,\ldots,n)$

where $g_j(x) = \sum_{k=1}^{N_j} \delta_{jk} c_{jk} \prod_{i=1}^{n} x_i^{\alpha_{jki}}$ $(j = 0,1,2,\ldots,m)$ g

$\delta_j = \pm 1$ $(j = 1,2,\ldots,m)$, $\delta_{jk} = \pm 1$ $\left(j = 0,1,2,\ldots,m;\ k = 1,2,\ldots,N_j\right)$, $x \equiv \left(x_1, x_2,\ldots\ldots,x_n\right)^T$

3.3 Dual Problem

The dual problem of the primal problem (5) is

$$Maximize\ d(w;\lambda) = \delta_0 \left(\prod_{j=0}^{m} \left(\frac{c_{jk} w_{j0}}{w_{jk}} \right)^{\alpha_{jk} w_{jk}} \right)^{\delta_0}$$

Subject to $\sum_{k=1}^{N_0} \sigma_{0k} w_{0k} = \delta_0$ (Normal condition)

$\sum_{j=0}^{m} \sum_{k=1}^{N_j} \delta_{jk} \alpha_{jki} w_{jk} = 0$ $(i = 1,2,\ldots,n)$ (Orthogonality condition).

where $\delta_j = \pm 1,\ (j = 1,2,\ldots,m)$, $\delta_{jk} = \pm 1$ $\left(j = 1,2,\ldots.,m; k = 1,2,\ldots,N_j\right)$ and $\delta_0 = +1,-1$ and non-negativity conditions, $w_{j0} = \delta_j \sum_{k=1}^{N_j} \delta_{jk} w_{jk} \ge 0,\ w_{jk} \ge 0,\ \left(j = 1,2,\ldots,m; k = 1,2,\ldots,N_j\right)$ and $w_{00} = 1$.

Case I: For $N \ge n+1$,the dual program presents a system of linear equations for the dual variables where the number of linear equations is either less than or equal to the number of dual variables. A solution vector exists for the dual variable (Beightler et al.,[15]).

Case II: For $N < n+1$,the dual program presents a system of linear equations for the dual variables where the number of linear equation is greater than the number of dual variables. In this case, generally, no solution vector exists for the dual variables. However, one can get an approximate solution vector for this system using either the least squares or the linear programming method.

IV. Fuzzy Mathematics Prerequisites

Fuzzy sets first introduced by Zadeh [2] in 1965 as a mathematical way of representing impreciseness or vagueness in everyday life.

Definition 4.1. Fuzzy Set

A fuzzy set $\tilde{A}$ in a universe of discourse X is defined as the following set of pairs $\tilde{A} = \left\{\left(x, \mu_{\tilde{A}}(x) / x \in X\right)\right\}$ .Here $\mu_{\tilde{A}} : X \to [0,1]$ is a mapping called the membership function of the fuzzy set $\tilde{A}$ and $\mu_{\tilde{A}}$ is called the membership value or degree of membership of $x \in X$ in the fuzzy set $\tilde{A}$. The larger $\mu_{\tilde{A}}(x)$ is the stronger the grade of membership form in $\tilde{A}$.

Definition 4.2. Fuzzy Number

A fuzzy number is a fuzzy set in the universe of discourse X. It is both convex and normal.

Definition 4.3. α -cut of a Fuzzy Number

The α -level of a fuzzy number $\tilde{A}$ is defined as a crisp set $A_\alpha = \left\{x : \mu_{\tilde{A}}(x) \ge \alpha, x \in X, \alpha \in [0,1]\right\}$. A_α is non-empty bounded closed interval contained in X and it can be denoted by $A_\alpha = [A_L(\alpha), A_R(\alpha)]$, $A_L(\alpha)$ and $A_R(\alpha)$ are the lower and upper bounds of the closed interval, respectively. Figure 1 shows a fuzzy number $\tilde{A}$ with α – cuts $A_{\alpha_1} = [A_L(\alpha_1), A_R(\alpha_1)]$, $A_{\alpha_2} = [A_L(\alpha_2), A_R(\alpha_2)]$. It is seen that if $\alpha_2 \geq \alpha_1$ then $A_L(\alpha_2) \geq A_L(\alpha_1)$ and $A_R(\alpha_2) \geq A_R(\alpha_1)$.

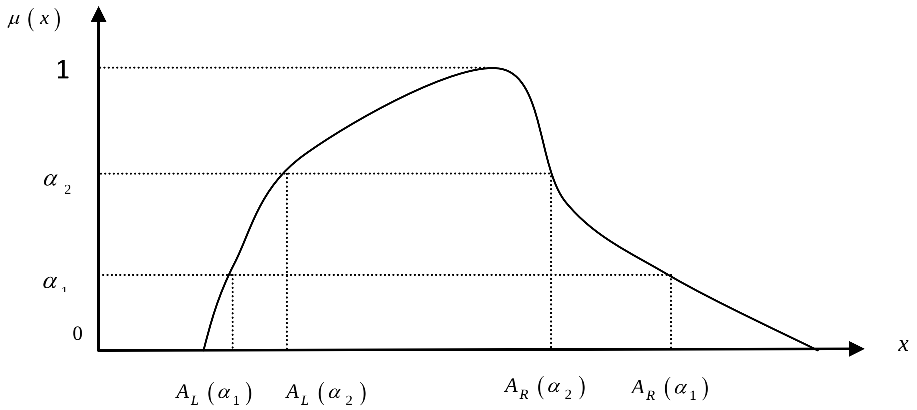

Fig. 1. Fuzzy number $\tilde{A}$ with α -cut

Definition 4.4. Convex fuzzy set

A fuzzy set $\tilde{A}$ of the universe of discourse X is convex if and only if for all x_1, x_2 in X,
$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda x_2) \geq \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)))$ when $0 \leq \lambda \leq 1$.

Definition 4.5. Normal fuzzy set

A fuzzy set $\tilde{A}$ of the universe of discourse X is called a normal fuzzy set implying that there exist at least one $x \in X$ such that $\mu_{\tilde{A}}(x) = 1$.

Definition 4.6. Generalized Fuzzy Number (GFN)

Generalized fuzzy number $\tilde{A}$ as $\tilde{A} = (a, b, c, d; w)$ where $0 < w \leq 1$ and a, b, c and d are real numbers. The generalized fuzzy number $\tilde{A}$ is a fuzzy subset of real line R, whose membership function $\mu_{\tilde{A}}(x)$ satisfies the following conditions:

(1) $\mu_{\tilde{A}}(x)$ is a continuous mapping from R to the closed interval $[0,1]$.

(2) $\mu_{\tilde{A}}(x) = 0$ where $-\infty < x \leq a$;

(3) $\mu_{\tilde{A}}(x)$ is strictly increasing with constant rate on $[a, b]$

(4) $\mu_{\tilde{A}}(x) = w$ where $b \leq x \leq c$;

(5) $\mu_{\tilde{A}}(x)$ is strictly decreasing with constant rate on $[c, d]$;

(6) $\mu_{\tilde{A}}(x) = 0$ where $d \leq x < \infty$.

Note: $\tilde{A}$ is a convex fuzzy set and it is a non-normalized fuzzy number till $w \neq 1$. It will be normalized for $w = 1$.

(i) If $a = b = c = d$ and $w = 1$, then $\tilde{A}$ is called a real number a.

Here $\tilde{A} = (x, \mu_{\tilde{A}}(x))$ with membership function $\mu_{\tilde{A}}(x) = \begin{cases} 1 & if \quad x = a \\ 0 & if \quad x \neq a \end{cases}$

(ii) If $a = b$ and $c = d$, then $\tilde{A}$ is called crisp interval $[a, b]$.

Here $\tilde{A} = (x, \mu_{\tilde{A}}(x))$ with membership function $\mu_{\tilde{A}}(x) = \begin{cases} 1 & if \quad a \le x \le d \\ 0 & otherwise \end{cases}$

(iii) If $b = c$, then $\tilde{A}$ is called a generalized triangular fuzzy number (GTFN) as $\tilde{A} = (a, b, c; w)$

(iv) If $b = c$, $w = 1$ then it is called a triangular fuzzy number (TFN) as $\tilde{A} = (a, b, c)$.

Here $\tilde{A} = (x, \mu_{\tilde{A}}(x))$ with membership function $\mu_{\tilde{A}}(x) = \begin{cases} w\left(\dfrac{x-a}{b-a}\right) & for \quad a \le x \le b \\ w\left(\dfrac{d-x}{d-b}\right) & for \quad b \le x \le d \\ 0 & otherwise \end{cases}$

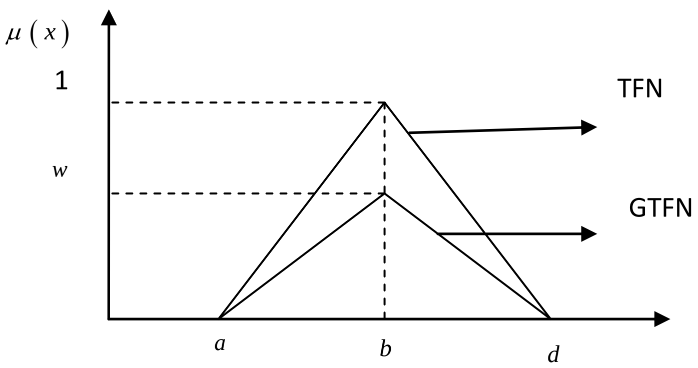

Fig. 2. TFN and GTFN

(v) If $b \ne c$, then $\tilde{A}$ is called a generalized trapezoidal fuzzy number (GTrFN) as $\tilde{A} = (a, b, c, d; w)$

(vi) If $b \ne c$, $w = 1$ then it is called a trapezoidal fuzzy number (TrFN) as $\tilde{A} = (a, b, c, d)$.

Here $\tilde{A} = (x, \mu_{\tilde{A}}(x))$ with membership function $\mu_{\tilde{A}}(x) = \begin{cases} w\left(\dfrac{x-a}{b-a}\right) & for \quad a \le x \le b \\ w & for \quad b \le x \le c \\ w\left(\dfrac{d-x}{d-c}\right) & for \quad c \le x \le d \\ 0 & otherwise \end{cases}$

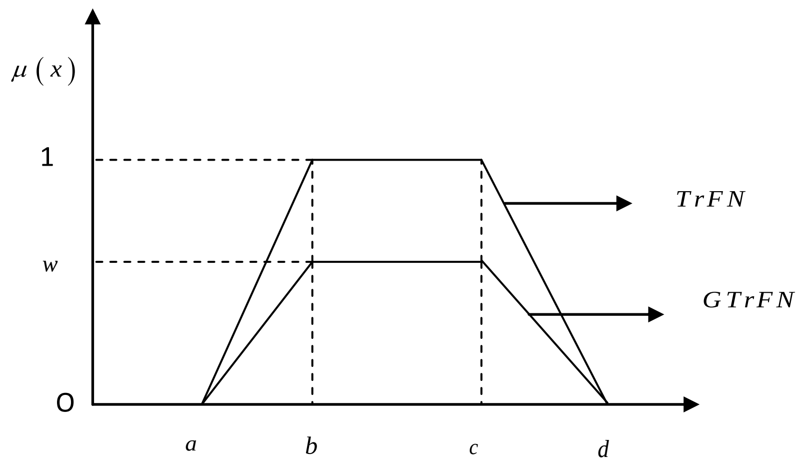

Fig. 3. TrFN and GTrFN

Fig. 3. shows GTrFNs $\tilde{A} \equiv (a,b,c,d;w)$ and TrFN $\tilde{A} \equiv (a,b,c,d)$ which indicate different decision maker's opinions for different values of w, $0 < w \le 1$.The values of w represents the degree of confidence of the opinion of the decision maker.

V. Different Methods for Defuzzification of Fuzzy Number

In real life, bulk of the information is assimilated as fuzzy numbers but there will be a need to defuzzify the fuzzy number. Actually defuzzification is the conversion of the fuzzy number to precise or crisp number. Several processes are used for such conversion. Here we have discussed four types of defuzzification method;

5.1. Type-I: Center of Mass (COM) Method

Let $\tilde{A}$ be a fuzzy number then the defuzzification of $\tilde{A}$ is given by $\hat{A} = \dfrac{\int_{a_l}^{a_u} x\mu_{\tilde{A}}(x)\,dx}{\int_{a_l}^{a_u} \mu_{\tilde{A}}(x)\,dx}$ where a_l and a_u are the lower and upper limits of the support of $\tilde{A}$.The value $\hat{A}$ represents the centroid of the fuzzy number $\tilde{A}$.

5.1.a. Defuzzification of $\tilde{A}_{GTFN} = (a,b,c;w)$ by COM method $\hat{A} = \frac{1}{3}(a+b+c)$

5.1.b. Defuzzification of $\tilde{A}_{GTrFN} = (a,b,c,d;w)$ by COM method $\hat{A} = \frac{1}{3}\dfrac{d^2+c^2-b^2-a^2+dc-ba}{d+c-b-a}$

Note: 5.1. For COM method, defuzzification of GTFN and GTrFN does not depend on w. In this case, defuzzification of generalized fuzzy number and normalized fuzzy number $(w=1)$ will be same.

5.2. Type-II: Mean of α -Cut (MC) Method

Let $\tilde{A}$ be a fuzzy number then the defuzzification of $\tilde{A}$ is given by $\hat{A} = \int_0^{\alpha_{max}} m\left[A_L(\alpha), A_R(\alpha)\right] d\alpha$ where α_{max} is the height of $\tilde{A}$, $A_\alpha = \left[A_L(\alpha), A_R(\alpha)\right]$ ia an $\alpha-$ cut, i.e. $m\left[A_L(\alpha), A_R(\alpha)\right] = \dfrac{A_L(\alpha)+A_R(\alpha)}{2}$ where $A_L(\alpha)$ and $A_R(\alpha)$ are the left and right limits of the $\alpha-$ cut of the a fuzzy number $\tilde{A}$.

5.2.a. Defuzzification of $\tilde{A}_{GTFN} = (a,b,c;w)$ by MC method $\hat{A} = \frac{w}{4}(a+2b+c)$.Here $A_L(\alpha) = a + \frac{\alpha}{w}(b-a)$ and $A_R(\alpha) = c - \frac{\alpha}{w}(c-b)$

5.2.b. Defuzzification of $\tilde{A}_{GTrFN} = (a,b,c,d;w)$ by MC method $\hat{A} = \frac{w}{4}(a+b+c+d)$.Here $A_L(\alpha) = a + \frac{\alpha}{w}(b-a)$ and $A_R(\alpha) = d - \frac{\alpha}{w}(d-c)$

Note: 5.2. For MC method, defuzzification of TFN and TrFN (normalized fuzzy number $(w=1)$ obtained by putting $w=1$ in the defuzzification rule of GTFN (5.2.a) and GTrFN (4.2.b) respectively.

5.3. Type III: Removal area (RA) method

According to Kaufmann and Gupta [25], let us consider an ordinary number $k \in \Re$ and a fuzzy number $\tilde{A}$. The left side removal of $\tilde{A}$ with respect to k, $R_l(\tilde{A},k)$,is defined as the area bounded by $x=k$ and the left side of the fuzzy number $\tilde{A}$. Similarly, the right side removal $R_r(\tilde{A},k)$ is defined. The removal of the fuzzy number $\tilde{A}$ with respect to $x=k$ is defined as the mean of $R_l(\tilde{A},k)$ and $R_r(\tilde{A},k)$. Thus $R(\tilde{A},k) = \frac{1}{2}\left(R_l(\tilde{A},k) + R_r(\tilde{A},k)\right)$.

5.3.a. Defuzzification of $\tilde{A}_{GTFN} = (a, b, c; w)$ by RA method, the removal number of $\tilde{A}$ with respect to origin is defined as the mean of two areas, $R_l(\tilde{A}, 0) = w\frac{a+b}{2}$ and $R_r(\tilde{A}, 0) = w\frac{b+c}{2}$, $\hat{A} = R(\tilde{A}, 0) = \frac{w}{4}(a + 2b + c)$.

5.3.b. Defuzzification of $\tilde{A}_{GTrFN} = (a, b, c, d; w)$ by RA method, the removal number of $\tilde{A}$ with respect to origin is defined as the mean of two areas, $R_l(\tilde{A}, 0) = w\frac{a+b}{2}$ and $R_r(\tilde{A}, 0) = w\frac{c+d}{2}$, $\hat{A} = R(\tilde{A}, 0) = \frac{w}{4}(a + b + c + d)$.

Note:5.3. For RA method, defuzzification of TFN and TrFN are obtained by putting $w = 1$ in the defuzzification rule of GTFN (4.3.a), GTrFN (5.3.b) respectively.

Note: 5.4. Defuzzification of GTFN and GTrFN by type-II and type-III method are same but these are different with type-I.

5.4. Mean of Expected Interval (MEI) Method

The α level set of $\tilde{A}$ is defined as $A_\alpha = [A_L(\alpha), A_R(\alpha)]$. According to Heilpern [28], the expected interval of fuzz number $\tilde{A}$,denoted as $EI(\tilde{A})$ is $EI(\tilde{A}) = \left[\int_0^1 A_L(\alpha)\,d\alpha, \int_0^1 A_R(\alpha)\,d\alpha\right]$. The approximated value of $\tilde{A}$ is given by $\tilde{A}_{MEI} = \frac{1}{2}\left[\int_0^1 A_L(\alpha)\,d\alpha + \int_0^1 A_R(\alpha)\,d\alpha\right]$.

5.4.a. Defuzzification of $\tilde{A}_{GTFN} = (a, b, c; w)$ by MEI method $\hat{A} = \frac{1}{2}\left[(a + c) - \frac{1}{2w}(a - 2b + c)\right]$.Here $A_L(\alpha) = a + \frac{\alpha}{w}(b - a)$ and $A_R(\alpha) = c - \frac{\alpha}{w}(c - b)$

5.4.b. Defuzzification of $\tilde{A}_{GTrFN} = (a, b, c, d; w)$ by MEI method $\hat{A} = \frac{1}{2}\left[(a + d) - \frac{1}{2w}(a - b - c + d)\right]$.Here $A_L(\alpha) = a + \frac{\alpha}{w}(b - a)$ and $A_R(\alpha) = d - \frac{\alpha}{w}(d - c)$

Note:4.6. For MEI method, defuzzification of TFN and TrFN are obtained by putting $w = 1$ in the defuzzification rule of GTFN (4.4.a), GTrFN (5.4.b) respectively.

VI. Geometric Programming With Fuzzy Coefficient

When all coefficients of (5) are generalized fuzzy number, then the geometric programming problem is of the form

$$Minimize\ \tilde{g}_0(x) = \sum_{k=1}^{T_0} \delta_{0k} \tilde{c}_{0k} \prod_{i=1}^{n} x_i^{\alpha_{0ki}}$$

$$\text{subject to } \tilde{g}_j(x) = \sum_{k=1}^{N_j} \delta_{jk} \tilde{c}_{jk} \prod_{i=1}^{n} x_i^{\alpha_{jki}} \le \delta_j \tilde{b}_j \text{ for } j = 1, 2, 3, \ldots, m. \quad (6)$$

$$x_i > 0 \text{ for } i = 1, 2, .., n.$$

where $\tilde{c}_{0k}$, $\tilde{c}_{jk}$ and $\tilde{b}_j$ are generalized fuzzy number.

Using different difuzzification methods, we transform all generalized fuzzy number into crisp number i.e. $\hat{c}_{0k}$, $\hat{c}_{jk}$ and $\hat{b}_j$.

The geometric programming problem with imprecise parameters is of the following form

$$Minimize\ \tilde{g}_0(x) = \sum_{k=1}^{T_0} \delta_{0k} \hat{c}_{0k} \prod_{i=1}^{n} x_i^{\alpha_{0ki}}$$

subject to $\tilde{g}_j(x) = \sum_{k=1}^{N_j} \delta_{jk} \hat{c}_{jk} \prod_{i=1}^{n} x_i^{\alpha_{jki}} \le \delta_j \hat{b}_j$ for $j = 1, 2, 3, \ldots, m.$ (7)

$x_i > 0$ for $i = 1, 2, .., n.$

This is a geometric programming problem.

VII. Two Bar Truss Structural Model

Two bar truss model is developed and work out under the following notations.

7.1. Notation

We define the following variables and parameters;
2P = applied load;
t = thickness of the bar;
d = mean diameter of the bar (decision variable);
2b= the distance between two hinged point.
WT= weight of the structure;
h = the perpendicular distance from applied load point to the base line (decision variable);
y = depends on b and h (decision variable);

7.2. Crisp Structural Model

The symmetric two-bar truss shown in Figure 4 has been studied by several researchers like [17,21]. Here we consider same model. The objective is to minimize the weight of truss system subject to the maximum permissible stress in each member is σ_0. There are two design variables- mean tube diameter (d) and height (h) of the truss.

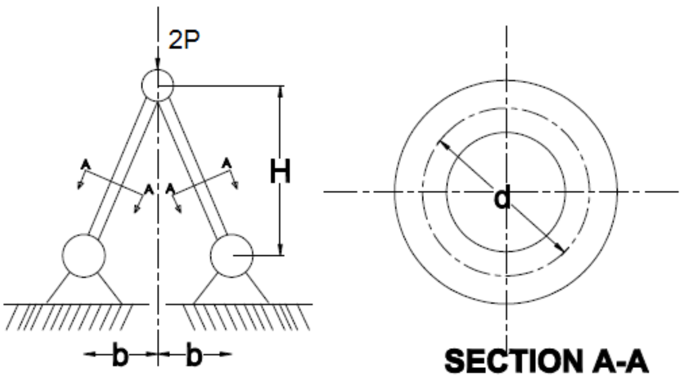

Fig. 4.Two bar truss under load

The weight of the structure is $\rho\left(2d\pi t\sqrt{b^2+h^2}\right)$ and stress is $\frac{\left(P\sqrt{b^2+h^2}\right)}{d\pi th}$.

The structural model can be written as

$$\begin{aligned} &Minimize\ WT(d,h) = \rho\left(2d\pi t\sqrt{b^2+h^2}\right) \\ &Subject\ to\ \sigma(d,h) \equiv \frac{P\sqrt{b^2+h^2}}{d\pi th} \le \sigma_0; \\ &d, h > 0; \end{aligned} \tag{8}$$

Let $\sqrt{b^2+h^2} = y \Rightarrow b^2+h^2 = y^2$. Hence the new constraint is $b^2+h^2 \le y^2 \Rightarrow b^2y^{-2} + h^2y^{-2} \le 1$.

Hence the structural model is

$$\begin{aligned} &Minimize\ \ WT(d,h,y) = 2\rho t d\pi y \\ &Subject\ \ to\ \ \sigma(d,h,y) \equiv \frac{Pyh^{-1}}{d\pi t} \le \sigma_0; \\ &\qquad b^2y^{-2} + h^2y^{-2} \le 1; \\ &\qquad d,h,y > 0; \end{aligned} \tag{9}$$

The above problem (9) can be treated as a Posynomial Geometric Programming problem with zero Degree of Difficulty.

7.3. Fuzzy Structural Model

The objective as well as constraint goal can involve many uncertain factors in a structural optimization problem. Therefore the structural optimization model can be represented in fuzzy environment to make the model more flexible and adoptable to the human decision process. If the coefficient of objective function and constraint goal of (9) are fuzzy in nature .Then the crisp model (9) is transformed into fuzzy model as follows

$$\begin{aligned} &Minimize\ \ WT(d,h,y) = 2\tilde{\rho}\, t\, d\,\pi\, y \\ &Subject\ \ to\ \ \frac{\tilde{P}\ y\ h^{-1}}{d\ \pi\ t} \le \tilde{\sigma}_0; \\ &\qquad b^2y^{-2} + h^2y^{-2} \le 1; \\ &\qquad d,h,y > 0; \end{aligned} \tag{10}$$

where $\tilde{P}, \tilde{\rho}$ and $\tilde{\sigma}_0$ are fuzzy in nature.

VIII. Solution Procedure of Fuzzy Structural Model throgh Geometric Programming

After defuzzificationof the fuzzy parameters, the fuzzy two bar truss structural model (10) reduces as

$$\begin{aligned} &Minimize\ \ WT(d,h,y) = 2\hat{\rho} d\pi yt \\ &Subject\ \ to\ \ \frac{\hat{P}yh^{-1}}{d\pi t\hat{\sigma}_0} \le 1; \\ &\qquad b^2y^{-2} + h^2y^{-2} \le 1; \\ &\qquad d,h,y > 0; \end{aligned} \tag{11}$$

Applying Geometric Programming Technique, the dual programming of the problem (11) is

$$\max g(w) = \left(\frac{2\pi\hat{\rho}t}{w_{01}}\right)^{w_{01}} \left(\frac{\hat{P}}{\pi t\hat{\sigma}_0}\right)^{w_{11}} \left(\frac{b^2}{w_{21}}\right)^{w_{21}} \left(\frac{1}{w_{22}}\right)^{w_{22}} (w_{21}+w_{22})^{(w_{21}+w_{22})} \tag{12}$$

$subject\ to\ \ w_{01} = 1$ (Normality condition)

For primal variable y : $1.w_{01} + w_{11} + (-2).w_{21} + (-2).w_{22} = 0$ (orthogonal condition)

For primal variable h : $0.w_{01} + (-1).w_{11} + 0.w_{21} + 2.w_{22} = 0$ (orthogonal condition)

For primal variable d : $1.w_{01} + (-1).w_{11} + 0.w_{21} + 0.w_{22} = 0$ (orthogonal condition)

$$w_{01}, w_{11}, w_{21}, w_{22} > 0$$

This is a system of four linear equation with four unknowns. Solving we get the optimal values as follows

$$w^*_{01} = 1, w^*_{11} = 1, w^*_{21} = 0.5\ \ and\ \ w^*_{22} = 0.5$$

From primal dual relation we get

$$2\hat{\rho} d\pi yt = w_{01} g^*(w)$$

$$\frac{\hat{P}}{\pi t\hat{\sigma}_0} yd^{-1}h^{-1} = \frac{w_{11}}{w_{11}}$$

$$b^2y^{-2} = \frac{w_{21}}{w_{21}+w_{22}} \text{ and } h^2y^{-2} = \frac{w_{22}}{w_{21}+w_{22}}$$

So the dual objective value is given by

$$g^*(w) = \left(\frac{2\pi\hat{\rho}t}{w_{01}}\right)^{w_{01}} \left(\frac{\hat{P}}{\pi t\hat{\sigma}_0}\right)^{w_{11}} \left(\frac{b^2}{w_{21}}\right)^{w_{21}} \left(\frac{1}{w_{22}}\right)^{w_{22}} (w_{21}+w_{22})^{(w_{21}+w_{22})}$$

$$y^* = \sqrt{\frac{b^2(w_{21}+w_{22})}{w_{21}}} \quad , \quad h^* = \sqrt{\frac{b^2 w_{22}}{w_{21}}}$$

$$d^* = \frac{\tilde{P}}{\pi t \tilde{\sigma}_0} \times \sqrt{\frac{b^2(w_{21}+w_{22})}{w_{21}}} \times \sqrt{\frac{w_{21}}{b^2 w_{22}}}$$

IX. Numerical Expose

We assume that material density of the bar, permissible stress of each bar and applied load are fuzzy in nature. We take two types of fuzzy generalized, GTFN, GTrFN as input data instead of crisp coefficient.

Table-1: Input data for fuzzy model (10) as TFN

$\tilde{P}$	$\tilde{\rho}$	$\tilde{\sigma}_0$
$(32000, 33000, 33750 : w)$	$(0.2, 0.3, 0.5; w)$	$(60000, 61000, 61750; w)$

Table-2: Input data for fuzzy model (10) as TrFN

$\tilde{P}$	$\tilde{\rho}$	$\tilde{\sigma}_0$
$(32000, 32600, 33250, 33750 : w)$	$(0.1, 0.2, 0.4, 0.55; w)$	$(58500, 59000, 60750, 61500; w)$

For COM defuzzification rule is not considered for different values of w (only $w = 1$ is considered) for numerical result of different types of generalized fuzzy number which are exhibited in table-3, table-4, table-5 and table-6.

Table 3. Optimal solution of Two Bar Truss Structural Model (10) by GP method when input data are GTFN

Weights	WT^*(lbs)	Diameter d^*(in)	Height h^*(in)	y^*(in)	Defuzzificztion Type
$w = 1$	21.39818441	2.431487813	30	42.42640687	Type-I
$w = 0.2$	4.21614703	2.432268425	30	42.42640687	Type-II&III
	22.36176586	2.430502722	30	42.42640687	Type-IV
$w = 0.5$	10.54007352	2.432200569	30	42.42640687	Type-II&III
	21.88113042	2.431114268	30	42.42640687	Type-IV
$w = 0.8$	16.86400000	2.432183604	30	42.42640687	Type-II&III
	21.40057448	2.431759397	30	42.42640687	Type-IV
$w = 1$	21.08014703	2.432200569	30	42.42640687	Type-II&III
	21.08014703	2.432200569	30	42.42640687	Type-IV

The table (3) gives the result of optimum weight for two bar truss using generalized triangular fuzzy number by defuzzification rule of COM method, MC method, RA method and MEI method. For MC method and RA method outcome are same.

Table 4. Optimal solution of Two Bar Truss Structural Model (10) by NLP method when input data are GTFN

Weights	WT^*(lbs)	Diameter d^*(in)	Height h^*(in)	y^*(in)	Defuzzificztion Type
$w = 1$	21.39818	2.431488	30	42.42641	Type-I
$w = 0.2$	4.216147	2.432268	30	42.42641	Type-II&III
	22.36177	2.430503	30	42.42641	Type-IV
$w = 0.5$	10.54007	2.432201	30	42.42641	Type-II&III
	21.88114	2.431114	30	42.42641	Type-IV
$w = 0.8$	16.86400	2.432184	30	42.42641	Type-II&III
	21.40057	2.431759	30	42.42641	Type-IV
$w = 1$	21.08015	2.432201	30	42.42641	Type-II&III
	21.08015	2.432201	30	42.42641	Type-IV

Table (4) displayed the result of two bar truss model by non-linear programming by Lingo software taking GTFN . It is notice that GP method gives batter result for some case otherwise almost same.

Table 5. Optimal solution of Two Bar Truss Structural Model (10) by GP method when input data are GTrFN

Weights	WT^* (lbs)	Diameter d^* (in)	Height h^* (in)	y^* (in)	Defuzzificztion Type
$w = 1$	20.68491787	2.469420081	30	42.42640687	Type-I
$w = 0.2$	4.114900358	2.468814258	30	42.42640687	Type-II&III
	21.21842370	2.466367467	30	42.42640687	Type-IV
$w = 0.5$	10.29105767	2.468937777	30	42.42640687	Type-II&III
	20.98022858	2.467749166	30	42.42640687	Type-IV
$w = 0.8$	16.46028766	2.468917190	30	42.42640687	Type-II&III
	20.74113428	2.469056683	30	42.42640687	Type-IV
$w = 1$	20.57518954	2.468896603	30	42.42640687	Type-II&III
	20.58376989	2.469926374	30	42.42640687	Type-IV

The table (5) gives the result of optimum weight for two bar truss using generalized trapezoidal fuzzy number by defuzzification rule of COM method, MC method, RA method and MEI method. For MC method and RA method outcome are same.

Table 6. Optimal solution of Two Bar Truss Structural Model (10) by NLP method when input data are GTFN

Weights	WT^* (lbs)	Diameter d^* (in)	Height h^* (in)	y^* (in)	Defuzzificztion Type
$w = 1$	20.68492	2.469420	30	42.42641	Type-I
$w = 0.2$	4.114900	2.468814	30	42.42641	Type-II&III
	21.21184	2.466367	30	42.42641	Type-IV
$w = 0.5$	10.29106	2.468938	30	42.42641	Type-II&III
	20.98023	2.467749	30	42.42641	Type-IV
$w = 0.8$	16.46029	2.468917	30	42.42641	Type-II&III
	20.74113	2.469057	30	42.42641	Type-IV
$w = 1$	20.57519	2.468897	30	42.42641	Type-II&III
	20.58377	2.469926	30	42.42641	Type-IV

Table (6) displayed the result of two bar truss model by non-linear programming by Lingo software taking GTrFN . It is notice that GP method gives batter result for some case otherwise almost same.

X. Conclusion

We have considered two bar truss structural model whose aim is to minimize the weight of truss system subjected to the maximum permissible stress of each member. We use geometric programming technique to solve structural problem with fuzzy coefficients. Here material density of the bar, permissible stress of each bar and applied load are generalized fuzzy numbers. In many situations, problem parameters are more competent to take as GFN for real life examples. Hence this work gives more significant for structural engineer for decision-making. This technique can be applied to solve the different decision making problems in other engineering and management sciences with different types of fuzzy number.

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