

On Q^*g closed sets in Supra Topological Spaces

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Abstract: The aim of this paper is to introduce and study some properties of supra topological spaces. We introduce the concepts of supra Q^*g closed sets, supra Q^*g open sets, supra Q^* closed sets and supra Q^* open sets.

Keywords : supra Q^*g closed sets, supra Q^*g open sets, supra Q^* closed sets, supra Q^* open sets.

I. Introduction

Levin[2] introduced generalized closed set in topological spaces in the year 1970. In 1983, Mashhour et al[3] introduced supra topological spaces. Closed sets are fundamental objects in a topological space. The notion of Q^* closed sets in a topological spaces was introduced by Murugalingam and lalitha[4] in 2010. In this paper introduced Q^*g closed sets in supra topological space. P.Padma and S.Udayakumar[6] introduced Q^*g closed sets in topological space in the year 2015.

II. Preliminaries

Definition 2.1 :[2] A topological Space (X, τ) is said to be generalized closed (briefly g -closed) set if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open (X, τ) .

Definition 2.2 :[8] A topological space (X, τ) is said to be generalized star closed (briefly g^* - closed) set if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g -open (X, τ) .

Definition 2.3 :[7] A topological space (X, τ) is said to be generalized star star closed (briefly g^{**} -closed) set if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g^* -open (X, τ) .

Definition 2.4 :[2] Let X be a non empty set. The subfamily $\mu \subseteq \mathcal{P}(X)$ where $\mathcal{P}(X)$ is the power set of X is said to be a supra topology on X if $X \in \mu$ and μ is closed under arbitrary unions.

The pair (X, μ) is called a supra topological space. The elements of μ are said to be supra open in (X, μ) . Complements of supra open sets are called supra closed sets.

Definition 2.5 :[9] Let A be a subset of (X, μ) . Then the supra closure of A is denoted by $cl^\mu(A) = \bigcap \{ B / B \text{ is a supra closed set and } A \subseteq B \}$.

Definition 2.6 :[9] Let A be a subset of (X, μ) . Then the supra interior of A is denoted by $int^\mu(A) = \bigcup \{ B / B \text{ is a supra open set and } A \supseteq B \}$.

Definition 2.7 :[1] Let (X, μ) be a topological space and μ be a supra topology on X . μ is supra topology associated with τ if $\tau \subseteq \mu$.

III. On Q^*G Closed Sets In Supra Topological Spaces

Definition 3.1 : A subset A of a supra topological space (X, μ) is called

- (1) a supra Q^* - closed if $int^\mu(A) = \emptyset$ and A is closed.
- (2) a supra Q^* - open if $cl^\mu(A) = X$ and A is open.

Definition 3.2 : A subset A of supra topological space (X, μ) is called a supra Q^*g closed if $cl^\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is Q^* - open in (X, μ) . The complement of a supra Q^*g closed set is called supra Q^*g open set.

Theorem 3.3 : Every supra closed set is supra Q^*g closed.

Proof : Let $A \subseteq X$ be a supra closed set and $A \subseteq U$ and U is supra Q^* open.
 Since A is supra closed, $cl^\mu(A) = A$ and hence $cl^\mu(A) = U$.
 Therefore A is supra Q^*g closed.

Converse part : Every supra Q^*g closed set is not supra closed.

Proof :

Let A be supra Q^*g closed set .since $cl^\mu(A) \subseteq U$. whenever $A \subseteq U$.
 And U is Q^* open in (X, μ) . Since the elements of μ are called supra open in (X, μ) .
 Therefore every supra Q^*g closed set is not supra closed.

Remark 3.3: The converse of the theorem is not true as shown in the following example.

Example 3.4 : Let $X = \{ a,b,c,d,e \}$, $\mu = \{ \emptyset, X, \{a,b\}, \{a,b,d\}, \{b,c,d\}, \{c,d,e\} \}$
 $A = \{ a,b,c \}$ is supra Q^*g closed but not supra closed.

Corollary 3.5: Every closed set is supra Q^*g closed.

Proof : Every closed set is supra closed. By the theorem “Every Supra closed set is supra Q^*g closed”.

Theorem 3.6: A subset A of X is supra Q^*g closed if and only if $cl^\mu(A) \setminus A$ contains no non empty supra Q^* -closed set.

Proof :

Necessity : Let F be a supra Q^* closed set of $cl^\mu(A) \setminus A$ that is $F \subseteq cl^\mu(A) \setminus A$.
 Now $A \subseteq F^c$. Where F^c is supra Q^* open.
 Since A is supra Q^*g closed. $cl^\mu(A) \subseteq F^c \Rightarrow F \subseteq [cl^\mu(A)]^c$.
 Therefore $F \subseteq cl^\mu(A) \cap [cl^\mu(A)]^c = \emptyset$.
 Hence F^c is supra Q^* open, $cl^\mu(A) = X$ and A is open.
 Therefore $F^c \neq \emptyset$.

Sufficiency : Suppose $A \subseteq U$ and U is supra Q^* open.

Suppose $cl^\mu(A) \not\subseteq U$. Then $cl^\mu(A) \cap U^c$ is supra Q^* closed subset of $cl^\mu(A) \setminus A$.
 Hence $cl^\mu(A) \cap U^c = \emptyset$. And hence $cl^\mu(A) \subseteq U$.
 Therefore A is supra Q^*g closed.

Corollary 3.7 : A supra Q^*g closed set A of X is supra closed if and only if $cl^\mu(A) \setminus A$ is supra Q^* closed.

Proof :

The supra Q^*g closed set A is supra closed.
 Then $cl^\mu(A) = A$. and hence $cl^\mu(A) \setminus A = \emptyset$ is supra Q^* closed.
 Conversely, Suppose that $cl^\mu(A) \setminus A$ is supra Q^* -closed.
 Since A is supra Q^*g - closed $cl^\mu(A) \setminus A$ does not contain a non empty Q^* closed set.
 But since $cl^\mu(A) \setminus A$ is itself supra Q^* -closed,
 $cl^\mu(A) \setminus A = \emptyset$. Which implies $cl^\mu(A) = A$.
 Therefore A is supra closed.

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