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An Inventory Model with Weibull Distribution Deterioration under Selling Price Demand Rate Using Partial Backlogging

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Abstract: *Weibull distribution is a life data analysis, have been developed for failure rate, it is often used to analyse the field or test failure data to understand how items are failing. In this paper, we have studied an inventory model for two parameter Weibull distribution deterioration with selling price demand rate, in which shortages are allowed and are partially backlogged*

Keywords: *Deterioration Products, Partial Backlogging, Selling price demand, Shortage, Time varying Holding Cost, Weibull Distribution.*

I. Introduction

Inventory management deals essentially with balancing the inventory levels. The challenge in any inventory control problem is to determine the quantity of products to be ordered and the moment for placing the order, which both affect the amount of the costs. The general solution to the inventory control problem cannot be obtained on the basis of one model. Therefore, most diverse models, describing different particular cases have been developed.

One of the determinatives in the development of an inventory management model is pattern on demand. Inventory is categorized into two types based on the demand pattern, which creates the need for inventory. The two types of demand are independent demand and dependent demand for inventories. An inventory of an item is said to be falling into the category of independent demand for such an item is not dependent upon the demand for another item. Finished goods items, which are ordered by external customers or manufactured for stock and sale, are called independent demand items. Independent demands for inventories are based on confirmed customer orders, forecasts, estimates and past historical data. If the demand for inventory of an item is dependent upon another item, such demands are categorized as dependent demand. Raw materials and component inventories are dependent upon the demand for finished goods are hence can be called as dependent demand inventories

The assumption of constant, linear, exponential demand rate is not always applicable to many inventory products like vegetables, food stuffs and fashionable clothes electronic rate. Introducing new items with fascinate more in demand during the growth phase of their life cycles. It is evident that, some items may decline due to the introduction of new products due to the choice or influencing customers. So the develop deteriorating inventory models with selling price demand problem is worth attempt. The equation for inventory model for deteriorating products is $\frac{dI(t)}{dt} + \theta I(t) = -f(t)$ where θ is the constant decay rate, I (t) the inventory level at time t, and f (t) the Selling price demand rate at time t.

In the classical inventory model the demand rate may be time dependent, price dependent and stock dependent. Ajantha Roy developed an inventory model where demand rate is a function of selling price. Vipin Kumar, S.R.Singh, Sanjay Sharma has developed a production inventory model for linear time. The demand is which implies a uniform change in the demand rate of the product per unit time. Time proportional demand was developed by Dave and Patel. S.K.Ghosh and K.S.Chaudhuri had discussed Quadratic time demand in their inventory model. Similarly the constant deterioration was considered by Zhao Pei-xin S.K.Ghosh, K.S.Chaudhuri, Azizul Baten and Anton Abdulbasah Kamil etc.

In general holding cost is assumed to be known and constant. But in realistic holding cost may not always be constant. Many researchers like C.K.Tripathy U.Mishra,, V.K.Mishra and L.S.Singh, Ajantha Roy etc have discussed with time dependent holding cost.

Inventory model with variable as backlogging rate was developed by V.K.Mishra and L.S.Singh. Liang-Yuh OUYAHG, K.S.W.U, M.C. Chany G.P Samantha has developed an inventory model with partial backlogging rate. R.Amutha was developed the inventory model with weibull distribution deterioration and time-varying demand.

In this paper, we have developed an inventory model for deteriorating with two parameter Weibull distribution, selling price demand and time dependent holding cost. Shortages are allowed and are partially backlogged.

II. Materials And Methods

Assumptions and Notations

- The inventory system deals with single item
- The lead time is zero
- Shortages are allowed and are partially backlogged. During stock out period, the backlogging rate is variable and is dependent on the length of the waiting time for next replenishment. So that the backlogging rate for negative inventory is B(t)=$e^{-\lambda(T-t)}$,λ is backlogging parameter and (T-t) is waiting time (t_1≤t≤T)
- Ɵ be the deterioration rate. It follows the two parameter Weibull distribution $\theta(t)=\alpha\beta\, t^{\beta-1}$ where 0<α<1 and β>0
- Holding cost h(t) per item-unit is time dependent and is assumed to be h(t)=c+dt when c>0,d>0
- T is the length of the cycle
- Replenishment is instantaneous at an infinite rate
- The demand rate is $D(t)=a-p$
- The planning horizon is finite
- A,C_1, C_2, C_3&C_4 denote the set up cost, inventory carrying cost, deterioration cost per unit time, shortage cost for backlogged items and the unit cost of lost sales respectively. All of the cost parameters are positive constants.
- Deteriorating takes places after the life time of items.
- At time t_1 the inventory level reaches zero.

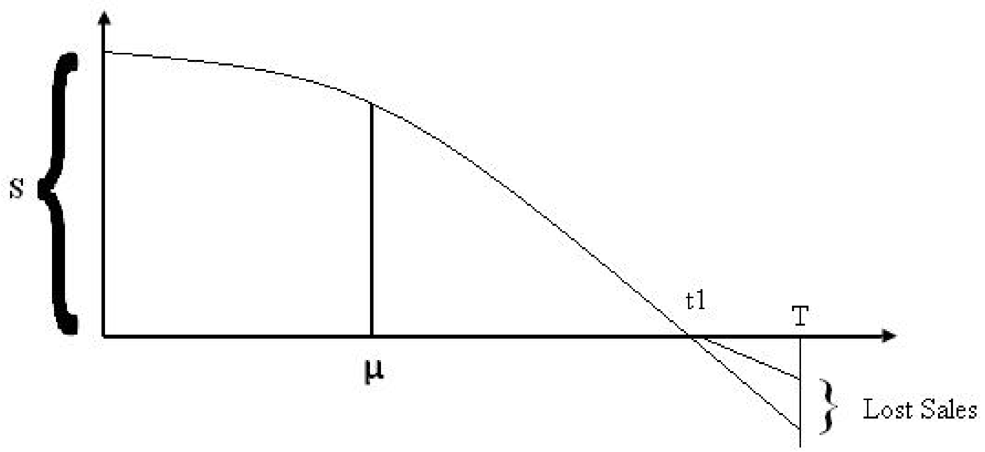

Fig.1 Graphical Representation of Inventory System

Mathematical model

During the period $(0,\mu)$ the inventory level is decreasing and at time t_1 the inventory reaches zero level, where the shortage starts, and in the period (t_1, T) some demands are backlogged.

The rate of change of inventory during, positive stock period (0,t_1) is governed by the differential equations.

$$\frac{dI(t)}{dt}=-(a-p), \qquad 0\le t\le\mu \tag{1}$$

$$\frac{dI(t)}{dt}+\alpha\beta t^{\beta-1}I(t)=-(a-p), \qquad \mu\le t\le t_1 \tag{2}$$

$$\frac{dI(t)}{dt}=-(a-p)e^{-\lambda(T-t)}, \qquad t_1\le t\le T \tag{3}$$

With boundary conditions $I(0)=S,\ I(t_1)=0$

Solving the equations, (1), (2) and (3) and neglecting higher powers of t, we get

$$I(t) = S-(a-p)t \qquad 0 \le t \le \mu \tag{4}$$

$$I(t) = (a-p)\{(t_1-t)+\frac{\alpha\beta}{\beta+1}(t_1^{\beta+1}-t^{\beta+1})\} \qquad \mu \le t \le t_1 \tag{5}$$

$$I(t) = (a-p)\{(t_1-t)-\lambda T(t_1-t)+\frac{\lambda(t_1^2-t^2)}{2}\} \qquad t_1 \le t \le T \tag{6}$$

From the equations (4) and (5) after replacing $t = \mu$, we get

$$I(\mu) = S-(a-p)\mu \tag{7}$$

$$I(\mu) = (a-p)(t_1-\mu)+\frac{\alpha\beta}{\beta+1}(t_1^{\beta+1}-\mu^{\beta+1}) \tag{8}$$

Equating (7) and (8)

$$S = (a-p)\mu+(a-p)t_1-(a-p)\mu+(a-p)\frac{\alpha\beta}{\beta+1}(t_1^{\beta+1}-\mu^{\beta+1})$$

$$S = (a-p)(t_1+\frac{\alpha\beta}{\beta+1}(t_1^{\beta+1}-\mu^{\beta+1})) \tag{9}$$

Using (9) in (4)

$$I(t) = S-(a-p)t$$

$$I(t) = (a-p)[(t_1-t)+\frac{\alpha\beta}{\beta+1}(t_1^{\beta+1}-\mu^{\beta+1})] \tag{10}$$

Total amount of lost sales I_L, during the period $(0,T)$ is

$$I_L = \int_{t_1}^{T}(1-e^{-\lambda(T-t)})(a-p)dt$$

$$I_L = (a-p)(\frac{\lambda T^2}{2}-\lambda Tt_1+\frac{\lambda t_1^2}{2}) \tag{11}$$

Total amount of shortage units I_s during the period $(0,T)$ is

$$I_S = -\int_{t_1}^{T} I(t)dt$$

$$I_S = (a-p)(\frac{T^2}{2}-Tt_1+\lambda T^2 t_1-\lambda Tt_1^2-\frac{\lambda T^3}{3}+\frac{t_1^2}{2}+\frac{\lambda t_1^3}{3}) \tag{12}$$

Total amount of deterioration items I_D, during the period $(0,T)$ is

$$I_D = \int_{\mu}^{t_1}\alpha\beta t^{\beta-1}(a-p)[(t_1-t)+\frac{\alpha\beta}{\beta+1}(t_1^{\beta+1}-t^{\beta+1})]$$

$$I_D = (a-p)(\frac{\alpha t_1^{\beta+1}}{(\beta+1)}+\frac{\alpha^2\beta t_1^{2\beta+1}}{(2\beta+1)}-\alpha t_1\mu^{\beta}+\frac{\alpha\beta\mu^{\beta+1}}{(\beta+1)}-$$
$$\frac{\alpha^2\beta^2 t_1^{\beta+1}\mu^{\beta}}{(\beta+1)}+\frac{\alpha^2\beta^2\mu^{2\beta+1}}{(\beta+1)(2\beta+1)}) \tag{13}$$

During period $(0,T)$ total number of units holding I_H is

$$I_H = \int_0^{\mu} (c+dt)I(t)dt + \int_{\mu}^{t_1} (c+dt)I(t)dt$$

$$= \int_0^{\mu} (c+dt)\{(a-p)[(t_1-t)+\frac{\alpha\beta}{\beta+1}(t_1^{\beta+1}-\mu^{\beta+1})]\} + \int_{\mu}^{t_1} (c+dt)\{(a-p)$$

$$[(t_1-t)+\frac{\alpha\beta}{\beta+1}(t_1^{\beta+1}-t^{\beta+1})]\}$$

$$I_H = (a-p)[\frac{ct_1^2}{2}+\frac{dt_1^3}{6}+\frac{\alpha\beta ct_1^{\beta+2}}{(\beta+2)}+\frac{\alpha\beta dt_1^{\beta+3}}{2(\beta+3)}-\frac{\alpha\beta c\mu^{\beta+2}}{\beta+2}-\frac{\alpha\beta d\mu^{\beta+3}}{2(\beta+3)}] \quad (14)$$

Therefore total unit cost per unit time is given by

TC = $\frac{1}{T}$ [ordering cost + carrying cost + backordering cost + lost sale cost + purchase cost]

$$TC = \frac{1}{T}[A + C_1 I_H + C_2 I_D + C_3 I_S + C_4 I_L]$$

$$= \frac{1}{T}[A + (a-p)\{(\frac{C_1ct_1^2}{2}+\frac{C_1dt_1^3}{6}+\frac{C_1\alpha\beta ct_1^{\beta+2}}{(\beta+2)}+\frac{C_1\alpha\beta dt_1^{\beta+3}}{2(\beta+3)}-$$

$$\frac{C_1\alpha\beta c\mu^{\beta+2}}{\beta+2}-\frac{C_1\alpha\beta d\mu^{\beta+3}}{2(\beta+3)})+(\frac{C_2\alpha t_1^{\beta+1}}{(\beta+1)}+\frac{C_2\alpha^2\beta t_1^{2\beta+1}}{(2\beta+1)}-C_2\alpha t_1\mu^{\beta}+$$

$$\frac{C_2\alpha\beta\mu^{\beta+1}}{(\beta+1)}-\frac{C_2\alpha^2\beta^2t_1^{\beta+1}\mu^{\beta}}{(\beta+1)}+\frac{C_2\alpha^2\beta^2\mu^{2\beta+1}}{(\beta+1)(2\beta+1)})+(\frac{C_3T^2}{2}-$$

$$C_3Tt_1 + C_3\lambda T^2t_1 - \frac{C_3\lambda T^3}{3}+\frac{C_3t_1^2}{2}-C_3\lambda Tt_1^2+\frac{C_3\lambda t_1^3}{3})+$$

$$(\frac{C_4\lambda T^2}{2}-C_4\lambda Tt_1+\frac{C_4\lambda t_1^2}{2})\}] \quad (15)$$

Optimal Value of t_1 can be obtained by solving the equation

$$\frac{\partial(TC)}{\partial t_1} = 0$$

$$\frac{\partial(TC)}{\partial t_1} = \frac{1}{T}\{(a-p)(C_1ct_1+\frac{C_1dt_1^2}{2}+C_1\alpha\beta ct_1^{\beta+1}+\frac{C_1\alpha\beta dt_1^{\beta+2}}{2}+$$

$$C_2\alpha t_1^{\beta}+C_2\alpha^2\beta t_1^{2\beta}-C_2\alpha\mu^{\beta}-C_2\alpha^2\beta^2t_1^{\beta}\mu^{\beta}-C_3T-2C_3\lambda Tt_1+$$

$$C_3\lambda T^2+C_3t_1+C_3\lambda t_1^2-C_4\lambda T+C_4\lambda t_1)\} \quad (16)$$

The minimum total average cost per unit time is obtained for those values of t_1 for which

$$\frac{\partial^2(TC)}{\partial t_1^2} > 0 \quad (17)$$

By solving equation (16), the value of t_1 (the inventory level reaches zero) scan be obtained and then using this in equations (15) and (9), the optimal value of total cost (TC) and maximum inventory level (S) can be found out respectively.

Numerical example: Consider an inventory system with following parameter in proper unit A=1000, a=25, p=400, c=0.5, d=0.02, μ=1, α=0.5, β=0.1, λ=0.8, $C_1 = 0.5,\ C_2 = 0.02,\ \ C_3 = 0.5,\ C_4 = 2,$ T=2 we get $t_1 = 1.76012$ and TC=411.8976

Sensitivity analysis:
Case (i): Using the above said parameters and only varying the deterioration parameter β, we get,

β	t_1	TC
0.1	1.76012	411.8976
0.2	1.74742	408.9778
0.3	1.73357	405.9724
0.4	1.71858	402.8876
0.5	1.70249	399.7331
0.6	1.68538	396.5215

Result: Increasing the value of deterioration parameter β, the value of t_1 and the Total Cost (TC) be decreased.
Case (ii): Using the above said parameters and only varying the deterioration parameter α, we get,

α	t_1	TC
0.4	1.76246	412.4829
0.5	1.76012	411.8976
0.6	1.75777	411.3028
0.7	1.75541	410.6988
0.8	1.75303	410.0855
0.9	1.75064	409.4630

Result: Increasing the value of deterioration parameter α, the value of t_1 and the Total Cost (TC) be decreased.

Numerical example:
Consider an inventory system with following parameter in proper unit A=1000, a=25, p=400, c=0.5, d=0.02, μ=1, α=0.5, β=0.1, λ=0.8, $C_1 = 0.5,\ C_2 = 0.02,\ \ C_3 = 0.5,\ C_4 = 2,$ T=3 we get $t_1 = 2.6254$ and TC=198.9561

Sensitivity analysis:
Case (i): Using the above said parameters and only varying the deterioration parameter β, we get,

β	t_1	TC
0.1	2.6254	198.9561
0.2	2.6029	192.9880
0.3	2.5765	186.4509
0.4	2.5460	179.3459
0.5	2.5113	171.6924
0.6	2.4723	163.5303

Result: Increasing the value of deterioration parameter β, the value of t_1 and the Total Cost (TC) be decreased.

Case (ii): Using the above said parameters and only varying the deterioration parameter α, we get,

α	t_1	TC
0.4	2.6292	200.0561
0.5	2.6254	198.9561
0.6	2.6216	197.8476
0.7	2.6177	196.7307
0.8	2.6138	195.6054
0.9	2.6099	194.4718

Result: Increasing the value of deterioration parameter α, the value of t_1 and the Total Cost (TC) be decreased.

III. Conclusion

In this paper, we have developed a model for deteriorating item with time dependent selling price demand and partial backlogging. From the analytical solutions of the above model it is to be concluded that the total inventory cost be minimized when we increase the deterioration rate.

Reference

[1]. R.Amutha and Dr.E.Chandrasekaran "An Inventory Model for Deteriorating Products with Weibull Distribution Deterioration, Time-Varying Demand and Partial Backlogging" ISSN [2229-5518], Vol.3,N0.10, (2012), pp.1-4

[2]. Azizul Baten and Anton Abdulbasah Kamil "Analysis of inventory-production systems with Weibull distributed deterioration", International Journal of Physical Sciences, Vol.4, No.11, (2009), pp 676-682

[3]. R.Begum, S.K.Sahu and R.R.Sahoo "An EOQ Model for Deteriorating Items with Weibull Distribution Deterioration, Unit Production Cost with Quadratic Demand and Shortages", Applied Mathematical Sciences, Vol.4,No.6, (2010), pp.271-288

[4]. S.K.Ghosh and K.S.Chaudhuri "An Order-Level Inventory Model for A Deteriorating Item with Weibull Distribution Deterioration, Time-Quadratic Demand and Shortages", AMO-Advanced Modeling and Optimization, Vol.6, No.1, (2004)

[5]. S.K.Goyal and B.C.Giri "Recent Trends in Modelling of Deteriorating Inventory", European Journal of Operational Research, Vol.134, (2001), pp. 1-16

[6]. Liang-Yuh OUYANG, Kun-Shan WU and Mei-Chaun CHENG "An Inventory Model for Deteriorating Items with Exponential Declining Demand and Partial Backlogging", Yugoslav Journal of Operations Research, Vol.15, No.2, (2005), pp.277-288

[7]. Ruxian Li, Hongjie LAN and John R.Mawhinney "A Review on Deteriorating Inventory Study", J.service Sciences & Management, Vol.3, (2010), pp.117-129

[8]. G.P.Samanta and Ajanta Roy "A Production Inventory Model with Deteriorating Items and Shortages", Yugoslav Journal of Operations Research, Vol.14, No.2, (2004), pp.219-230

[9]. C.K.Tripathy and L.M.Pradhan "An EOQ Model for Weibull Deteriorating Items with Power Demand and Partial Backlogging", Int.J.Contemp, Math.sciences, Vol.5, No.38, (2010), 1895-1904

[10]. C.K.Tripathy and U.S.Mishra "An Inventory Model for Weibull Deteriorating Items with Price Dependent Demand and Time-varying holding cost", Applied Mathematical Sciences, Vol.4, No.44, (2010), 2171-2179

[11]. P.K.Tripathy and S.Pradhan "An Integral Partial Backlogging Inventory Model having Weibull Demand and Variable Deterioration rate with the Effect of Trade Credit", International Journal Scientific & Engineering Research, Vol.2, No.4, (2011)

[12]. Vinod Kumar Mishra and Lal Sahan Singh "Deteriorating Inventory Model with Time Dependent Demand and Partial Backlogging", Applied Mathematical Sciences, Vol. 4 No.72, (2010), pp.3611-3619

[13]. Vinod kumar Mishra and lal sahib Singh "Deteriorating Inventory Model for Time Dependent Demand and Holding cost with Partial Backlogging" International Journal of Management Science & Engineering Research, Vol.2, No.4, (2011)

[14]. Vipin Kumar, S.R.Singh and Sharma "Profit Maximization Production Inventory Models with Time Dependent Demand and Partial Backlogging", International Journal of Operations Research and Optimization, Vol.1, No.2, (2010), pp.367-375

[15]. Zhao Pei-xin "An EOQ Model for Items with Weibull Distribution Deterioration", IEEE, [4244-0737], (2007), pp-451-454.