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Mhd Flow Past An Exponentially Accelerated Isothermal Vertical Plate With Variable Mass Diffusion In The Presence Of Thermal Radiation

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Abstract: *Hydromagnetic and thermal radiation effects on unsteady free convective flow of a viscous incompressible flow past an exponentially accelerated infinite isothermal vertical plate with variable mass diffusion has been considered. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. The plate temperature is raised to* T_w *and the concentration level near the plate is raised linearly with time. An exact solution to the dimensionless governing equations has been obtained by the Laplace transform method, when the plate is exponentially accelerated with a velocity* $u = u_0 \exp(a't')$ *in its own plane against gravitational field. The effects of velocity, temperature and concentration are studied for different physical parameters like magnetic field parameter, thermal radiation parameter, thermal Grashof number, mass Grashof number and Schmidt number. It is observed that the velocity increases with decreasing magnetic field parameter or radiation parameter. But the trend is just reversed with respect to* a *or* t*.*
Key words *: isothermal, radiation, exponentially, accelerated, vertical plate, magnetic field.*

I. Introduction

MHD finds applications in ion propulsion, electromagnetic pumps, MHD power generators, controlled fusion research, plasma jets and chemical synthesis, Magnetoconvection plays an important role in agriculture,petroleum industries, geophysics and astrophysics. MHD flow has application in metrology, solar physics and in motion of earths core. Also it has applications in the field of stellar and planetary magnetospheres, aeronautics, chemical engineering and electronics. The effects of transversely applied magnetic field, on the flow of an electrically conducting fluid past an impulsively started infinite isothermal vertical plate was studied by Soundalgekar et al.(1979). MHD effects on impulsively started vertical infinite plate with variable temperature in the presence of transverse magnetic field were studied by Soundalgekar et al. (1979). The dimensionless governing equations were solved using Laplace transform technique.

Radiative heat and mass transfer play an important role in manufacturing industries for the design of fins, steel rolling, nuclear power plants, gas turbines and various propulsion device for aircraft, missiles, satellites and space vehicles are examples of such engineering applications. England and Emery(1969) have studied the thermal radiation effects of a optically thin gray gas bounded by a stationary vertical plate. Raptis and Perdikis (1999) studied the effects of thermal radiation and free convection flow past a moving vertical plate. The governing equations were solved analytically. Radiation effect on mixed convection along a isothermal vertical plate were studied by Hossain and Takhar(1996). The governing equations were solved analytically. Das et al.(1996) have analyzed radiation effects on flow past an impulsively started infinite isothermal vertical plate.

Free convection effects on flow past an exponentially accelerated vertical plate was studied by Singh and Naveen Kumar (1984). The skin friction for accelerated vertical plate has been studied analytically by Hossain and Shayo (1986). Basant Kumar Jha *et al* (1991) analyzed mass transfer effects on exponentially accelerated infinite vertical plate with constant heat flux and uniform mass diffusion.

However the combined study of MHD and thermal radiation effects on infinite oscillating isothermal vertical plate with variable mass diffusion is not studied in the literature. It is proposed to study the thermal radiation effects on unsteady flow past an exponentially accelerated isothermal vertical plate, in the presence of transverse magnetic field. The dimensionless governing equations are tackled using the Laplace transform technique and the resultant solutions are in terms of exponential and complementary error function.

II. Basic Equations And Analysis

Here the unsteady flow of a viscous incompressible fluid which is initially at rest and surrounds an infinite vertical plate with temperature T_∞ and concentration C'_∞ . Here, the x -axis is taken along the plate in the vertically upward direction and the y -axis is taken normal to the plate. Initially, it is assumed that the plate and the fluid are of the same temperature and concentration. At time $t' > 0$, the plate is exponentially accelerated with a velocity $u = u_0 \exp(a't')$ in its own plane against gravitational force. At the same time the temperature of the plate is raised to T_w as well as the concentration level near the plate are also raised linearaly with time 't'. The plate is also subjected to a uniform magnetic field of strength B_0 . The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. It is assumed that the effect of viscous dissipation is negligible in the energy equation. Then by usual Boussinesq's approximation, the unsteady flow is governed by the following

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u \qquad (1)$$

$$\rho\, C_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} \qquad (2)$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y^2} \qquad (3)$$

With the following initial and boundary conditions:

$$\begin{array}{lll} t' \le 0: \quad u = 0, & T = T_\infty, & C' = C'_\infty \quad \text{for all} \quad y \\ t' > 0: \quad u = u_0 \exp(a't'), & T = T_\infty, & C' = C'_\infty + (C'_w - C'_\infty)\, A\, t' \quad \text{at} \quad y = 0 \\ \qquad\quad\; u = 0, & T \to T_\infty, & C' \to C'_\infty \quad \text{as} \quad y \to \infty \end{array} \qquad (4)$$

Where, $A = \dfrac{u_0{}^2}{\nu}$.

The local radiant for the case of an optically thin gray gas is defined as

$$\frac{\partial q_r}{\partial y} = -4a^* \sigma\left(T_\infty^4 - T^4\right) \qquad (5)$$

It is assume that the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms, thus

$$T^4 \cong 4T_\infty^3\, T - 3T_\infty^4 \qquad (6)$$

By using equations (5) and (6), equation (2) reduces to

$$\rho\, C_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y^2} + 16 a^* \sigma T_\infty^3 \,(T_\infty - T) \qquad (7)$$

On introducing the following non-dimensional quantities:

$$U = \frac{u}{u_0},\, t = \frac{t' u_0^2}{\nu},\, Y = \frac{y u_0}{\nu},\, \theta = \frac{T - T_\infty}{T_w - T_\infty},$$
$$Gr = \frac{g\beta\nu(T_w - T_\infty)}{u_0^3},\, C = \frac{C' - C'_\infty}{C'_w - C'_\infty},\, Gc = \frac{\nu g \beta^* (C'_w - C'_\infty)}{u_0^3},\, a = \frac{a'\nu}{u_0^2}, \qquad (8)$$
$$R = \frac{16 a^* \nu^2 \sigma T_\infty^3}{k u_0^2},\, Pr = \frac{\mu C_p}{k},\, Sc = \frac{\nu}{D},\, M = \frac{\sigma B_0^2 \nu}{\rho u_0^2}.$$

in equations (1), (3) and (7), reduces to

$$\frac{\partial U}{\partial t} = Gr\,\theta + Gc\,C + \frac{\partial^2 U}{\partial Y^2} - M\,U \qquad (9)$$
$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial Y^2} - \frac{R}{Pr}\theta \qquad (10)$$
$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} \qquad (11)$$

The initial and boundary conditions in non-dimensional form are

$$\begin{array}{llllll} & U = 0, & \theta = 0, & C = 0, & for\ all & Y, t \le 0 \\ t > 0: & U = \exp(at), & \theta = 1, & C = t, & at & Y = 0 \\ & U = 0, & \theta \to 0, & C \to 0 & as & Y \to \infty \end{array} \qquad (12)$$

All the physical variables are defined in the nomenclature. The solutions are obtained for hydrodynamic flow field in the presence of first order chemical reaction. The equations (9) to (11), subject to the boundary conditions (12), are solved by the usual Laplace-transform technique and the solutions are derived as follows:

$$\theta = \frac{1}{2}\left[\exp(2\eta\sqrt{Rt})\, erfc(\eta\sqrt{Pr} + \sqrt{at}) + \exp(-2\eta\sqrt{Rt})\, erfc(\eta\sqrt{Pr} - \sqrt{at})\right] \qquad (13)$$
$$C = t\left[(1 + 2\eta^2 Sc) erfc(\eta\sqrt{Sc} - \frac{2\eta}{\sqrt{\pi}}\sqrt{Sc}\exp(-\eta^2 Sc)\right] \qquad (14)$$

$$U = \frac{\exp(at)}{2}\left[\exp(2\eta\sqrt{(M+a)t}) erfc(\eta + \sqrt{(M+a)t}) + \exp(-2\eta\sqrt{(M+a)t}) erfc(\eta - \sqrt{(M+a)t})\right]$$
$$+ \frac{e}{2}\left[\begin{array}{l} \exp\left(2\eta\sqrt{Mt}\right) erfc\left(\eta + \sqrt{Mt}\right) + \exp\left(-2\eta\sqrt{Mt}\right) erfc\left(\eta - \sqrt{Mt}\right) \\ + \exp\left(2\eta\sqrt{Rt}\right) erfc\left(\eta\sqrt{\Pr} + \sqrt{bt}\right) + \exp\left(-2\eta\sqrt{Rt}\right) erfc\left(\eta\sqrt{\Pr} - \sqrt{bt}\right) \end{array}\right]$$

$$-g\frac{\exp(dt)}{2}\begin{bmatrix}\exp\left(-2\eta\sqrt{(M+d)t}\right)erfc\left(\eta-\sqrt{(M+d)t}\right)+\exp\left(2\eta\sqrt{(M+d)t}\right)erfc\left(\eta+\sqrt{(M+d)t}\right)\\-\exp\left(2\eta\sqrt{(Sc.d)t}\right)erfc\left(\eta\sqrt{Sc}+\sqrt{dt}\right)-\exp\left(-2\eta\sqrt{(Sc.d)t}\right)erfc\left(\eta\sqrt{Sc}-\sqrt{dt}\right)\end{bmatrix}$$

$$-e\frac{\exp(ct)}{2}\begin{bmatrix}\exp\left(-2\eta\sqrt{(M+c)t}\right).erfc\left(\eta-\sqrt{(M+c)t}\right)+\exp\left(2\eta\sqrt{(M+c)t}\right).erfc\left(\eta+\sqrt{(M+c)t}\right)\\-\exp\left(2\eta\sqrt{\Pr(b+c)t}\right).erfc\left(\eta\sqrt{\Pr}+\sqrt{(b+c)t}\right)-\exp\left(-2\eta\sqrt{\Pr(b+c)t}\right).erfc\left(\eta\sqrt{\Pr}-\sqrt{(b+c)t}\right)\end{bmatrix}$$

$$+f\,t\left[(1+2\eta^2 Sc)erfc(\eta\sqrt{Sc})-\frac{2\eta\sqrt{Sc}}{\sqrt{\pi}}\exp(-\eta^2 Sc)\right]$$

$$-f\begin{Bmatrix}\frac{t}{2}\left[\exp\left(2\eta\sqrt{Mt}\right)erfc\left(\eta+\sqrt{Mt}\right)+\exp\left(-2\eta\sqrt{Mt}\right)erfc\left(\eta-\sqrt{Mt}\right)\right]\\-\frac{\eta\sqrt{t}}{2\sqrt{M}}\left[\exp\left(-2\eta\sqrt{Mt}\right)erfc\left(\eta-\sqrt{Mt}\right)-\exp\left(2\eta\sqrt{Mt}\right)erfc\left(\eta+\sqrt{Mt}\right)\right]\end{Bmatrix} \qquad (15)$$

Where,

$$b=\frac{R}{Pr},c=\frac{R-M}{(1-\Pr)},d=\frac{M}{(1-Sc)},e=\frac{Gr}{c(1-\Pr)},\ f=\frac{Gc}{d(1-Sc)},g=\frac{Gc}{d^2(1-Sc)}$$

$\eta = Y/2\sqrt{t}$ and erfc is called complementary error function.

III. Discussion Of Results

The numerical values of the velocity and skin-friction are computed for different parameters like Magnetic field parameter, chemical reaction parameter, Schmidt number, thermal Grashof number and mass Grashof number. The purpose of the calculations given here is to assess the effects of the parameters M, K, Gr, Gc and Sc upon the nature of the flow and transport. The value of Prandtl number Pr is chosen such that they represent air (*Pr* = 0.71). The solutions are in terms of exponential and complementary error function.

The temperature profiles are calculated for different values of thermal radiation parameter($R = 2,5,7,10$) at time $t = 0.4$ and these are shown in figure 1. The effect of thermal radiation parameter is important in temperature profiles. It is observed that the temperature increases with decreasing radiation parameter.

Figure 2 represents the effect of concentration profiles for different values of time (t = 0.2, 0.4, 0.6, 0.8, 1) and Sc = 2.01. The trend shows that the wall concentration increases with increasing values of time t. The effect of concentration profiles at time t = 1 for different Schmidt number (Sc = 0.16, 0.3, 0.6, 2.01) are presented in figure 3. The effect of Schmidt number plays an important role in concentration field. The profiles have the common feature that the concentration decreases in a monotone fashion from the surface to a zero value far away in the free stream. It is observed that the wall concentration increases with decreasing values of the Schmidt number**.**

Figure 4. demonstrates the velocity profiles for different thermal radiation parmeter ($R = 2,5,15$), $Gr = 5, Gc = 10, a = 1, M = 0.2$ and $t = 0.6$. It is observed that the velocity increases with decreasing thermal radiation parameter. This shows that there is dip in velocity in the presence of radiation. The effect of velocity for different values of ($a = 0.2,0.6,1$), $R = 2, Gr = Gc = 5$ at $t = 0.2$ are studied and presented in figure 5. It is observed that the velocity increases with increasing values of a .

Figure 6. illustrates the velocity profiled for different values of the magnetic filed parameter (M=0.2,5,10), Gr=5, Gc=10, a=1, R=15 and t=0.2. Is is observed that the velocity increases with decreasing values of the magnetic filed parameter. This shows that there is fall in velocity in the presence of higher magnetic intensity. This agrees with the expectations, since the magnetic filed exerts retarding force on the free convective flow. The effect of velocity for different values of thermal Grashof number(Gr=5,10), mass Grashof

number(Gc=5,10), R=5, a=1, M=0.2 and t=0.6 are presented in figure 7. It is observed that the velocity increases with increasing thermal Grashof number or mass Grashof number.

IV. Conclusion

Theoretical solution of hydromagnetic and thermal radiation effects on unstaedy flow past an exponentially accelerated infinite isothermal vertical plate with variable mass diffusion has been studied in detail. The dimensionless equations are solved using Laplace transform technique. The effect of velocity, temperature and concentration for different parameters like M, R, Gr, Gc, Sc and t are studied. The study concludes that the velocity increases with decreasing magnetic field parameter or radiation parameter. It is also observed that the velocity increases with increasing thermal Grashof number or mass Grashof number.

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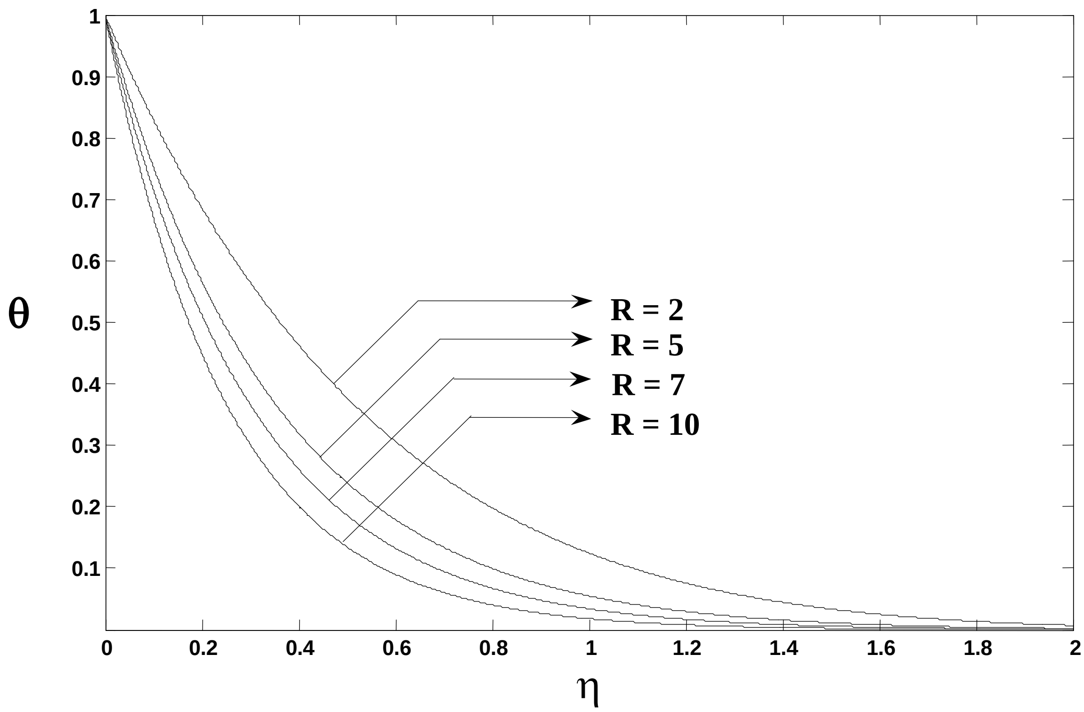

Figure 1.Temperature profiles for different values of R

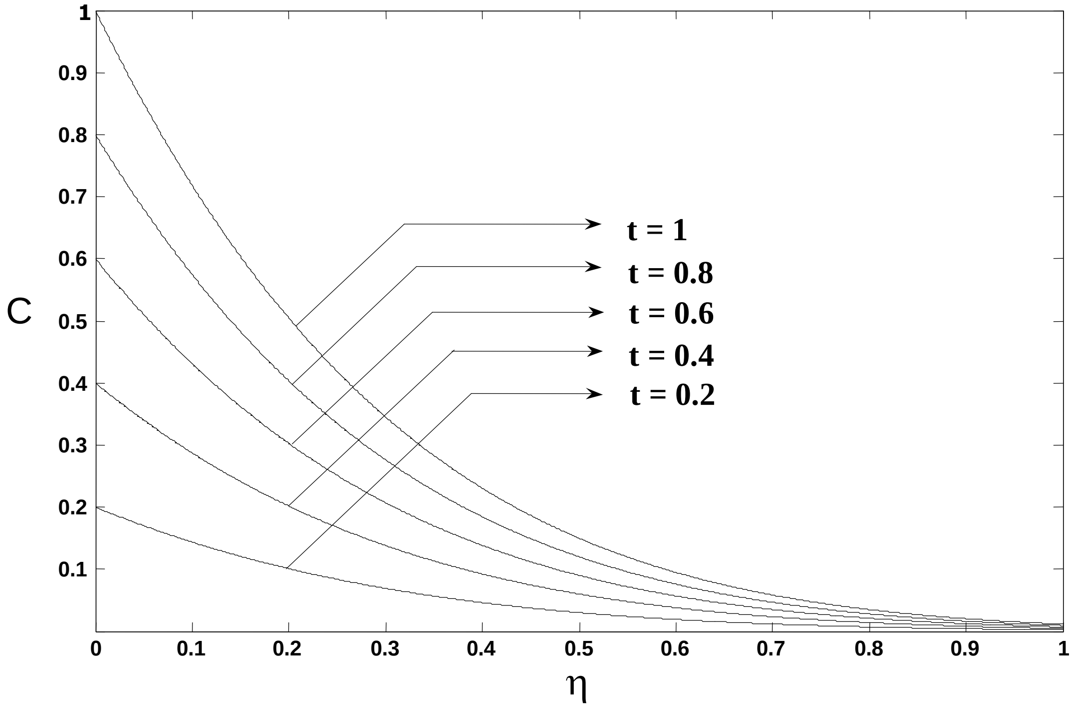

Figure 2. Concentration profiles for different values of t

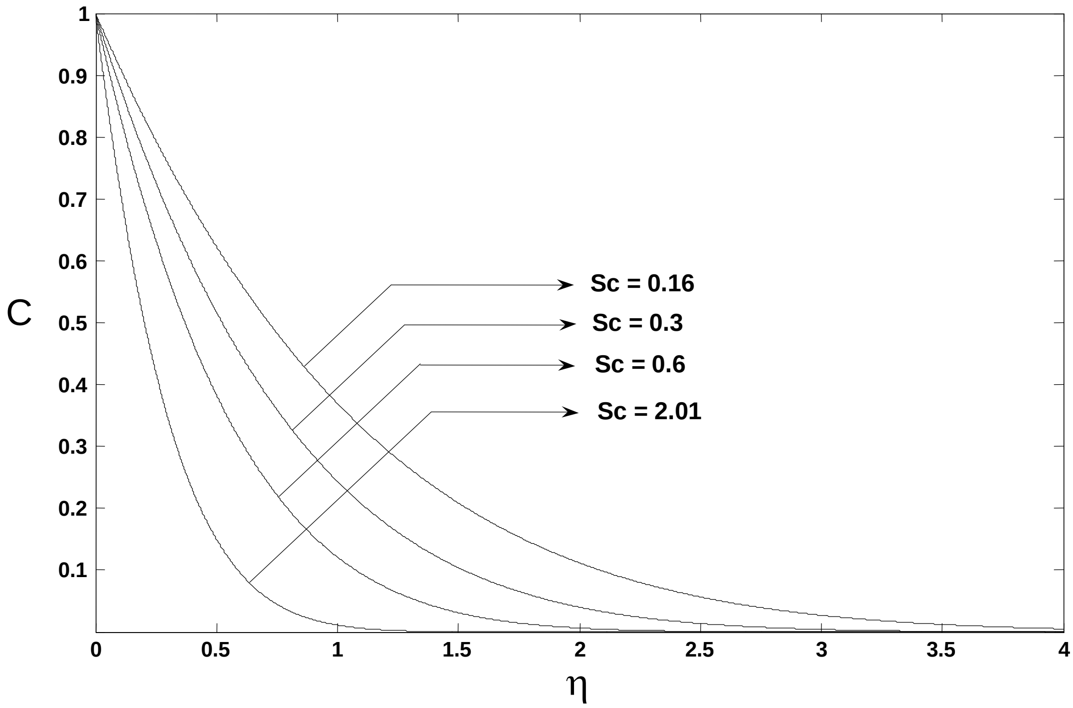

Figure 3. Concentration profiles for different values of Sc

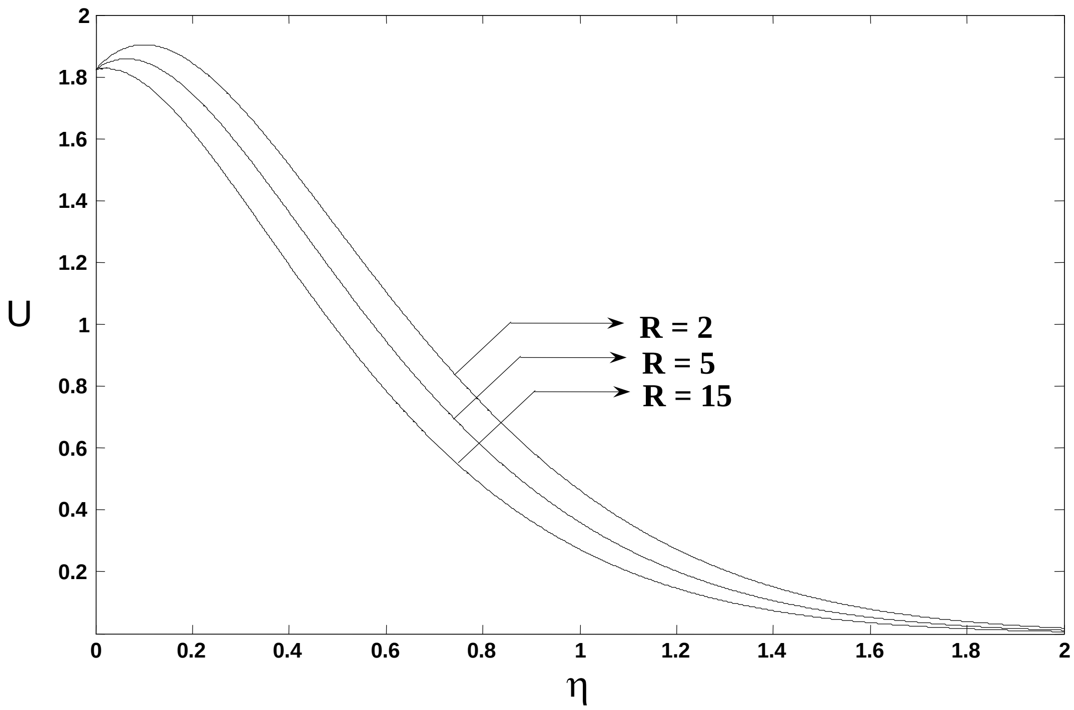

Figure 4.Velocity profiles for different values of R

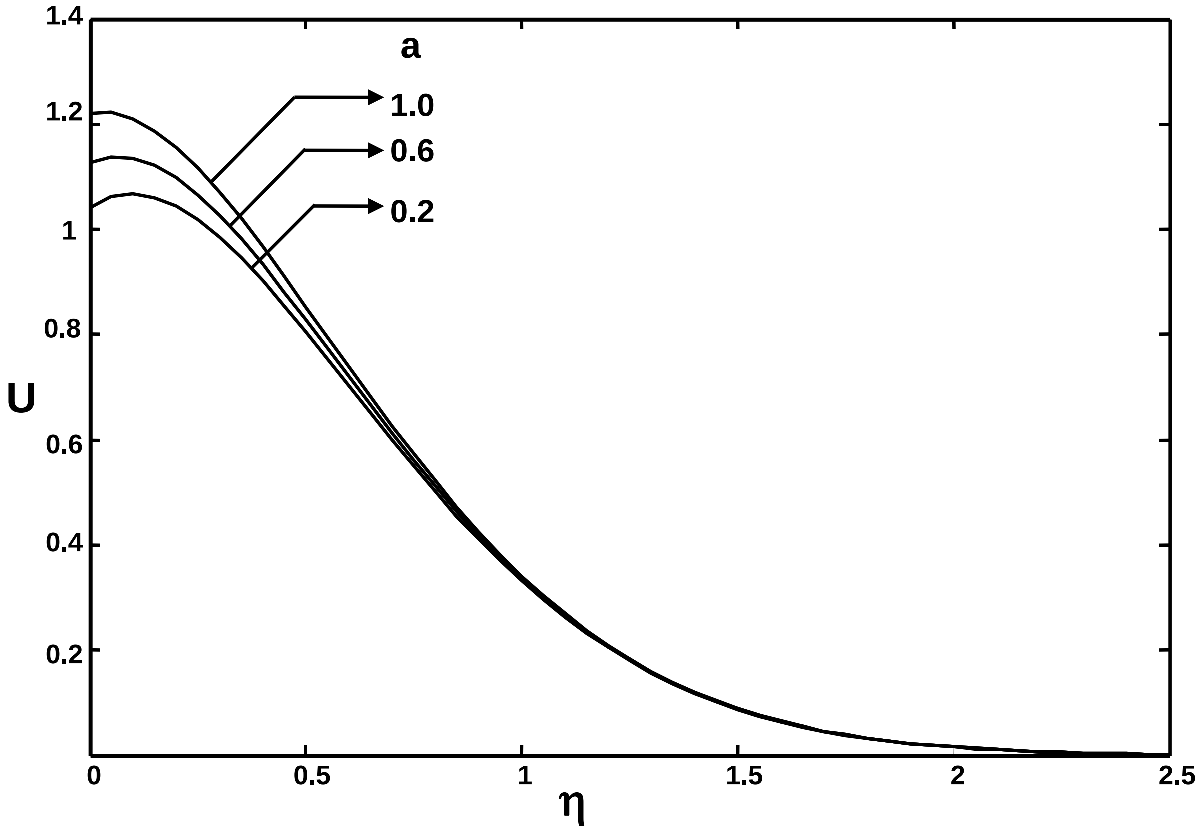

Figure 5. Velocity profiles for different a

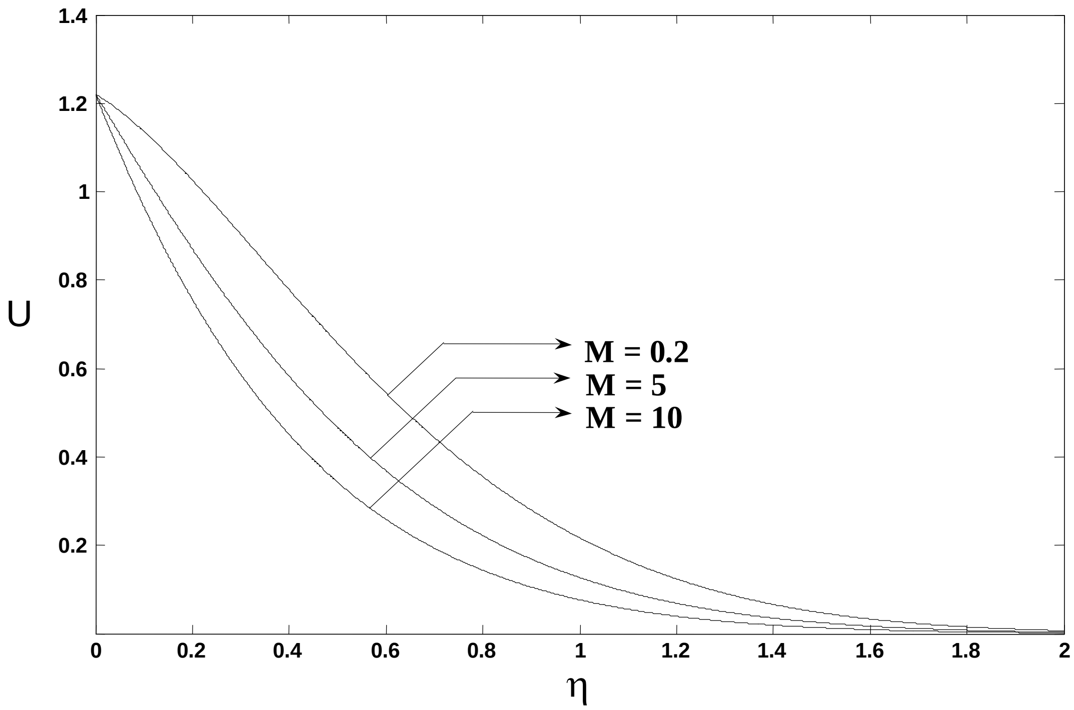

Figure 6.Velocity profiles for different values of M

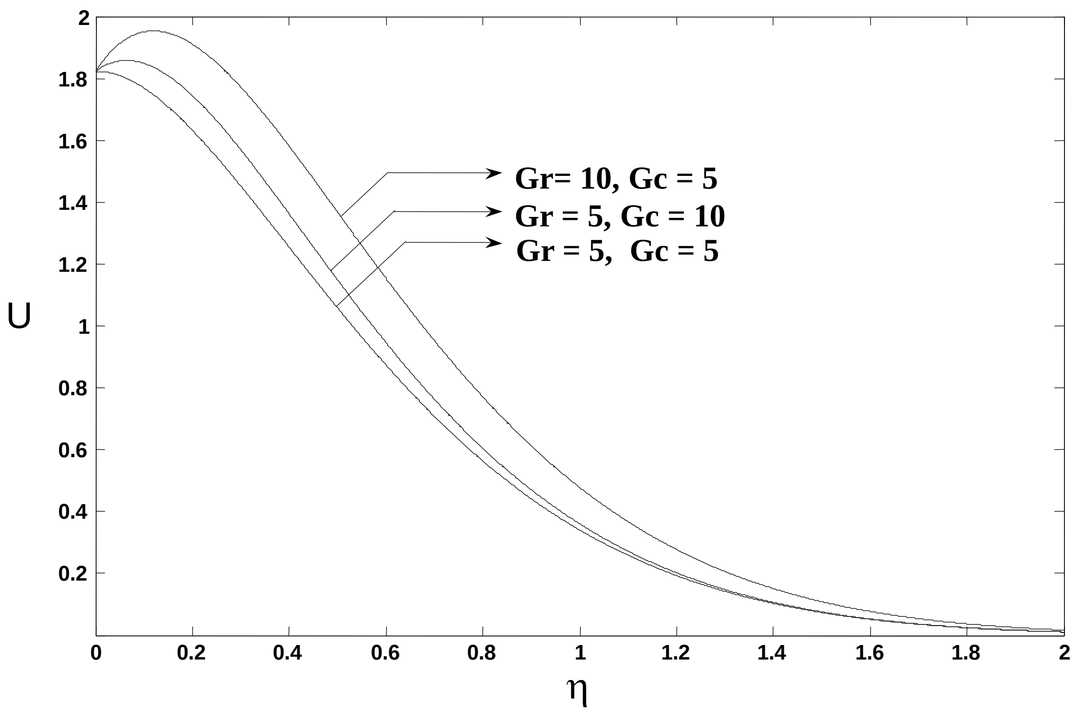

Figure 7.Velocity profiles for different values of Gr and Gc.