IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 11, Issue 5 Ver. IV (Sep. - Oct. 2015), PP 05-13
www.iosrjournals.org

On Pure PO-Ternary Γ-Ideals in Ordered Ternary Γ-Semirings

V. Syam Julius Rajendra[1], Dr. D. Madhusudhana Rao[2] and M. Sajani Lavanya[3]
[1]*Lecturer, Department of Mathematics, A.C. College, Gunture, A.P. India.*
[2]*Head, Department of Mathematics, V. S. R & N.V.R. College, Tenali, A.P. India.*
[3]*Lecturer, Department of Mathematics, A.C. College, Guntur, A.P. India.*

Abstract*: In this paper, we introduce the concepts of pure po-ternary Γ-ideal, weakly pure po-ternary Γ-ideal and purely prime po-ternary Γ-ideal in an ordered ternary Γ-semiring. We obtain some characterizations of pure po-ternary Γ-ideals and prove that the set of all purely prime po-ternary Γ-ideals is topologized.*
Keywords*: ternary Γ-semiring; ordered ternary Γ-semiring; weakly regular; pure po-ternaryΓ-ideal; weakly pure po-ternary Γ-ideal; purely prime po-ternary Γ-ideal; topology.*

I. Introduction

In [1], Ahsan and Takahashi introduced the notions of pure ideal and purely prime ideal in a semigroup. Recently, Bashir and Shabir [2] defined the concepts of pure ideal, weakly pure ideal and purely prime ideal in a ternary semigroup without order. The authors gave some characterizations of pure ideals and showed that the set of all purely prime ideals of a ternary semigroup is topologized. In this paper, we introduce the concepts of purepo-ternary Γ-ideal, weakly pure po-ternary Γ-ideal and purely prime po-ternary Γ-ideal in an ordered ternary Γ-semiring. We characterize pure po-ternary Γ-ideals and prove that the set of all purely prime po-ternary Γ-ideals of an ordered ternary Γ-semiring is topologized. Note that the results on ternary Γ-semiring without order become then special cases.

II. Preliminaries

Definition 2.1: Let T and Γ be two additive commutative semigroups. T is said to be a ***Ternary Γ-semiring*** if there exist a mapping from T ×Γ× T ×Γ× T to T which maps $(x_1, \alpha, x_2, \beta, x_3) \to [x_1\alpha x_2\beta x_3]$ satisfying the conditions :

i) $[[a\alpha b\beta c]\gamma d\delta e] = [a\alpha[b\beta c\gamma d]\delta e] = [a\alpha b\beta[c\gamma d\delta e]]$
ii) $[(a + b)\alpha c\beta d] = [a\alpha c\beta d] + [b\alpha c\beta d]$
iii) $[a\alpha (b + c)\beta d] = [a\alpha b\beta d] + [a\alpha c\beta d]$
iv) $[a\alpha b\beta(c + d)] = [a\alpha b\beta c] + [a\alpha b\beta d]$ for all $a, b, c, d \in$ T and $\alpha, \beta, \gamma, \delta \in \Gamma$.

Obviously, every ternary semiring T is a ternary Γ-semiring. Let T be a ternary semiring and Γ be a commutative ternary semigroup. Define a mapping T ×Γ× T ×Γ× T ⟶ T by $a\alpha b\beta c = abc$ for all $a, b, c \in$ T and $\alpha, \beta \in \Gamma$. Then T is a ternary Γ-semiring.

Note 2.2 : Let (T,Γ, +, [ ]) be a ternary Γ-semiring. For nonempty subsets A_1,A_2 and A_3 of T, let [AΓBΓC]= $\{\Sigma a\alpha b\beta c : a \in A, b \in B, c \in C, \alpha, \beta \in \Gamma\}$ .For $x \in$ T, let $[x\Gamma A_1\Gamma A_2] = [\{x\}\Gamma A_1\Gamma A_2]$. The other cases can be defined analogously.

Note2.3 : Let T be a ternary semiring. If A, B are two subsets of T , we shall denote the set A + B = $\{a + b : a \in A, b \in B\}$ and 2A = $\{ a + a : a \in A\}$.

Definition 2.4 :A ternary Γ-semiring T is called an ordered ternary Γ-semiring if there is a partial order ≤ on T such that x ≤ y implies that (i) $a + c \leq b + c$ and $c + a \leq c + b$
(ii) $[a\alpha c\beta d] \leq [b\alpha c\beta d]$, $[c\alpha a\beta d] \leq [c\alpha b\beta d]$ and $[c\alpha d\beta a] \leq [c\alpha d\beta b]$for all $a, b, c, d \in$ T and $\alpha, \beta, \gamma, \delta \in \Gamma$.

Note2.5 : For the convenience we write $x_1\alpha x_2\beta x_3$ instead of $[x_1\alpha x_2\beta x_3]$

III. PO-Ternary Γ-Ideals :

Definition 3.1: Let T be PO-ternary Γ-semiring. A nonempty subset 'S' is said to be a PO-***ternary Γ-subsemiring*** of T if

(i) S is an additive subsemigroup of T,
(ii) $a\alpha b\beta c \in$ S for all $a, b, c \in$ S, $\alpha, \beta \in \Gamma$.

(iii) $T \in$ T, $s \in$ S, $t \leq s \Rightarrow t \in$ S.

Example 3.2 : Let$T$= M_2 (Z) and Γ = M_2 (Z_0^-) define the ordering as $a_{ii} \leq b_{ii}$. Then T be the PO-ternary Γ-semiring of the set of all 2x2 square matrices over Z , the set of all non-positive integers. Let $S = \left\{ \begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} / a \in Z \right\}$. Then *S* is a PO-ternary Γ-subsemiring of *T.*

Notation 3.3 : LetT be PO-ternary Γ-semiring and S be a nonempty subset of T. If H is a nonempty subset of S, we denote {s∈S :$s \leq h$ for some h∈ H} by $(H]_S$.

Notation 3.4: Let T be PO-ternary Γ-semiring and S be a nonempty subset of T. If H is a nonempty subset of S, we denote {s∈S :$h \leq s$ for some h∈ H} by $[H)_S$.

Note 3.5 : $(H]_T$ and $[H)_T$ are simply denoted by (H] and [H) respectively.

Note 3.6: A nonempty subset S of a po-ternary Γ-semiring T is apo-ternary Γ-subsemiring of T iff (1)S + S ⊆ S (2) SΓSΓS $\subseteq$ S, (2) (S] ⊆ S.

Theorem 3.7 : Let S be po-ternary Γ-semiring and A ⊆ S, B ⊆ S. Then (i) A ⊆ (A], (ii) ((A]] = (A], (iii) (A]Γ(B]Γ(C] ⊆ (AΓBΓC] and (iv) A ⊆ B ⇒ A ⊆ (B], (v) A ⊆ B ⇒(A] ⊆ (B], (vi) (A ∩ B] = (A] ∩ (B], (vii) (A ∪ B] = (A] ∪ (B].

Definition 3.8 : A nonempty subset A of a PO-ternary Γ-semiring T is said to be ***left PO-ternary Γ-ideal*** of T if
(1) a, b ∈ A implies $a + b$∈ A.
(2) b, $c \in$ T, $a \in$ A, $\alpha, \beta \in \Gamma$ implies $b\alpha c\beta a \in$ A.
(3) $t \in T$, $a \in A$, $t \leq a \Rightarrow t \in$ A.

Note 3.9 : A nonempty subset A of a PO-ternary Γ-semiring T is a left PO-ternary Γ-ideal of T if and only if A is additive subsemigroup of T, TΓTΓA $\subseteq$ A and (A] ⊆ A.

Note 3.10: Let T be a PO-ternary Γ-semiring.

Then the set (TΓTΓa] = {t∈ T / $t \leq \sum_{i=1}^{n} x_i \alpha_i y_i \beta_i a$ for some $x_i, y_i \in$ T, $\alpha_i, \beta_i \in \Gamma$ and n∈ N}.

Example 3.11 : In the PO-ternary Γ-semiring Z^0 , nZ^0 is a left PO-ternary Γ-ideal for any n ∈N.

Theorem 3.12: Let Tbe a PO-ternary Γ-semiring. Then (TΓTΓ*a*] is a left PO-ternary Γ-ideal of T for all *a*∈ T.

Definition 3.13: A left PO-ternary Γ-ideal A of a PO-ternary Γ-semiring T is said to be the ***principal left PO-ternary Γ-ideal generated by a*** if A is a left PO-ternary Γ-ideal generated by $\{a\}$ for some $a \in$ T. It is denoted by L (a) or <a>$_l$.

Theorem 3.14 : If T is a PO-ternary Γ-semiring and *a*∈ T then

L(*a*) = (A] where A=$\left\{ \sum_{i=1}^{n} r_i \alpha_i t_i \beta_i a + na : r_i, t_i \in T, \alpha_i, \beta_i \in \Gamma \text{ and } n \in z_0^+ \right\}$**, and Σ denotes a finite sum and z_0^+ is the set of all positive integer with zero.**

***Proof* :**Given that A = $\left\{ \sum_{i=1}^{n} r_i \alpha_i t_i \beta_i a + na : r_i, t_i \in T, \alpha_i, \beta_i \in \Gamma \text{ and } n \in z_0^+ \right\}$. Let a, b∈ A.

a, b∈ A . Then a = $\sum r_i \alpha_i t_i \beta_i a + na$ and b = $\sum r_j \alpha_j t_j \beta_j a + na$ for r_i, t_i, r_j, t_j∈ T, $\alpha_i, \beta_i, \alpha_j, \beta_j \in \Gamma$ and $n \in z_0^+$.

Now $a + b$ = $\sum r_i \alpha_i t_i \beta_i a + na + \sum r_j \alpha_j t_j \beta_j a + na$ ⇒$a + b$ is a finite sum.
Therefore $a + b$∈ A and hence A is a additive subsemigroup of T.
For t_1,t_2∈ T and a∈ A.
Then $t_1 \alpha t_2 \beta a = t_1 \alpha t_2 (\sum r_i \alpha_i t_i \beta_i a + na) = \sum r_i \alpha_i t_i \beta_i (t_1 \alpha t_2 \beta a) + n(t_1 \alpha t_2 \beta a)$ ∈ A

Therefore $t_1\alpha t_2\beta a \in A$ and hence A is a left ternary Γ-ideal of T. By theorem 3.18,we have (A] is a left ordered ternary Γ-ideal of T containing a. Thus L(a) ⊆(A].On the other hand, L(a) is also aleft ordered Γ-ideal of T containing a, so we have A ⊆ L(a). Thus (A] ⊆L(a) since (A] is a left ordered ternary ideal of T generated by A. Therefore L(a) = (A], as required.

Definition 3.15 : A nonempty subset of a PO-ternary Γ-semiring T is said to be a ***lateral PO-ternary ideal*** of T if

(1) $a, b \in A$ implies $a + b \in A$.
(2) $b, c \in T$, $\alpha, \beta \in \Gamma$, $a \in A$ implies $b\alpha a\beta c \in A$.
(3) $t \in T, a \in A, t \leq a \Rightarrow t \in A$.

Note 3.16: A nonempty subset of A of a PO-ternary semiring T is a lateral PO-ternary Γ-ideal of T if and only if A is additive subsemigroup of T, TΓAΓT ⊆ A and (A] ⊆ A.

Theorem 3.18: Let T be a PO-ternary Γ-semiring. Then (TΓ*a*ΓT] is a lateral PO-ternary Γ-ideal of T for all *a*∈ T.

Theorem 3.18: Let T be a PO-ternary Γ-semiring. Then (TΓTΓ*a*ΓTΓT] is a lateral PO-ternary Γ-ideal of T for all *a*∈ T.

Definition 3.19 : A lateral PO-ternary Γ-ideal A of a PO-ternary Γ-semiring T is said to be the ***principal lateral PO-ternary Γ-ideal generated by a*** if A is a lateral PO-ternary Γ-ideal generated by $\{a\}$ for some $a \in T$. It is denoted by M (a) (or) $\langle a \rangle_m$.

Theorem 3.20 : If T is a PO-ternary Γ-semiring and $a \in$ T then
M(a) = (A], where A =

$$\left\{ \sum_{i=1}^{n} r_i\alpha_i a\beta_i t_i + \sum_{j=1}^{n} u_j\alpha_j v_j\beta_j a\gamma_j p_j\delta_j q_j + na \, : r_i, t_i, u_j v_j p_j q_j \in T, \ \alpha_i, \beta_i, \alpha_j, \beta_j, \gamma_j, \delta_j \in \Gamma \text{ and } n \in z_0^+ \right\}$$

, and Σ denotes a finite sum and z_0^+ is the set of all positive integer with zero.

Definition 3.21 : A nonempty subset A of a PO-ternary Γ-semiring T is a ***right PO-ternary Γ-ideal***of T if
(1) $a, b \in A$ implies $a + b \in A$.
(2) $b, c \in T$, $\alpha, \beta \in \Gamma$, $a \in A$ implies $a\alpha b\beta c \in A$.
(3) $t \in T, a \in A, t \leq a \Rightarrow t \in A$.

Note 3.22 : A nonempty subset A of a PO-ternary Γ-semiring T is a rightPO-ternary Γ-ideal of T if and only if A is additive subsemigroup of T, AΓTΓT ⊆ A and (A] ⊆ A.

Definition 3.23 : A right PO-ternary Γ-ideal A of a PO-ternary Γ-semiring T is said to be a ***principal right PO-ternary Γ-ideal generated by a*** if A is a right PO-ternary Γ-ideal generated by $\{a\}$ for some $a \in T$. It is denoted by R (a) (or) $\langle a \rangle_r$.

Theorem 3.24 : If T is a po-ternary Γ-semiring and $a \in$ T then

R(a) = (A], where A = $\left\{ \sum_{i=1}^{n} a\alpha_i r_i\beta_i t_i + na \, : r_i, t_i \in T, \alpha_i, \beta_i \in \Gamma \text{ and } n \in z_0^+ \right\}$ **, Σ denotes a finite sum and**

z_0^+ is the set of all positive integer with zero.

Definition 3.25 : A nonempty subset A of a PO-ternary Γ-semiring T is a ***two sided PO-ternary Γ-ideal***of T if

1. $a, b \in A$ implies $a + b \in A$
2. $b, c \in T$, $\alpha, \beta \in \Gamma$, $a \in A$ implies $b\alpha c\beta a \in A$, $a\alpha b\beta c \in A$.
3. $t \in T, a \in A, t \leq a \Rightarrow t \in A$.

Note 3.26: A nonempty subset A of a PO-ternary Γ-semiring T is a two sided PO-ternary Γ-ideal of T if and only if it is both a left PO-ternary Γ-ideal and a right PO-ternary Γ-ideal of T.

Definition 3.27 : A two sided PO-ternary Γ-ideal A of a PO-ternary Γ-semiring T is said to be the ***principal two sided PO-ternary Γ-ideal*** provided A is a two sided PO-ternary Γ-ideal generated by $\{a\}$ for some $a \in$ T. It is denoted by T (a) (or) $<a>_t$.

Theorem 3.28 : If T is a PO-ternary Γ-semiring and $a \in$ T then T(a) = (A], where

$$A = \left\{ \sum_{i=1}^{n} r_i\alpha_i s_i\beta_i a + \sum_{j=1}^{n} a\alpha_j t_j\beta_j u_j + \sum_{k=1}^{n} l_k\alpha_k m_k\beta_k a\gamma_k p_k\delta_k q_k + na : r_i, s_i, t_j, u_j, l_k m_k, p_k, q_k \in T, \alpha_i, \beta_i, \alpha_j, \beta_j, \alpha_k, \beta_k, \gamma_k, \delta_k \in \Gamma \text{ and } n \in Z_0^+ \right\}$$

and Σ denotes a finite sum and z_0^+ is the set of all positive integer with zero.

Definition 3.29 : A nonempty subset A of a PO-ternary Γ-semiring T is said to be PO-***ternary Γ-ideal***of T if

(1) $a, b \in$ A implies $a + b \in$ A

(2) $b, c \in$ T ,$\alpha, \beta \in \Gamma, a \in$ A implies $b\alpha c\beta a \in$ A, $b\alpha a\beta c \in$ A, $a\alpha b\beta c \in$ A.

(3) $t \in T$, $a \in A$, $t \leq a \Rightarrow t \in$ A.

Note 3.30 : A nonempty subset A of a PO-ternary Γ-semiring T is aPO-ternary Γ-ideal of T if and only if it is left PO-ternary Γ-ideal, lateral PO-ternary Γ-ideal and right PO-ternary Γ-ideal of T.

Definition 3.31 : An element a of a PO-ternary Γ-semiring. T is said to be ***regular*** if there exist $x, y \in$ T such that $a \leq a\alpha x\beta a\gamma y\delta a$ for all $\alpha, \beta, \gamma, \delta \in \Gamma$.

IV. Pure po-ternaryΓ-ideals in ordered ternary Γ-semiring

In this section we define pure po-ternary Γ-ideals in ordered ternary Γ-semiring.

Definition 4.1: Let T be an ordered ternary Γ-semiring. A two-sided po-ternary Γ-ideal A of T is called a ***left (respectively, right) pure two-sided po-ternary Γ-ideal*** if for each $x \in A$ there exist $y_i, z_i \in A$, α_i , $\beta_I \in \Gamma$ where $i \in \Delta$such that $x \leq \sum_{i=1}^{n} y_i\alpha_i z_i\beta_i x$ (respectively, $x \leq \sum_{i=1}^{n} x\alpha_i y_i\beta_i z_i$). Apo-ternary Γ- ideal A of $T$is called ***left (respectively, right) pure po-ternary Γ-ideal*** if for each $x \in A$ there exist y_i ,$z_i \in A$, α_i , $\beta_I \in \Gamma$ where $i \in \Delta$such that $x \leq \sum_{i=1}^{n} y_i\alpha_i z_i\beta_i x$ (respectively $x \leq \sum_{i=1}^{n} x\alpha_i y_i\beta_i z_i$). Similarly, we define one-sided left and right pure po-ternary Γ-ideals.

Theorem 4.2: Let T be an ordered ternary Γ-semiring and A a two-sided po-ternaryΓ-ideal of T. Then A is right pure po-ternary two-sided Γ-ideal if and only if B\A = ([BΓAΓA]] for all right po-ternary Γ-ideals B of T.

Proof:Assume that A is right pure two-sided po-ternary Γ-ideal. Let B be a right po-ternary Γ-ideal of T. We have $[B\Gamma A\Gamma A] \subseteq [B\Gamma T\Gamma T] \subseteq B$. Then $([B\Gamma A\Gamma A]] \subseteq (B] = B$. Since $[B\Gamma A\Gamma A] \subseteq [T\Gamma T\Gamma A] \subseteq A$, so $([B\Gamma A\Gamma A]] \subseteq (A] = A$.Hence $([B\Gamma A\Gamma A]] \subseteq B\cap A$.To prove the reverse inclusion, let $x \in B\cap A$. By assumption, there exist y_i ,$z_i \in A$, α_i , $\beta_i \in \Gamma$ where $i \in \Delta$such that $x \leq \sum_{i=1}^{n} x\alpha_i y_i\beta_i z_i$. Since $\sum_{i=1}^{n} x\alpha_i y_i\beta_i z_i \in [B\Gamma A\Gamma A]$, we obtain $x \in ([B\Gamma A\Gamma A]]$.

Thus $B\cap A \subseteq ([B\Gamma A\Gamma A]]$.

Conversely, suppose that $B \cap A = ([B\Gamma A\Gamma A]]$ for all right po-ternary Γ-ideals B of T. Let $x \in A$. Since $(\{x\}\cup[x\Gamma T\Gamma T]]$ is a right po-ternary Γ-ideal of T and $[T\Gamma T\Gamma A] \subseteq A$, we have $(\{x\}\cup[x\Gamma T\Gamma T]] \cap A = ([(\{x\}\cup[x\Gamma T\Gamma T]]\Gamma A\Gamma A]] \subseteq ([x\Gamma A\Gamma A] \cup [[x\Gamma T\Gamma T]\Gamma A\Gamma A]] \subseteq ([x\Gamma A\Gamma A]]$. Since $x \in (\{x\}\cup[x\Gamma T\Gamma T]] \cap A$, $x \in ([x\Gamma A\Gamma A]]$. Hence A is a right pure two-sided po-ternaryΓ-ideal of T.

Definition 4.3: An ordered ternary Γ-semiring T is said to be ***right weakly regular*** if for any $x \in$ T, $x \in ([[x\Gamma T\Gamma T]\Gamma[x\Gamma T\Gamma T]\Gamma[x\Gamma T\Gamma T]]]$.

Note that every regular ordered ternary Γ-semiring is right weakly regular.

Theorem 4.4: Let T be an ordered ternary Γ-semiring. The following are equivalent.

(i) T is right weakly regular.

(ii) ([AΓAΓA]] = A for all right po-ternary Γ-ideals A of T.

(iii) B∩A = ([BΓAΓA]] for all right po-ternary Γ-ideals B and all two-sided po-ternary Γ-ideals A of T.

(iv) B∩A = ([BΓAΓA]] for all right po-ternaryΓ-ideals B and all po-ternary Γ-ideals A of T.

***Proof*:**(i) ⇒ (ii). Assume that T is right weakly regular.
Let A be a right po-ternary Γ-ideal of T.
Since [AΓAΓA] ⊆ [AΓTΓT] ⊆A, we have ([AΓAΓA]] ⊆ A.
Let x ∈ A. By assumption, x∈ ([[xΓTΓT]Γ[xΓTΓT]Γ[xΓTΓT]]] ⊆ ([AΓAΓA]].
Then A ⊆ ([AΓAΓA]], whence ([AΓAΓA]] = A.
(ii) ⇒ (i). Assume that ([AΓAΓA]] = A for all right po-ternary Γ-ideals A of T.
Let x ∈ T. Since ({x}∪[xΓTΓT]] is a right po-ternary Γ-ideal of T, we have
({x}∪[xΓTΓT]] = ([({x}∪[xΓTΓT]]Γ({x}∪[xΓTΓT]]Γ({x}∪[xΓTΓT]]]
⊆({[xΓxΓx]}∪[[xΓxΓx]TΓT]∪[[xΓxΓT]ΓTΓx]∪[[xΓxΓT]ΓTΓ[xΓTΓT]]∪[[xΓTΓT]ΓxΓx]
∪[[xΓTΓT]ΓxΓ[xΓTΓT]]∪[[xΓTΓT]Γ[xΓTΓT]Γx]∪[[xΓTΓT]Γ[xΓTΓT]Γ[xΓTΓT]]].
Since x∈ ({x}∪ [xΓTΓT]], we obtain (by calculations) x∈ ([[xΓTΓT]Γ[xΓTΓT]Γ[xΓTΓT]]].
Hence T is right weakly regular.
(i)⇒(iii). Assume that T is right weakly regular. Let B and A be a right po-ternary Γ-ideal and a two-sided po-ternary Γ-ideal of T, respectively. Since [BΓAΓA] ⊆ [BΓTΓT] ⊆ B, ([BΓAΓA]] ⊆ B.
Similarly, ([BΓAΓA]] ⊆ A.Then ([BΓAΓA]] ⊆ B ∩ A.
Let x∈ B ∩ A. We have ([[xΓTΓT]Γ[xΓTΓT]Γ[xΓTΓT]]] ⊆ ([BΓAΓA]].
By assumption, we get x∈ ([[xΓTΓT]Γ[xΓTΓT]Γ[xΓTΓT]]], hence x∈ ([BΓAΓA]].
Thus B∩A ⊆ ([BΓAΓA]], whence B∩A = ([BΓAΓA]].
That (iii) ⇒(iv) is clear.
(iv) ⇒ (i). Assume that B∩A = ([BΓAΓA]] for all right po-ternary Γ-ideals B and all po-ternary Γ-ideals A of T.
To prove that T is right weakly regular, let x∈ T.
We have ({x}∪[xΓTΓT]] and ({x}∪[xΓTΓT]∪[TΓTΓx]∪[TΓxΓT]∪[TΓ[TΓxΓT]ΓT]] are rightpo-ternary Γ- ideal and ideal of T, respectively.
Then ({x}∪[xΓTΓT]]∩({x}∪[xΓTΓT]∪[TΓTΓx]∪[TΓxΓT]∪[TΓ[TΓxΓT]ΓT]]
=(({x}∪[xΓTΓT]]Γ({x}∪[xΓTΓT]∪[TΓTΓx]∪[TΓxΓT]∪[TΓ[TΓxΓT]ΓT]]Γ
({x}∪[xΓTΓT]∪[TΓTΓx]∪[TΓxΓT]∪[TΓ[TΓxΓT]ΓT]]]
⊆ ({[xΓxΓx]}∪[[xΓxΓx]ΓTΓT]∪[xΓ[xΓTΓT]x]∪[xΓ[xΓTΓx]ΓT]
∪[xΓ[xΓTΓT]Γ[xΓTΓT]]∪[xΓ[TΓTΓx]Γx]∪[[xΓTΓT]Γ[xΓxΓT]ΓT]
∪[[xΓTΓT]Γ[xΓTΓT]Γx]∪[[xΓTΓT]Γ[xΓTΓx]ΓT]∪[[xΓTΓT]Γ[xΓTΓT]Γ[xΓTΓT]]
∪[[xΓTΓx]ΓTΓx]∪[[xΓTΓx]Γ[TΓxΓT]ΓT]∪[[xΓTΓx]Γ[TΓTΓx]ΓT]]
⊆ ([[xΓTΓT]Γ[xΓTΓT]Γ[xΓTΓT]]].
Thus x ∈([[xΓTΓT]Γ[xΓTΓT]Γ[xΓTΓT]]].
Hence T is right weakly regular ordered ternary Γ-semiring.

Theorem 4.5.Let T be an ordered ternary Γ-semiring. The following areequivalent.
(i) T is right weakly regular.
(ii) Every two-sided po-ternaryΓ-ideal A of T is right pure.
(iii) Every po-ternaryΓ-ideal A of T is right pure.
***Proof*:**This follows from Theorem 4.2, and Theorem 4.4.

Definition4.6: An element a of a po-ternary Γ-semigroup T is said to be ***zero*** of T provided $a\alpha b\beta c = b\alpha a\beta c = b\alpha c\beta a = a$ and $a \leq t\ \forall\ b, c \in$ T, $\alpha, \beta \in$ Γ.

Theorem 4.7.Let T be an ordered ternary Γ-semiring with zero 0.
(i) {0}is a right pure po-ternaryΓ-ideal of T.
(ii) Union of any right pure two-sided po-ternaryΓ-ideals (respectively, po-ternary Γ-ideal) of T is a right pure two-sided po-ternaryΓ-ideal (respectively, po-ternary Γ-ideals) of T.
(iii) Finite intersection of right pure two-sided ideals (respectively, ideal) of T isa right pure two-sided po-ternaryΓ-ideal (respectively, po-ternary Γ-ideals) of T.
Proof:(i) This is obvious.
(ii) Let A_i, $i \in I$ be right pure two-sided po-ternary Γ-ideals of T. We have $\bigcup_{i\in I} A_i$ is a two-sided po-ternary Γ-ideal of T. Let $x \in \bigcup_{i\in I} A_i$. Then $x \in A_j$ for some $j \in I$.
Since A_j is right pure two-sided po-ternary Γ-ideal, there exist $y, z \in A_j$, $\alpha, \beta \in \Gamma$ such that $x \leq [x\alpha y\beta z]$. Since $y, z \in A_j \subseteq \bigcup_{i\in I} A_i$, we have $\bigcup_{i\in I} A_i$ is right pure.
(iii) Let $A_1, A_2, \ldots\ldots A_n$ be right pure two-sided po-ternary Γ-ideals of T.

Then $\bigcap_{i=1}^{n} A_i$ is a two-sided po-ternary Γ-ideal of *T*.

Let $x \in \bigcap_{i=1}^{n} A_i$. For $k \in \{1,2, \ldots, n\}$, there exist $y_k, z_k \in A_k, \alpha, \beta \in \Gamma$ such that $x \leq [x\alpha y_k \beta z_k]$.

We have $x \leq [[x\alpha y_n \beta z_n] \ldots\ldots [y_2 \alpha z_2 \beta y_1]\gamma z_1]$.Since $[[y_n \alpha z_n \beta y_{n-1}] \ldots\ldots [y_2 \gamma z_2 \delta y_1]\varepsilon z_1] \in \bigcap_{i=1}^{n} A_i$, we have $\bigcap_{i=1}^{n} A_i$ is a right pure two-sided po-ternary Γ-ideal of *T*.

Theorem 4.8.Let T be an ordered ternary Γ-semiring with zero 0 and A is a two-sided po-ternary Γ-ideal of T. Then A contains the largest right pure two-sided po-ternary Γ-ideal of T, denoted by S(A). S(A) is called the pure part of A.

***Proof*:** Since {0}is a right pure two-sided po-ternary ideal of *T* contained in *A*, it follows thatthe union of all right pure two-sided po-ternary Γ-ideals of *T* contained in *A* exists, and henceit is the largest right pure two-sided po-ternary Γ-ideal of *T* contained in *A*.

Similarly, we have the following.

Theorem 4.9.Let T be an ordered ternary Γ-semiring with zero 0 and A is a po-ternary Γ-ideal of T. Then A contains the largest right pure po-ternary Γ-ideal of T.

Theorem 4.10.Let T be an ordered ternary Γ-semiring with zero 0. Let A, B and A_i, $i \in I$ be two-sided po-ternary Γ-ideals of T.

(i) S(A⋂B) = S(A)⋂S(B).

(ii) $\bigcup_{i \in I} S(A_i) \subseteq S(\bigcup_{i \in I} A_i)$ **.**

***Proof*:**(i) Since $S(A) \subseteq A$ and $S(B) \subseteq B$, we have $S(A) \cap S(B) \subseteq A \cap B$.

Hence$S(A) \cap S(B) \subseteq S(A \cap B)$. Since $S(A \cap B) \subseteq A \cap B \subseteq A$, we get $S(A \cap B) \subseteq S(A)$.

Similarly, $S(A \cap B) \subseteq S(B)$. Then $S(A \cap B) \subseteq S(A) \cap S(B)$, whence $S(A \cap B) = S(A) \cap S(B)$.

(ii) Since $S(A_i) \subseteq A_i$ for all $i \in I$, we have $\bigcup_{i \in I} S(A_i) \subseteq \bigcup_{i \in I} A_i$. Then $\bigcup_{i \in I} S(A_i) \subseteq S(\bigcup_{i \in I} A_i)$.

Definition4.11: A right pure two-sided po-ternary Γ-ideal A of an ordered ternary Γ-semiring T is said to be ***purely maximal*** if for any proper right pure two-sided po-ternary Γ-ideal B of T, A ⊆ B implies A = B.

Definition 4.12: A proper right pure two-sided po-ternary Γ-ideal A of an ordered ternary Γ-semiring T is said to be ***purely prime*** if for any right pure two-sided po-ternary Γ-ideals B_1, B_2 of T, $B_1 \cap B_2 \subseteq A$ implies $B_1 \subseteq A$ or $B_2 \subseteq A$.

Theorem 4.13:**Every purely maximal two-sided po-ternary Γ-ideal of an ordered ternary Γ-semiring T is purely prime.**

***Proof*:** Let A be a purely maximal two-sided po-ternary Γ-ideal of T. Let B and C be right pure two-sided po-ternary Γ-ideals of T such that B∩C ⊆ A and B ⊈ A. Since B∪A is a right pure two-sided po-ternary Γ-ideal such that A ⊂ B∪A, so T = B∪A.

We have C = C ∩T = C ∩(B∪A) = (C ∩B)∪(C ∩A) ⊆ A. Then A is purely prime.

Theorem 4.14:The pure part of any maximal two-sided po-ternary Γ-ideal of an ordered ternary Γ-semiring T with zero is purely prime.

Proof: Let A be a maximal two-side po-ternary Γ-ideal of T. To show thatS(A) is purely prime, let B,C be right pure two-side po-ternary Γ-ideals of T such that B ∩C ⊆ S(A). If B ⊆ A, then B ⊆S(A). Suppose that B ⊈ A. We have B∪A is a two-side po-ternary Γ-ideal of T. By maximality of A, T = B∪A, and hence C ⊆ A.

Thus C ⊆S(A).

Theorem 4.15: Let T be an ordered ternary Γ-semiring and Aa right pure two-sided po-ternary Γ-ideal of T. If *x*∈ T \A, then there exists a purely prime two-sided po-ternary Γ-ideal B of T such that A ⊆ B and *x*∉ B.

***Proof*:** Let P = {B | B is a right pure two-sided po-ternary Γ-ideal of T, A ⊆ B and *x*∉ B}. Since A ∈ P, P ≠∅. Under the usual inclusion, P is a partially ordered set. Let $B_k, k \in K$ be a totally ordered subset of P. By Theorem 4.7, $\bigcup_{k \in K} B_k$ is a right pure two-sided po-ternary Γ-ideal of T. Since $A \subseteq \bigcup_{k \in K} B_k$ and $x \notin \bigcup_{k \in K} B_k$, we have $\bigcup_{k \in K} B_k \in P$.

By Zorn's Lemma, P has a maximal element, say M. Then M is a right pure two-sided po-ternary Γ-ideal, A⊆ M and *x*∉ M. We shall show that M is purely prime. Let A_1 and A_2 be right pure two-sided po-ternary Γ-ideals of T such that $A_1 \nsubseteq M$ and $A_2 \nsubseteq M$. Since A_1, A_2 and M are right pure two-sided po-ternary Γ-ideals of T, we obtain $A_1 \cup M$ and $A_2 \cup M$ are right pure two-sided po-ternary Γ-ideals of T such that $M \subset A_1 \cup M$ and

$M \subset A_2 \cup M$. Thus $x \in A_i \cup M$ (k = 1,2). Since $x \notin M$, $x \in A_1 \cap A_2$. Hence $A_1 \cap A_2 \nsubseteq M$. This shows that M is purely prime.

Theorem 4.16:Any proper right pure two-sided po-ternary Γ-ideal A of an ordered ternary Γ-semiring T is the intersection of all the purely prime two-sided po-ternary Γ-ideals of T containing A.

Proof:By Theorem 4.15, there exists purely prime po-ternary Γ-ideals containing *A*.

Let $\{B_i : I \in I\}$ be the set of all purely prime two-sided po-ternary Γ-ideals of *T* containing *A*. We have $A \subseteq \bigcap_{i \in I} B_i$. To show that $\bigcap_{k \in K} B_k \subseteq A$. Let $x \notin A$. By Theorem 4.15, there exists purely prime po-ternary Γ-ideal B_j such that $A \subseteq B_j$ and $x \notin B_j$. Hence $x \notin \bigcap_{i \in I} B_i$.

V. Weakly pure ideals in ordered ternary PO-semi-rings

In this section, we introduce the concept of weakly pure po-ternary Γ-ideal in ordered ternary Γ-semiring.

Definition 5.1. Let *T* be an ordered ternary Γ-semiring. A two-sided po-ternary Γ-ideal *A* of *T* is called *left (respectively, right) weakly pure* if $A \cap B = ([A\Gamma A\Gamma B]]$(respectively,$A \cap B = ([B\Gamma A\Gamma A]]$) for all two-sided po-ternary Γ-ideals *B* of *S*.

In an ordered ternary Γ-semiring, every left (right) pure two-sided po-ternary Γ-ideals is left(right) weakly pure.

Theorem 5.2:Let Tbe an ordered ternary semigroup with zero 0. If A and B are two-sidedpo-ternary Γ-ideals of T, then

$$B\Gamma A^{-1} = \{t \in T \mid \forall\, x, y \in A,\ \alpha, \beta \in \Gamma, [x\alpha y\beta t] \in B\}$$
$$A_{-1}\Gamma B = \{t \in T \mid \forall\, x, y \in A,\ \alpha, \beta \in \Gamma, [t\alpha x\beta y] \in B\}$$

are two-sided po-ternary Γ-ideals of T.

Proof: We shall show that $B\Gamma A^{-1}$ is a two-sided po-ternary Γ-ideal of T. That $A_{-1}\Gamma B$ is a twosided po-ternary Γ-ideal of T can be proved similarly. Clearly, $0 \in B\Gamma A^{-1}$.Let $u, v \in T$, $\alpha, \beta \in \Gamma$ and $t \in B\Gamma A^{-1}$. To show that $[u\alpha v\beta t] \in B\Gamma A^{-1}$, let$x, y \in A$. Since $[y\gamma u\alpha v] \in A$ for $\gamma, \delta \in \Gamma$, we have $[x\varepsilon y[u\delta v\beta t]] = [x\varepsilon[y\gamma u\delta v]\beta t] \in B$. Thus $[u\alpha v\beta t] \in B\Gamma A^{-1}$. Let $x \in B\Gamma A^{-1}$ and $y \in T$ be such that $y \leq x$. Let $z, w \in A$. Since $[z\alpha w\beta y] \leq [z\alpha w\gamma x]$ and $[z\alpha w\gamma x] \in B$, we have $[z\alpha w\beta y] \in B$. Hence $y \in B\Gamma A^{-1}$. Therefore, $B\Gamma A^{-1}$ is a two-sided po-ternary Γ-ideal of T.

Theorem 5.3:**Let T be an ordered ternary Γ-semiring and Aa two-sided po-ternary Γ-ideal of T. Then A is left (right) weakly pure two-sided po-ternary Γ-ideal if and only if $(B\Gamma A^{-1}) \cap A = A \cap B$ ($(B\Gamma A_{-1}) \cap A = A \cap B$) for all po-ternary Γ-ideals B of T.**

***Proof*:**Suppose that A is left weakly pure two-sided po-ternary Γ-ideal. Let B be a po-ternary Γ-ideal of T. By Theorem 5.2, $B\Gamma A^{-1}$ is a two-side po-ternary Γ-ideal of T, and thus $A \cap B\Gamma A^{-1} = ([A\Gamma A\Gamma(B\Gamma A^{-1})]]$. Since $[A\Gamma A\Gamma(B\Gamma A^{-1})] \subseteq [A\Gamma T\Gamma T] \subseteq A$, we have $([A\Gamma A\Gamma(B\Gamma A^{-1})]) \subseteq (A] = A$. Let $t \in ([A\Gamma A\Gamma(B\Gamma A^{-1})]]$ be such that $t \leq [x\alpha y\beta z]$ for some $x,y \in A, \alpha, \beta \in \Gamma, z \in B\Gamma A^{-1}$. By Definition of $B\Gamma A^{-1}$, $[x\alpha y\beta z] \in B$. Thus $t \in B$. This proves that $A \cap B\Gamma A^{-1} \subseteq A \cap B$. For the reverse inclusion, let $a \in A \cap B$. Since $[x\alpha y\beta a] \in B$ for any $x,y \in A, \alpha, \beta \in \Gamma$, we have $a \in B\Gamma A^{-1}$. We get $a \in B\Gamma A^{-1} \cap A$, and then $A \cap B \subseteq B\Gamma A^{-1} \cap A$.

Conversely, assume that $(B\Gamma A^{-1}) \cap A = A \cap B$ for all po-ternary Γ-ideal B of T. To showthat A is left weakly pure two-sided po-ternary Γ-ideal, let C be any po-ternary Γ-ideal of T. To show that $A \cap C = ([A\Gamma A\Gamma C]]$. By assumption, $A \cap C = C\Gamma A^{-1} \cap A$. Since $[A\Gamma A\Gamma C] \subseteq [A\Gamma T\Gamma T] \subseteq A$, $([A\Gamma A\Gamma C]] \subseteq A$.Let $t \in ([A\Gamma A\Gamma C]]$ such that $t \leq [x\alpha y\beta z]$ for some $x,y \in A, \alpha, \beta \in \Gamma, z \in C$ and let $a,b \in A$. Since $[a[b\beta x\gamma y]\delta z] = a\alpha b\beta[x\gamma y\delta z] \in C$, we obtain $[x\gamma y\delta z] \in C\Gamma A^{-1}$, and so $t \in C\Gamma A^{-1}$. Then $([A\Gamma A\Gamma C]] \subseteq C\Gamma A^{-1}$.This proves that $([A\Gamma A\Gamma C]] \subseteq A \cap C$. For the reverse inclusion, we have $C \subseteq ([A\Gamma A\Gamma C]]\Gamma A^{-1}$ because $c \in C, a,b \in A$, $\alpha, \beta \in \Gamma$ implies $[a\alpha b\beta c] \in [A\Gamma A\Gamma C] \subseteq ([A\Gamma A\Gamma C]]$. Then $A \cap C \subseteq ([A\Gamma A\Gamma C]]\Gamma A^{-1} \cap A = A \cap ([A\Gamma A\Gamma C]] \subseteq ([A\Gamma A\Gamma C]]$.

Theorem 5.4: Let T be an ordered ternary Γ-semiring. The following are equivalent.

(i) Every two-sided po-ternary Γ-ideal is left weakly pure two-sided po-ternary Γ-ideal.

(ii) For every two-sided po-ternary Γ-ideal A of T, [AΓAΓA] = A. i.e. each two-sided po-ternary Γ-ideal is idempotent.

(iii) Every two-sided po-ternary Γ-ideal is right weakly pure two-sided po-ternary Γ-ideal.

Proof **:** (i) ⇒ (ii) Suppose that each two-sided po-ternary Γ-ideal of T is left weakly pure. Let A be the two sided po-ternary Γ-ideal of T, then for each two- sided po-ternary Γ-ideal B of T we have $A \cap B = A\Gamma A\Gamma B$. In particular $A = A \cap A = A\Gamma A\Gamma A$. Therefore each two-sided po-ternary Γ-ideal of T is idempotent.

(ii) ⇒ (i) Suppose that each two-sided po-ternary Γ-ideal of T is idempotent. Let A be a two-sided po-ternary Γ-ideal of T, then for any two-sided po-ternary Γ-ideal B of T we always have AΓAΓB = A ∩ B. On the other hand, A ∩ B = (A ∩ B)Γ(A ∩ B)Γ(A ∩ B) ⊆ AΓAΓB. Hence we have A ∩ B = AΓAΓB. Thus A is left weakly pure.
(ii) ⇒ (iii) Similarly as (ii) ⇒ (i)
(iii) ⇒ (ii) Suppose that each two-sided po-ternary Γ-ideal of T is right weakly pure two-sided po-ternary Γ-ideal. Let A be any two-sided po-ternary Γ-ideal of T. Then A is right weakly pure. Therefore for each two-sided po-ternary Γ-ideal B of T, we have A ∩ B = BΓAΓA. In particular A ∩ A = AΓAΓA. Thus each two-sided po-ternary Γ-ideal of T is idempotent.

6. Pure spectrum of an ordered ternary Γ-semiring

Notation 6.1 : Let *T* be an ordered ternary Γ-semiring with zero such that [*T*Γ*T*Γ*T*] = *T*.The set of all right pure po-ternary Γ-ideals of *T* and the set of all proper purely prime po-ternary
Γ-ideals of *T* will be denoted by *P*(*T*) and *P′*(*T*), respectively. For A ∈ P(T), let

$$I_A = \{J \in P'(T) \mid A \not\subseteq J\} \text{ and } \tau(T) = \{I_A \mid A \in P(T)\}.$$

Theorem 6.2: τ(T) forms a topology on P′(T).
Proof: Since {0} is a right pure po-ternary Γ-ideal of T and $I_{\{0\}} = \emptyset$, we have ∅ ∈ τ(T). Since T is a right pure po-ternary Γ-ideal of T such that $I_T = P'(T)$, we get P′(T) ∈ τ(T).

Let $\{ I_{A_\alpha} \mid \alpha \in \Lambda\} \subseteq \tau(T)$. We have $\bigcup_{\alpha\in\Lambda} I_{A_\alpha} = \{J \in P(T) : A_\alpha \not\subseteq J \text{ for some } \alpha \in \Lambda\} = \{J \in P(T) : \cup A_\alpha \not\subseteq J\} = I_{\bigcup_{\alpha\in\Lambda} A_\alpha}$. Whence $\bigcup_{\alpha\in\Lambda} I_{A_\alpha} \in \tau(T)$. Let $I_{A_1}, I_{A_2} \in \tau(T)$. We shall show that $I_{A_1} \cap I_{A_2} = I_{A_1\cap A_2}$, therefore let $J \in I_{A_1} \cap I_{A_2}$. We have J ∈ P′(T), $A_1 \not\subseteq J$ and $A_2 \not\subseteq J$. Suppose that $A_1 \cap A_2 \subseteq J$. Since J is purely prime, $A_1 \subseteq J$ or $A_2 \subseteq J$. A contradiction. Then $J \in I_{A_1} \cap I_{A_2}$, hence $I_{A_1} \cap I_{A_2} \subseteq I_{A_1\cap A_2}$. For the reverse inclusion, let $J \in I_{A_1\cap A_2}$. Since $A_1 \cap A_2 \not\subseteq J$, $A_1 \not\subseteq J$ and $A_2 \not\subseteq J$. This implies that $J \in I_{A_1} \cap I_{A_2}$, thus $I_{A_1\cap A_2} \subseteq I_{A_1} \cap I_{A_2}$. Consequently, $I_{A_1} \cap I_{A_2} = I_{A_1\cap A_2}$, which implies $I_{A_1} \cap I_{A_2} \in \tau(T)$. Therefore τ(T) forms a topology on P′(T).

VI. Conclusion

In this paper mainly we start the study of pure po-ternary Γ-ideals, weakly pure po-ternary Γ-ideals and purely prime po-ternary Γ-ideals in po-ternary Γ-semirings. We characterize po-ternary Γ-semirings by the properties of pure and weakly pure po-ternary Γ-ideals.

Acknowledgments

The first and third authors express their warmest thanks to the University Grants Commission(UGC), India, for doing this research under Faculty Development Programme.

The authors would like to thank the experts who have contributed towards preparation and development of the paper and the referees, Chief Editor for the valuable suggestions and corrections for improvement of this paper.

References

[1]. Ahsan. J, Takahashi. M, Pure spectrum of a monoid with zero, Kobe J. Mathematics 6 (1989) 163–182.
[2]. Bashir. S, Shabir. M, Pure ideals in ternary semigroups, Quasigroups and Related Systems 17 (2009) 149–160.
[3]. Changphas. T, Bi-ideals in ordered ternary semigroups, Applied Mathematical Sciences 6 (2012) 5943–5946.
[4]. Changphas. T, Bi-ideals in ternary semigroups, Applied Mathematical Sciences 6 (2012) 5939–5942.
[5]. Chinram. R, Saelee. S, Fuzzy ideals and fuzzy filters of ordered ternary semigroups, Journal of Mathematics Research 2 (2010) 93–97.
[6]. Dixit. V. N, Dewan. S, A note on quasi and bi-ideals in ternary semigroups, Internat. J. Math. Math. Sci. 18 (3) (1995) 501–508.
[7]. Iampan. I, On ordered ideal extensions of ordered ternary semigroups, Lobachevskii J. Math. 31 (1) (2010) 13–17.
[8]. Los. J, On the extending of models I, Fundam. Math. 42 (1995) 38–54.
[9]. Santiago. M. L, Sri Bala. S, Ternary semigroups, Semigroup Forum 81 (2010) 380–388.
[10]. Sioson F. M, Ideal theory in ternary semigroups, Math. Japan 10 (1965) 63– 84.
[11]. Syam julius rajendra. V, Dr. Madhusudhana rao. D, Sajani lavanya. M-A Study on Completely Regular po-Ternary **Γ**-Semiring, Asian Academic Research Journal of Multidisciplinary, Volume 2, Issue 4, September 2015, PP: 240-246.

Authors's Brief Biography:

[2]**Dr. D. MadhusudhanaRao:** He completed his M.Sc. from Osmania University, Hyderabad, Telangana, India. M. Phil. from M. K. University, Madurai, Tamil Nadu, India. Ph. D. from Acharya Nagarjuna University, Andhra Pradesh, India. He joined as Lecturer in Mathematics, in the department of Mathematics, VSR & NVR College, Tenali, A. P. India in the year 1997, after that he promoted as Head, Department of Mathematics, VSR & NVR College, Tenali. He helped more than 5 Ph. D's. At present he is guiding 7 Ph. D. Scholars and 3 M. Phil., Scholars in the department of Mathematics, AcharyaNagarjuna University, Nagarjuna Nagar, Guntur, A. P.
A major part of his research work has been devoted to the use of semigroups, Gamma semigroups, duo gamma semigroups, partially ordered gamma semigroups and ternary semigroups, Gamma semirings and ternary semirings, Near rings ect. He acting as peer review member to (1) "**British Journal of Mathematics & Computer Science", (2)** "**International Journal of Mathematics and Computer Applications Research", (3) "Journal of Advances in Mathematics"**and Editorial Board Member of (4) **"International Journal of New Technology and Research".** He is life member of (1) **Andhra Pradesh Society for Mathematical Sciences**, (2) Heath Awareness Research Institution Technology Association, (3) **Asian Council of Science Editors, Membership No: 91.7347, (4) Council for Innovative Research for Journal of Advances in Mathematics".** He published more than **62 research papers** in different International Journals to his credit in the last four academic years.

[1]**Mr. V. Syam Julius Rajendra**: He completed his M.Sc. from Madras Christian College, under the jurisdiction of University of Madras, Chennai,Tamilnadu. After that he did his M.Phil. from M. K. University, Madurai, Tamilnadu, India. He joined as lecturer in Mathematics, in the department of Mathematics, A. C. College, Guntur, Andhra Pradesh, India in the year 1998. At present he is pursuing Ph.D. under guidance of Dr. D. Mathusudhana Rao, Head, Department of Mathematics, VSR & NVR College, Tenali, Guntur(Dt), A.P. India in Acharya Nagarjuna University. His area of interests are ternary semirings, ordered ternary semirings, semirings and topology. He published more than **02 research papers** in different International Journals to his credit. Presently he is working on Partially Ordered Ternary Γ-Semirings.

[3]**Mrs. M. Sajani Lavanya** : She completed her M.Sc. from Hindu College, Guntur, under the jurisdiction of Acharya Nagarjuna University, Guntur, Andhra Pradesh, India. She joined as lecturer in Mathematics, in the department of Mathematics, A. C. College, Guntur, Andhra Pradesh, India in the year 1998. At present she pursuing Ph.D. under guidance of Dr. D. Mathusudhana Rao, Head, Department of Mathematics, VSR & NVR College, Tenali, Guntur(Dt), A.P. India in Acharya Nagarjuna University. Her area of interests are ternary semirings, ordered ternary semirings, semirings and topology. He published more than **02 research papers** in different International Journals to his credit. Presently she is working on Ternary Γ-Semirings.