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Great Algebra or simply Gr-Algebra

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Abstract: *A new algebraic structure Great Algebra or simply Gr-Algebra with three binary operations is defined. Additive Commutative Gr-Algebra, Multiplicative Commutative Gr-Algebra, Bi- Commutative Gr-Algebra, Tri-commutative Gr-Algebra, Gr-Algebra with multiplicative identity, Multiplicative Gr-Algebra Unit, Division Gr-Algebra, Field Gr-Algebra, R-identity and L-identity and identity of a Gr-Algebra are defined.*
Keywords : *Gr-Algebra, Division Gr-Algebra, Field Gr-Algebra, R-identity and L-identity and identity, Right-Unit, Left-Unit.*

I. Introduction

Algebraists say group theory is one of the richest branches of algebra. W e study group with one binary operation. But in rings there are two binary operations. If (R, + , .) is a ring, then with respect to the first operation, it should be an abelian group and with respect to the second operation it should be a monoid. Distributive property is also satisfied. After defining ring, commutative ring, ring with identity and division ring are defined. This motivated me to define the new algebraic structure Great Algebra or simplyGr-Algebra. The study of the new algebraic structure Great Algebra or Gr-Algebra with three binary operations will motivate the researcher.

II. Great Algebra

2.1 Definition : Great Algebra or simply Gr-Algebra: A non-empty subset A together with three binary operations denoted by + , **.** and / is called a Great Algebra or simply Gr-Algebra if1. (A , +) is a group
2. (A , **.**) is a semigroup
3. **.** is distributive over + and
4. / is closed in A.
A Great Algebra or Gr-Algebra is always denoted by (A, +, **.** , /).

2.1.1 Example : Let Q be the set of all rational numbers. Then (Q , + , **.** ÷), where + is the usual addition, . is the usual multiplication and ÷ is the division, is a Great Algebra or simply Gr-Algebra.

2.1.2 Example : Let R be the set of all real numbers. Then (R , + , **.** ÷), where + is the usual addition, . is the usual multiplication and ÷ is the division, is a Gr-Algebra.

2.1.3 Example : Let S be any set. Consider P(S), the power set of S. Then (P(S) , ∆ , ∩ , ∪), where ∆ is the symmetric difference of sets ,∩ is the intersection of sets and ∪ is the union of sets, is a Gr-Algebra.

2.2 Definition : Additive Commutative Gr-Algebra : A Gr-Algebra (A, +, **.**, /) is said to be additive commutative if a + b = b + a, for all a,bϵA.

2.2.1 Example : (Q , + , **.** ÷), where + is the usual addition, . is the usual multiplication and ÷ is the division, is an additive commutative Gr-Algebra.

2.3 Definition : Multiplicative Commutative Gr-Algebra: A Gr-Algebra (A, +, ., /) is said to be multiplicative commutative if a.b = b. a, for all a,bϵA.

2.3.1 Example : (Q , + , . ÷), where + is the usual addition, . is the usual multiplication and ÷ is the division, is a multiplicative commutative Gr-Algebra.

2.4 Definition : Bi- Commutative Gr-Algebra : A Gr-Algebra (A, +, **.**, /) is said to be a bi-commutativeGr-Algebra if (i) a + b = b + a, for all a,bϵA and (ii) a.b = b. a, for all a,bϵA.

2.4.1 Example : (Q , + , . ÷), where + is the usual addition, . is the usual multiplication and ÷ is the division, is a bi- commutative Gr-Algebra.

2.4.2 Example : (R , + , . ÷), where + is the usual addition, . is the usual multiplication and ÷ is the division, is a bi- commutative Gr-Algebra.

2.5 Definition : Tri-commutative Gr-Algebra : A Gr-Algebra (A, +, **.**, /) is said to be a Tri-commutative Gr-Algebra if (i) a + b = b + a, for all a,bϵA and (ii) a.b = b. a, for all a,bϵA and (iii) a / b = b / a, for all a,bϵA

2.5.1 Example : Let S be any set. Consider P(S), the power set of S. Then (P(S) , Δ , ∩ , ∪), where Δ is the symmetric difference of sets ,∩ is the intersection of sets and ∪ is the union of sets, is a Tri-commutative Gr-Algebra.

2.6 Definition : Gr-Algebra with multiplicative identity : A Gr-Algebra (V, +, **.**, /) is said to be a Gr-Algebra with multiplicative identity if there exists an element denoted by 1 in A such that a.1 = a = 1.a, for all aϵA.

2.6.1 Example : (Q , + , . ÷), where + is the usual addition, . is the usual multiplication and ÷ is the division, is a Gr-Algebra with multiplicative identity

2.6.2 Example : (R , + , . ÷), where + is the usual addition, . is the usual multiplication and ÷ is the division, is a Gr-Algebra with multiplicative identity.

2.7 Definition : Multiplicative Gr-Algebra Unit : Let (A, +, ., /) be a Gr-Algebra with multiplicative identity 1. An element a ≠ 0 in A is said to be a multiplicative Gr-Algebra unit if there exists an element b ≠ 0 in A such that a.b = 1 = b.a.

2.8 Definition : Division Gr-Algebra : A Gr-Algebra (A, +, ., /) with multiplicative identity 1 is said to be a Division Gr-Algebra if every non-zero element in A is a multiplicative Gr-Algebra unit in A.

2.8.1 Example :. Every non-zero element in (Q , + , . ÷), where + is the usual addition, . is the usual multiplication and ÷ is the division, is a multiplicative Gr-Algebra unit in Q.

2.8.2 Example : Every non-zero element in (R , + , . ÷), where + is the usual addition, . is the usual multiplication and ÷ is the division, is a multiplicative Gr-Algebra unit in R.

2.9 Definition : Field Gr-Algebra : A Bi-commutative Division Gr-Algebra (A, +, ., /) is said to be a Field Gr-Algebra.

2.9.1 Example : (Q , + , . ÷), where + is the usual addition, . is the usual multiplication and ÷ is the division, is a Field Gr-Algebra.

2.9.2 Example : (R , + , . ÷), where + is the usual addition, . is the usual multiplication and ÷ is the division, is a Field Gr-Algebra.

2.10 Definition : A Gr-Algebra (A, +, ., /) is said to have a **right-identity** or simply a **R-identity** if there exists an element 1' in A such that a / 1' = a, for all aϵA.
The element 1' is called right-identity or R-identity of the Gr-Algebra(A, +, ., /).

2.10.1 Example : In the Gr-Algebra (R , + , . ÷), where + is the usual addition, . is the usual multiplication and ÷ is the division, 1 is the R-identity.

2.11 Definition : A Gr-Algebra (A, +, ., /) is said to have a **left-identity** or simply a **L-identity** if there exists an element 1' in A such that 1' / a = a, for all aϵA.
The element 1' is called left-identity or L-identity of the Gr-Algebra(A, +, ., /).

2.12 Definition :A Gr-Algebra (A, +, ., /) is said to have an i**dentity** if there exists an element 1' in A such that a / 1' = a = 1' / a , for all aϵA.

2.12.1 Example : Let S be any set. Consider P(S), the power set of S. Then (P(S) , Δ , ∩ , ∪), where Δ is the symmetric difference of sets ,∩ is the intersection of sets and ∪ is the union of sets, is a Gr-Algebra. In this Gr-Algebra (P(S) , Δ , ∩ , ∪), ϕ, the empty set, is the R-identity and L-identity and hence the identity of the Gr-Algebra.

2.13 Definition : Right-Unit : An element a ≠ 0 in a field Gr-Algebra (A, +, ., /) with R-identity is said to be a Right-Unit of the Gr-Algebra or simply a R-Unit of the Gr-Algebra if there exists an element a'≠ 0 in A such that a / a' = 1'.

2.13.1 Example : (R , + , . ÷), where + is the usual addition, . is the usual multiplication and ÷ is the division, is a Field Gr-Algebra. Every non-zero element in the Gr-Algebra (R , + , . ÷) is a R-unit.

2.14 Definition : Left-Unit : An element a ≠ 0 in a field Gr-Algebra (A, +, ., /) with L-identity is said to be a Left-Unit of the Gr-Algebra or simply a L-Unit of the Gr-Algebra if there exists an element a'≠ 0 in A such that a'/ a = 1'.

2.14.1 Example : (R , + , . ÷), where + is the usual addition, . is the usual multiplication and ÷ is the division, is a Field Gr-Algebra. Every non-zero element in the Gr-Algebra (R , + , . ÷) is a L-unit.

2.15 Definition : Unit : An element a ≠ 0 in a field Gr-Algebra (A, +, ., /) is said to be a Unit 0f the Gr-Algebra if it is both Left-unit and Right-unit..

2.15.1 Example : (R , + , . ÷), where + is the usual addition, . is the usual multiplication and ÷ is the division, is a Field Gr-Algebra. Every non-zero element in the Gr-Algebra (R , + , . ÷) is unit.

2.16 Definition : Let (A, +, ., /) be a Bi-commutative Gr-Algebra with multiplicative identity 1.
Define A^2 = { a.b / a, b ϵ A }
Define / on A^2 by the following ;
For x, y ϵ A, where x= ab and y = cd, x / y = ab/cd = (a/c).(b/d) = (a/d).(b/c)

2.17 Definition : Let (A, +, ., /) be a Bi-commutative Gr-Algebra with multiplicative identity 1. An element a ≠ 0 in A is said to be a **multiplicative zero divisor** if there exists an element b ≠ 0 in A such that a.b = 0.

2.18 Propositions

2.18.1 Proposition : Let (A, +, ., /) be a Bi-commutative Gr-Algebra with multiplicative identity 1, R-identity 1' and without multiplicative zero divisor. Then the set of all R-units of A is closed under **.** in A.

Proof : Let (A, +, ., /) be a Bi-commutative Gr-Algebra with multiplicative identity 1, R-identity 1'and without multiplicative zero divisor.
Let R be the set of all R-units of A.
Let a, b ϵ R
Then a and b are R-units of A.
Therefore, a ≠ 0 and b ≠ 0
Also there exist elements c ≠ 0 and d ≠ 0 in A such that a / c = 1' and b / d = 1'.
Since c ≠ 0 and d ≠ 0 and A has no multiplicative zero divisor , cd ≠ 0.
Since a ≠ 0 and b ≠ 0 and A has no multiplicative zero divisor , ab ≠ 0.
Now ab / cd = (a/c).(b/d)
= 1'.1'
= 1'
Therefore, ab is a R-unit in A.
Therefore, ab ϵ R.

2.18.2 Proposition : Let (A, +, ., /) be a Bi-commutative Gr-Algebra with multiplicative identity1, L-identity 1'and without multiplicative zero divisor. Then the set of all L-units of A is closed under **.** in A.

Proof : Let (A, +, ., /) be a Bi-commutative Gr-Algebra with multiplicative identity 1, L-identity 1'and without multiplicative zero divisor.
Let L be the set of all L-units of A.
Let a, b ϵ L

Then a and b are L-units of A.
Therefore, a $\neq 0$ and b $\neq 0$
Also there exist elements c $\neq 0$ and d $\neq 0$ in A such that c / a = 1' and d / b = 1'.
Since c $\neq 0$ and d $\neq 0$ and A has no multiplicative zero divisor , cd $\neq 0$.
Since a $\neq 0$ and b $\neq 0$ and A has no multiplicative zero divisor , ab $\neq 0$.
Now cd / ab = (c/a).(d/b)
= 1'.1'
= 1'
Therefore, ab is a L-unit in A.
Therefore, ab ϵ L.

III. Conclusion

The study of the new algebraic structure Great Algebra or simply Gr-Algebra with three binary operations will motivate the researcher. Additive Commutative Gr-Algebra, Multiplicative Commutative Gr-Algebra, Bi- Commutative Gr-Algebra, Tri-commutative Gr-Algebra are very interesting nto study. Gr-Algebra with multiplicative identity, Multiplicative Gr-Algebra Unit, Division Gr-Algebra, Field Gr-Algebra, R-identity and L-identity and identity of a Gr-Algebra are also interesting and thought provoking.

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