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Symmetric Left Bi-Derivations in Semiprime Rings

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Abstract: *Let R be a 2-torsion free semiprime ring. Let $D(.,.): R \times R \to R$ be a symmetric left bi-derivation such that if (i) $xy \pm d(xy) = yx \pm d(yx)$, for all $x, y \in R$ and (ii) $[x,y] - d(xy) + d(yx) \in Z(R)$ or $[x,y] + d(xy) - d(yx) \in Z(R)$ for all $x, y \in R$, where d is a trace of D. Thenboth the cases of R is commutative.*
***Key Words**: Semiprime ring, Symmetric mapping, Trace,Derivation, Symmetric bi-derivation, Symmetric left bi-derivation.*

I. Introduction

The concept of bi-derivation was introduced by Maksa[3]. It is shown in [4] thatsymmetric bi-derivations are related to general solution of some functional equations. Vukman[6], [7] has studied some results concerning symmetric bi-derivations in prime and semiprime rings.The study ofcentralizing and commuting mappings was initiated by a well known theorem due to Posner[5] which states that the existence of nonzero centralizing derivation on a primering R implies that R is commutative. Daif and Bell [2] proved that if asemiprime ring R admits a derivation d such that $xy \pm d(xy) = yx \pm d(yx)$, for all $x, y \in R$,then R is commutative. Ashraf [1] proved that commutativity of a ring R which admits a symmetric bi-derivation $D(.,.): R \times R \to R$ such that (i) $xy \pm d(xy) = yx \pm d(yx)$, for all $x, y \in R$ and (ii) $[x,y] - d(xy) + d(yx) \in Z(R)$ or $[x,y] + d(xy) - d(yx) \in Z(R)$ for all $x, y \in R$, where d is a trace of D. Then both the cases of R is commutative.In this paper we proved some results on symmetric left bi-derivations in semiprime rings.

Throughout this paper R will be an associative ring with center Z(R). Recall that a ring R is prime if $aRb = (0)$ implies that $a = 0$ or $b = 0$, and is a semiprime if $axa = 0$ implies $a = 0$. We shall writecommutator$[x,y]$ for $xy - yx$ and use the identities$[xy,z] = [x,z]y + x[y,z]$, $[x,yz] = [x,y]z + y[x,z]$.An additive mapping $d: R \to R$ is called derivation if $d(xy) = d(x)y + xd(y)$ holds for all $x, y \in R$ .A mapping $B(.,.): R \times R \to R$ is said to be symmetric if $B(x,y) = B(y,x)$ holds for all $x, y \in R$ .A mapping $f: R \to R$ defined by $f(x) = B(x,x)$, where $B(.,.): R \times R \to R$ is a symmetric mapping, is called a trace of B. It is obvious that, in case $B(.,.): R \times R \to R$ is symmetric mapping which is also bi-additive (i. e. additive in both arguments) the trace of B satisfies the relation $f(x+y) = f(x) + f(y) + 2B(x,y)$, for all $x, y \in R$.We shall use also the fact that the trace of a symmetric bi-additive mapping is an even function.A symmetric bi-additive mapping $D(.,.): R \times R \to R$ is called a symmetric bi-derivation if $D(xy,z) = D(x,z)y + xD(y,z)$ is fulfilled for all $x, y, z \in R$ .Obviously, in this case also the relation $D(x,yz) = D(x,y)z + yD(x,z)$ for all $x, y, z \in R$. A symmetric bi-additive mapping $D(.,.): R \times R \to R$ is called a symmetric left bi-derivation if $D(xy,z) = xD(y,z) + yD(x,z)$ for all $x, y, z \in R$. Obviously, in this case also the relation $D(x,yz) = yD(x,z) + zD(x,y)$ for all $x, y, z \in R$. A mapping $f: R \to R$ is said to be commuting on R if $[f(x),x] = 0$ holds for all $x \in R$. A mapping $f: R \to R$ is said to be centralizing on R if $[f(x),x] \in Z(R)$ is fulfilled for all $x \in R$. A ring R is said to bea n-torsion free if whenever $na = 0$, with $a \in R$, then $a = 0$, where n is nonzero integer.

Theorem 1: Let R be a 2-torsion freesemiprime ring. Suppose that there exists a symmetric left bi-derivation $D(.,.): R \times R \to R$ such that $xy \pm d(xy) = yx \pm d(yx)$ for all $x, y \in R$, where d is a trace of D. Then R is commutative.

Proof: We have $xy - d(xy) = yx - d(yx)$, for all $x, y \in R$.

$$xy - yx = d(xy) - d(yx)$$
$$[x,y] = D(xy,xy) - D(yx,yx)$$
$$[x,y] = \big(xD(y,xy) + yD(x,xy)\big) - \big(yD(x,yx) + xD(y,yx)\big)$$
$$[x,y] = \big(x^2D(y,y) + xyD(y,x) + yxD(x,y) + y^2D(x,x)\big) - \big(y^2D(x,x) + yxD(x,y) + xyD(y,x) + x^2D(y,y)\big)$$
$$[x,y] = x^2d(y) + xyD(y,x) + yxD(x,y) + y^2d(x) - y^2d(x) - yxD(x,y) - xyD(y,x) - x^2d(y)$$

$[x,y] = 0$, for all $x, y \in R$, and hence R is commutative.

Use the similar arguments if R satisfies the property $xy + d(xy) = yx + d(yx)$, for all $x, y \in R$,we can prove that R is commutative. □

Theorem 2: Let R be a 2-torsion free semiprime ring. Suppose that there exists a symmetric left bi-derivation $D(.,.): R \times R \to R$ such that either $[x,y] - d(xy) + d(yx) \in Z(R)$ or $[x,y] + d(xy) - d(yx) \in Z(R)$ for all $x, y \in R$, where d is a trace of D.Then R is commutative.

Proof: We have $[x, y] - d(xy) + d(yx) \in Z(R)$, for all $x, y \in R$.

$$[x, y] - D(xy, xy) + D(yx, yx) \in Z(R)$$

$$[x, y] - \big(xD(y, xy) + yD(x, xy)\big) + \big(yD(x, yx) + xD(y, yx)\big) \in Z(R)$$

$$[x, y] - \big(x^2D(y, y) + xyD(y, x) + yxD(x, y) + y^2D(x, x)\big) + \big(y^2D(x, x) + yxD(x, y) + xyD(y, x) + x^2D(y, y)\big) \in Z(R)$$

$$[x, y] - x^2d(y) - xyD(y, x) - yxD(x, y) - y^2d(x) + y^2d(x) + yxD(x, y) + xyD(y, x) + x^2d(y) \in Z(R)$$

$[x, y] \in Z(R)$, for all $x, y \in R$. (1)

We replace y by yx in (1), we get

$$[x, yx] \in Z(R)$$

$$y[x, x] + [x, y]x \in Z(R)$$

$$[x, y]x \in Z(R)$$

$[x, y][x, r] = 0$, for all $x, y, r \in R$. (2)

We replace r by ry in (2), we get

$$[x, y][x, ry] = 0$$

$$[x, y]r[x, y] + [x, y][x, r]y = 0$$

By using (2) in the above equation, we get

$$[x, y]r[x, y] = 0$$

By the semiprimeness of R the above equation gives that $[x, y] = 0$ for all $x, y \in R$, and hence R is commutative.

Use the similar arguments if R satisfies the property $[x, y] + d(xy) - d(yx) \in Z(R)$ we can prove that R is commutative. □

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