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Transient free convective MHD flow through porous medium in slip flow regime

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Abstract: *The unsteady free convective viscous incompressible flow of an electrically conducting fluid, under uniform magnetic field, through porous medium near vertical insulated wall with variable suction in slip flow has been investigated. The governing equation in non-dimensional form is solved with the help of Perturbation technique. An approximated solution for velocity, skin friction, temperature and Nusselt number are obtained. The effects of various non-dimensional parameters on velocity profile, skin friction and Nusselt number are shown graphically.*

Nomenclature

B_o	magnetic field
C_p	specific heat at constant pressure
Gr	Grashof number
h	velocity slip parameter
K	permeability of the porous medium
M	magnetic parameter
Nu	Nusselt number
Pr	Prandtl number
t	dimensionless time
t'	dimensionless time
u'	velocities along to the plate
V_0	scale of suction velocity
y'	perpendicular to the plate, respectively
Rv	dimensionless ratio of viscosity
T	temperature of the fluid
T_∞	temperature of the free steam
T_w	temperature of the wall

Greek symbols

β_0	coefficient of thermal induction
ε	scalar constant (1)
υ	kinematic viscosity
ρ	fluid density
θ	Ratio of temperature difference
κ	thermal conductivity
μ	dynamic viscosity of fluid
μ_{eff}	effective viscosity of porous medium

I. Introduction

In recent decay, The fluid flow through porous media has attracted the attention of many researchers in recent years because it is encountered in many industrial applications such as optimization of solidification processes of metals and alloys, waste nuclear processing, dissemination control of chemical waste and pollutants, and design of MHD power generators.

The Ingham and Pop [1990], and Nield and Bejan [1998] have worked on porous media. Siegel [1958], Berezovsky et al. [1977], and Martynenko et al. [1984] have studied the free convective flow on vertical flat plate whereas Sharma and Chaudhary [2003] investigated the effect of variable suction on transient free convective viscous incompressible flow past a vertical plate with periodic temperature variations in slip flow regime. Later on, Das et al. [1999] have discussed on transient free convection flow past an infinite vertical plate with periodic temperature variation. Sahin [2010] studied oscillatory three dimensional flows and heat and mass transfer through a porous medium in presence of periodic suction. Uwanta et al. [2011] have examined heat and mass transfer flow through a porous medium with variable permeability and periodic suction. Mishra et al. [2014] have found some result on transient free convection flow past a vertical plate through porous medium with variation in slip flow regime obtained by perturbation method.

Research works in the magnetohydrodynamics (MHD) have been advanced significantly during the last four decades in sciences and engineering disciplines after the pioneer work of Hartmann [1937] in liquid metal duct flows under the influence of a strong external magnetic field. This fundamental investigation provides basic knowledge for the development of several MHD devices such as MHD pumps, generators, breaks, and flow meters. Further, Cramer and Pai [1973] investigated several results on MHD convective flows.

Jha [1990] have analytically studied the natural convective flow along a vertical infinite plate under a constant magnetic field. Singh and their collaborators have carried out a number of studies on hydromagnetic free convection covering several aspects such as effects of moving boundaries, rotation, asymmetrical heating, porous boundary, etc. [Chandran et al [1993, 1996, 1998, 2001]]. Kim [2000] presented the unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction.

In recent years, Singh et al. [2010], and Singh & Singh [2012] have studied effect of induced magnetic field on natural convection in vertical wall and concentric annuli respectively whereas Singh & Singh [2012] considered magnetohyrdynamic free convective flow past on semi infinite permeable vertical wall. Thermal radiation and oscillating plate temperature effects on MHD unsteady flow past a semi-infinite porous vertical plate under suction and chemical reaction investigated by Sharma and Deka [2012]. Recently, Singh et al. [2014] have studied effect of induced magnetic field on mixed convective flow.

In present study, we have discussed effect of magnetic field on unsteady free convective transient flow and heat transfer of a viscous incompressible fluid near a vertical flat plate embedded in a porous medium. The governing equations are solved by using perturbation technique. The behaviour of velocity, Nusselt numbers and skin friction for the non-dimensional parameters such as permeability(K), slip parameter(h),ratios of viscosities (Rv), Grashof number(Gr) and Hartmann number(M) are shown graphically.

II. Mathematical model

Consider a viscous, incompressible, electrically conducting fluid flow through a porous media occupying a semi-infinite region bounded by a vertical wall with slip boundary condition. Surface of wall is considered along X-axis in upward direction and Y-axis is considered direction perpendicular to wall. A uniform magnetic field of strength B_0 is applied perpendicular to the wall. The magnetic Reynolds number is assumed to be small enough to neglect the induced magnetic field. Then governing equations, after neglecting viscous dissipation and under usual Boussinesq's approximation, can be given as follows (Mishra et al. [2014]):

$$\frac{\partial u'}{\partial t'} - V_0'\left(1+\varepsilon e^{-n't'}\right)\frac{\partial u'}{\partial y'} = g\beta(T'-T_\infty') + \frac{\mu_{eff}}{\rho}\frac{\partial^2 u'}{\partial y'^2} - \frac{\mu_f}{\rho}\frac{u}{k} - \frac{\sigma B_0^2}{\rho}u' \qquad (1)$$

$$\frac{\partial T'}{\partial t'} - V_0'\left(1+\varepsilon e^{-n't'}\right)\frac{\partial T'}{\partial y'} = \frac{k}{\rho C_p}\frac{\partial^2 u'}{\partial y'^2} \qquad (2)$$

The boundary conditions are as follows

$$u' = L'\left(\frac{\partial u'}{\partial y'}\right), \quad T' = T_w' \quad at\ y' = 0$$
$$u' \to 0, \quad T' \to T_\infty' \quad at\ y' \to \infty \qquad (3)$$

In non-dimensional process of the governing equations, the following dimensionless quantities are used

$$y = \frac{y'V_0'}{\upsilon}, \quad t = \frac{t'V_0'}{\upsilon}, \quad u = \frac{u'}{V_0'}, \quad n = \frac{\upsilon n'}{V_0'^2}, \quad \theta = \frac{T'-T_\infty'}{T_w'-T_\infty'},$$
$$Gr = \frac{g\beta\upsilon(T_w'-T_\infty')}{V_0'^2}, \quad Rv = \frac{\mu_{eff}}{\mu_f}, \quad K = \frac{K'V_0'^2}{\upsilon^2}, \quad Pr = \frac{\rho C_p}{k}, \quad M = \frac{\sigma B_0^2 u'}{\rho V_0'^2}.$$

The governing equations in the non-dimensional form are as follows:

$$\frac{\partial u}{\partial t} - (1+\varepsilon e^{-nt})\frac{\partial u}{\partial y} = Rv\frac{\partial^2 u'}{\partial y'^2} + Gr\theta - Nu \qquad (4)$$

$$\frac{\partial \theta}{\partial t} - (1+\varepsilon e^{-nt})\frac{\partial \theta}{\partial y} = \frac{1}{Pr}\frac{\partial^2\theta}{\partial y^2} \qquad (5)$$

The boundary conditions in dimensionless form are as follows:

$$u = h\left(\frac{\partial u}{\partial y}\right), \quad \theta = 1, \quad at\ y = 0,$$
$$u \to 0, \quad \theta \to 0, \quad at\ y \to \infty. \qquad (6)$$

where $h = L'\mathrm{v}_0'/\upsilon$

III. Solution of the problem

For the small amplitude oscillations ($\varepsilon \ll 1$), the velocity u and temperature θ near the wall using perturbation technique

$$u(y,t) = u_0(y) + \varepsilon e^{-nt}u_1(y) \qquad (7)$$

$$\theta(y,t) = \theta_0(y) + \varepsilon e^{-nt}\theta_1(y) \quad (8)$$

Substituting Eqs. 7 & 8 in Eqs. 4-6 to get zeroth order, first order and the boundary conditions, respectively.

Zeroth order equations

$$Rv\frac{\partial^2 u_0}{\partial y^2} + \frac{du_0}{dy} - Nu_0 + Gr\theta_0 = 0 \quad (9)$$

$$\frac{\partial^2 \theta_0}{\partial y^2} + \frac{1}{Pr}\frac{d\theta_0}{dy} = 0 \quad (10)$$

Boundary conditions for the zeroth order

$$u_0 = h\left(\frac{\partial u_0}{\partial y}\right), \qquad \theta_0 = 1, \; at\; y = 0$$

$$u_0 \to 0, \qquad \theta_0 \to 0, \; at\; y \to \infty. \quad (11)$$

By solving equations subjected to boundary conditions, the solution for zero order is obtained as follows:

$$u_0 = C_1 e^{-m_1 y} + A_1 e^{-Pry}, \quad (12)$$

$$\theta_0 = e^{-Pry}, \quad (13)$$

where $C_1 = -\frac{PrA_1h}{1+m_1h}$, $A_1 = -\frac{Gr}{(RvPr^2-Pr-N)}$, $m_1 = \left(\frac{1}{Rv} + \sqrt{\frac{1}{Rv^2}+\frac{4N}{Rv}}\right)/2.$

First order equations

$$Rv\frac{\partial^2 u_1}{\partial y^2} + \frac{du_1}{dy} - Nu_1 + Gr\theta_1 = 0 \quad (14)$$

$$\frac{\partial^2 \theta_1}{\partial y^2} + \frac{1}{Pr}\frac{d\theta_1}{dy} = 0 \quad (15)$$

Boundary conditions for the first order

$$u_1 = h\left(\frac{\partial u_0}{\partial y}\right), \qquad \theta_1 = 1 \qquad at\;\; y = 0$$

$$u_1 \to 0, \qquad \theta_1 \to 0 \qquad at\;\; y \to \infty \quad (16)$$

By solving equations subjected to boundary conditions, the solution for zero order is obtained as follows:

$$u_1 = C_3 e^{-m_3 y} + A_5 e^{-Pry} + A_6 e^{-m_1 y} + A_7 e^{-m_2 y} \quad (17)$$

$$\theta_1 = \frac{Pr}{n} e^{-Pry} - C_2 e^{-m_2 y} \quad (18)$$

where

$A_2 = \frac{A_1 Pr - Gr\,C_2}{Rv}$, $\qquad C_2 = \frac{Pr}{n}$, $\qquad m_2 = \left(Pr + \sqrt{Pr^2 - 4nPr}\right)/2,$

$A_3 = \frac{m_1 C_1}{Rv}$, $\qquad A_4 = \frac{GrC_2}{Rv}$,

$A_5 = A_2 \Big/ \left(Pr^2 - \frac{Pr}{Rv} - \frac{(N-n)}{Rv}\right)$, $\qquad A_6 = A_3 \Big/ \left({m_1}^2 - \frac{m_1}{Rv} - \frac{(N-n)}{Rv}\right)$,

$A_7 = A_4 \Big/ \left({m_2}^2 - \frac{m_2}{Rv} - \frac{(N-n)}{Rv}\right)$,

$C_3 = \frac{-h(m_1A_6 + m_2A_7 + Pr\,A_5) - m_4}{1+hm_3}$, $\qquad m_3 = \left(\frac{1}{Rv} + \sqrt{\frac{1}{Rv^2} + \frac{4(N-n)}{Rv}}\right)/2,$

$m_4 = A_5 + A_6 + A_7.$

The Nusselt number is

$$Nu = -\left(\frac{\partial\theta}{\partial y}\right)_{y=0} = Pr + \varepsilon C_2(Pr + m_2) \quad (19)$$

The skin-friction (τ) at the wall is,

$$\tau = \left(\frac{du_0}{dy}\right)_{y=0} + \;\varepsilon\left(\frac{du_1}{dy}\right)_{y=0} e^{-nt}, \quad (20)$$

$$\tau = -(m_1C_1 + PrA_1) - \varepsilon(m_3C_3 + PrA_5 + m_1A_6 + m_2A_7)e^{-nt} \quad (21)$$

IV. Result and discussion:

In order to obtained the effects of various non-dimensional parameters such as Permeability (K), velocity slip parameter (h), Grashof Number(Gr), Ratio of Viscosity (Rv) and Prandtl number (Pr) on velocity profile and various results are shown graphically.

In Figure 1, the velocity profile are shown graphically for different values of permeability (K) & magnetic field(M), when other parameters are kept constant ($h = 1,\; Gr = 5, Rv = 1, Pr = 0.71, t = 1, n = 1$ & $\varepsilon = 0.1$). The results show that velocity increases with increases in value of Permeability(K) and it decrease with magnetic field(M).

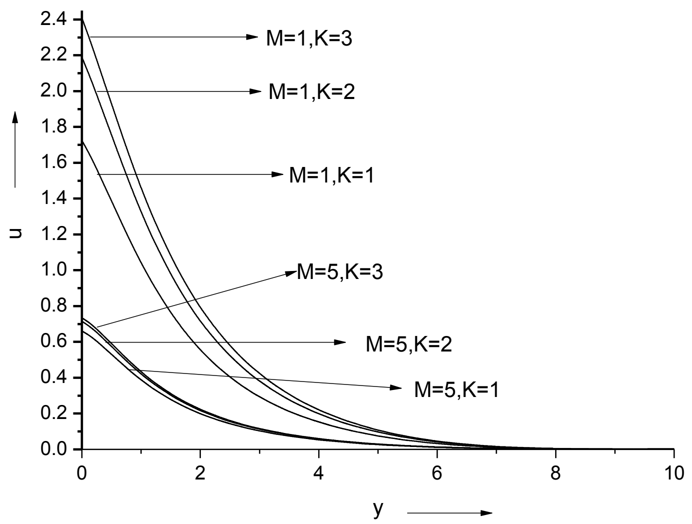

Fig.1 Velocity profile for different values of permeability (K) & magnetic parameter (M)

In Fig. 2, velocity profile are shown graphically for different values slip parameter (h) and magnetic field (M), while other parameters, $\mathrm{K} = 1, \mathrm{Gr} = 5, \mathrm{Rv} = 1, \mathrm{Pr} = 0.71, \mathrm{t} = 1, \mathrm{n} = 1$ and $\varepsilon = 0.1$, are kept constant. We have observed that magnitude of velocity decreases with increases of slip parameter (h) as well as magnetic parameter. For large value of magnetic field, we have observed that there is no significant effect of increasing slip parameter on velocity magnitude.

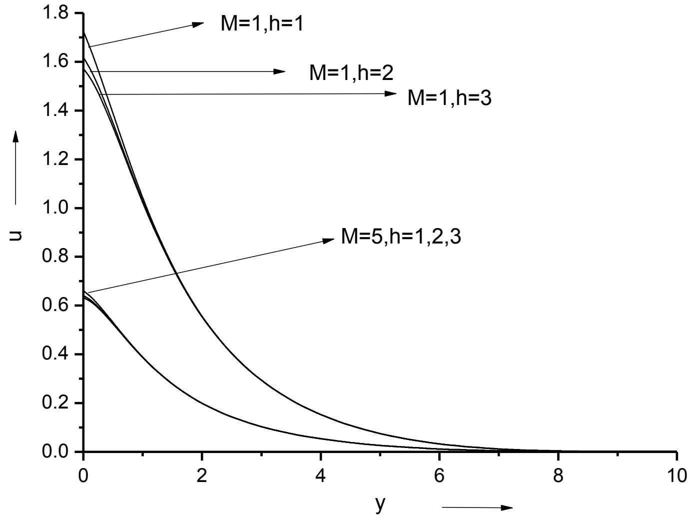

Fig.2. Velocity profile for different values of slip parameter (h) &
magnetic parameter (M)

Fig.3 display velocity profile with $\mathrm{K} = 1, \mathrm{Gr} = 5, \mathrm{h} = 1, \mathrm{Pr} = 0.71, \mathrm{t} = 1, \mathrm{n} = 1$ & $\varepsilon = 0.1$ for different values of as Ratio of viscosities Rv and magnetic parameter M. The results shows that velocity increases with increases in value of as Ratio of viscosities (Rv) and decrease with increases magnetic parameter M.

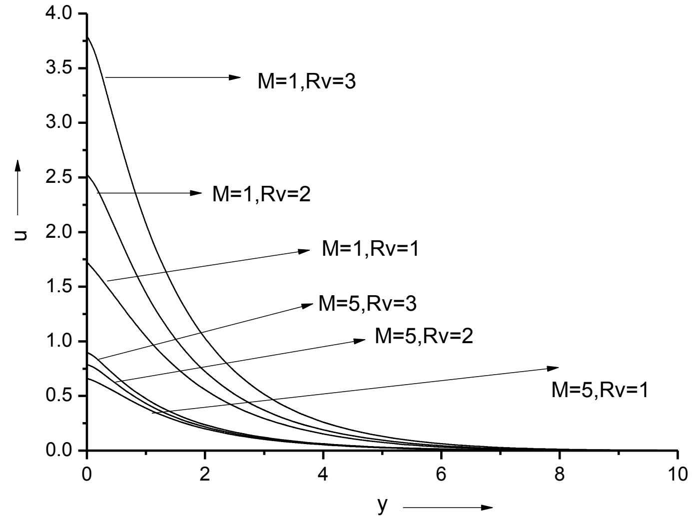

Fig.3. Velocity profile for different values of Ratios of viscositiesRv magnetic parameter (M)

In Fig. 4, the velocity profile are shown graphically for different values of as Grashof number (Gr) where as other parameters, $\mathrm{K} = 1, \mathrm{Rv} = 1, \mathrm{h} = 1, \mathrm{Pr} = 0.71, \mathrm{t} = 1, \mathrm{n} = 1$ & $\varepsilon = 0.1$, are kept constant. The results shows that velocity increases with increases in value of as Grashof number (Gr) where as it decreases with magnetic field.

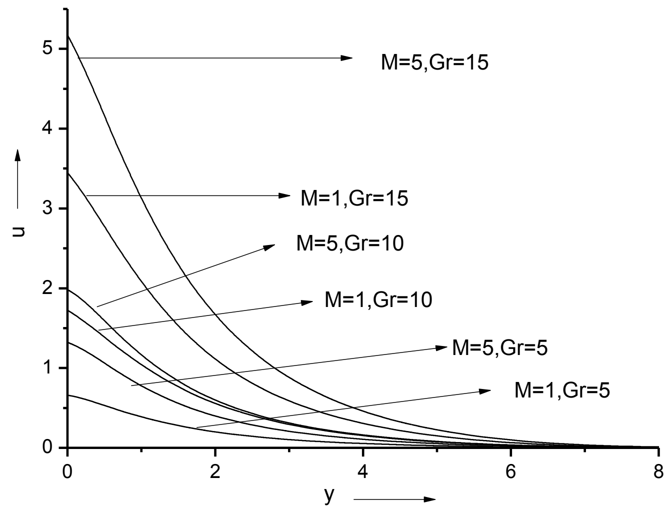

Fig.4. Velocity profile for different values of Grash of number (Gr) & magnetic parameter M.

Fig. 5 depicts variation skin-friction for different values M & ε with $\mathrm{K} = 1, \mathrm{Rv} = 1, \mathrm{h} = 1, \mathrm{Pr} = 7, \mathrm{Gr} = 5$ and $\mathrm{n} = 1$. Since velocity having decreasing tendency with magnetic field as it increases, then skin friction also having decreasing tendency with increase of magnetic field.

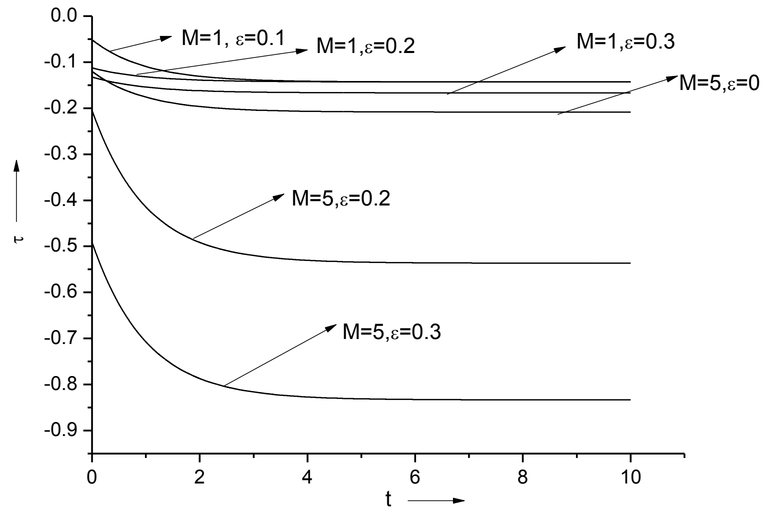

Fig.5. Skin friction for different values of scalar constant ε and magnetic parameter M.

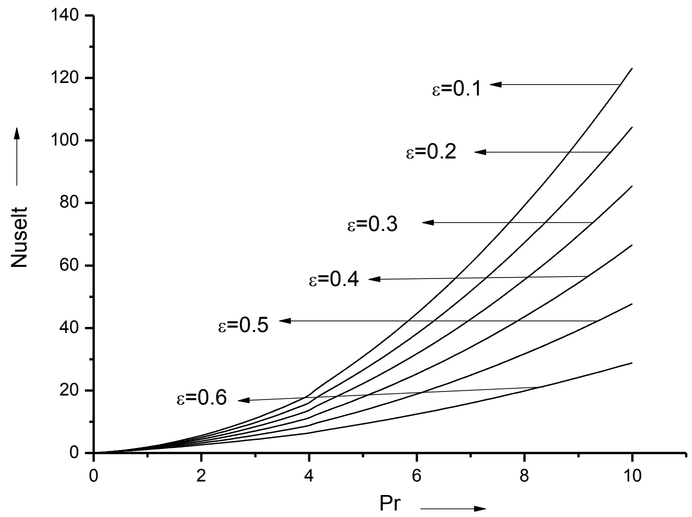

Fig.6. Nusselt numbers for different values of scalar constant ε

In Fig. 6, Nusselt numbers are shown graphically for different values ε when other parameters $(\mathrm{Pr} = 0.71 \;\&\; \mathrm{n} = 1)$ are kept constant. The results show that Nusselt number decreases with increases in value of ε .

V. Concluding remark

As per the solution of problem, the velocity is decreasing as magnetic field increase. By this theme it will be small step to control salinity and can be produced pollution free energy.

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References:

[1]. Berezovsky, A. A., Martynenko, O. G. and Yu, A., Sokovishin, Free convective heat transfer on a vertical semi infinite plate, J. Engng. Phys., 33, 32-39, 1977.
[2]. Chandran, P., Sacheti, N. C., and Singh, A. K., Effects of rotation on unsteady hydrodynamic Couette flow, Astrophysics and Space Science, 202, 1–10, 1993.
[3]. Chandran, P., Sacheti, N. C., and Singh, A. K. Haydromagnetic flow and heat transfer past a continuously moving porous boundary, International Communication in Heat and Mass Transfer, 23, 889–898, 1996.
[4]. Chandran, P., Sacheti, N. C., and Singh, A. K. Unsteady hydromagnetic free convection flow with heat flux and accelerated boundary motion, Journal of Physical Society of Japan, 67, 124–129, 1998.
[5]. Chandran, P., Sacheti, N. C., and Singh, A. K. An undefined approach to analytical solution of a hydromagnetic free convection flow, Scientiae Mathematicae Japonicae, 53, 467–476, 2001.

[6]. Cramer, K. R. and Pai, S. I., Magnetofluid Dynamics for Engineering and Applied Physicists, Mc Graw-Hill, New York, 1973.

[7]. Das, U. N., Deka, R.K. and Soundalgekar, V. M., Transient free convection flow past an infinite vertical plate with periodic temperature variation, J. Heat Transfer (ASME), 121, 1091-1094, 1999.

[8]. Jha, B. K., MHD free-convection and mass transform through a porous media, Astrophysics and Space Science, 175, 283- 289, 1990.

[9]. Hartmann, J., Hg-dynamics I theory of the laminar flow of an electrically conductive liquid in a homogenous magnetic field, Det Kongelige Danske Videnskabernes Selskab Mathematisk-fysisk Meddeleser, XV, 1–27, 1937.

[10]. Ingham, B. D., and Pop, I., Convection in porous media, (2nd Ed.) Springer, New York, 1990.

[11]. Kim J., Unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction had studied, International J. Engg. Sci, 38 833-845, 2000.

[12]. Martynenko, O. G., Berezovsky, A. A. and Yu. A., So-kovishin, Laminar free convection from a vertical plate, Int. J. Heat Mass Transfer, 27, 869-881, 1984.

[13]. Mishra, A.K., Abdullahi, A., and Manjak, N. H, Transient free convection flow past a vertical plate through porous medium with variation in slip flow regime, International Journal of Advanced Technology and Engineering Research, Vol. 4(2), 2250-3536 Jan.2014.

[14]. Nield, D. A., and Bejan, A., Transport phenomena in porous media, Oxford (1998).

[15]. Sharma P. K, and Chaudhaury, R. C., Effect of variable sunction on transient free convective viscous incompressible flow past a vertical plate with periodic temperature variations in slip-flow regime, Emirates Journal for Engineering Research, 8(2), 33-38, 2003.

[16]. Shahin, A., Oscillatory three dimensional flow and heat and mass transfer through a porous medium in presence of periodic sunction Emirates Journal for Engineering Research, 15(2), 49-61, 2010.

[17]. Siegel R., Transient free convection from a vertical flat plate, Transactions of the ASME, 30, 349, 1958.

[18]. Singh, R. K., Singh, A. K., Chandran, P., Sacheti, N. C.: Effect of Induced magnetic field on hydromagnetic free convective flow between vertical walls, Heat and Mass transfer, 46, 523-529. 2010.

[19]. Singh, R. K., Singh, A. K.: Effect of induced magnetic field on natural convection in vertical concentric annuli, Acta Mechanica Sinica, 28(2), 315-323, 2012.

[20]. Singh, R. K., Singh A. K.: MHD free convective flow past a semi-infinite vertical permeable wall, Applied Mathematics and Mechanics, 33(9), 1207-1222, 2012.

[21]. Sharma, S. and Deka, R. K., Thermal radiation and oscillating plate temperature effects on MHD unsteady flow past a semi-infinite porous vertical plate under suction and chemical reaction. International Journal available at http://www.cibtech.org/jpms.htm, Vol. 292, 33-52, 2012.

[22]. Singh R. K., Singh T. R., and Kataria H. R., Induced magnetic field effects on mixed convection, IOSR Journal of Mathematics (IOSR-JM), 10(6), 76-85, 2014.

[23]. Uwanta, I. J., Hanmza, M. M., and Ibrahim, A. O., Heat and Mass Transfer Flow through a Porous Medium with Variable Permeability and Periodic Suction, International Journal of Computer Applications, Volume 36, No.2, 0975–8887, 2011.