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Pre- Operator Compact Space

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Abstract: *The main object of this paper to introduce T-pre-compact space. And a good pre operator.*
Key words: *T-pre compact, good pre-operator.*

I. Introduction

In 1979 Kasahara [1] introduce the concept of operator associated with a topology Γ of a space X as amap from P(X) to P(X) such that $u\subseteq \alpha(u)$ for every $u\in \Gamma$. And introduced the concept of an operator compact space on a topological space(X,Γ) as a subset A of X is α-compact if for every open covering $\prod$ of A tyere exists a finite sub collection $\{c_1,c_2,\dots,c_n\}$ of $\prod$ such that $A\subseteq \cup^{n}_{i=1}\alpha(c_i)$. In 1999 Rosas and Vielma[3] modified the definition by allowing the operator α to be defined in P(X) as a map α from Γ to P(X).And properties of α-compact spaces has been in vestigated in [1,3].And [4] gives some theorems about α-compact. In 2013 Mansur and Moussa [2] introduce the concept of an operator T on pre-open set in topological space(X, Γ_{pre}) namely T-pre-operator and studied some of their properties.

In this paper we introduce the concept of T-pre-open set with compact space. Asubset A of X is called T-pre-compact if for any T-pre-open cover $\{U_\alpha:\alpha\in\Omega\}$ of A, has a finite collection that covers A and $A\subseteq \bigcup_{i=1}^{n} T(U_{\alpha_i})$ .In §2 Using the pre-operator T ,we introduce the concept pre-operator compact space, good pre-operator. And we study some of their properties and optained new results. In §3 we introduce some properties about pre-operator compact space and give some results in relation to a pre-operator separation axioms.

II. Pre- Operator Compact Space

2.1 Definition:

Let (X,Γ,T) be a pre-operator topological space. A subset A of X is said to be pre-operator compact(T-Pre Compact) if for any T-pre-open cover $\{U_\alpha : \alpha \in \Omega\}$ of A, has a finite collection that covers A and $A \subseteq$

$$\bigcup_{i=1}^{n} T(U_{\alpha_i})$$

2.2 Definition:

Let (X,Γ,T) be a α-operator topological space. A subset A of X is said to be α-operator compact if for any T-α-open cover $\{U_\alpha : \alpha \in \Omega\}$ of A, has a finite collection that covers A and $A \subseteq \bigcup_{i=1}^{n} T(U_{\alpha_i})$.

In the following theorem, we present the relationship between T-pre compact and T-compact spaces:

2.3 Theorem:

Every T-pre compact space is T- compact space.

2.4 Proposition:

Every T- pre compact space is T-α compact space.

2.5 Theorem:

The union of two T-pre compact sets is T-pre- compact set.

Proof:

Suppose that (X,Γ,T) be a pre-operator topological space and A, B be two T-pre compact subsets of (X,Γ,T)
Let $W = \{U_\alpha : \alpha \in \Omega\}$ be T-pre-open cover of $A \cup B$
Since $A \subseteq A \cup B$ and $B \subseteq A \cup B$

Hence, W is T-pre-open cover of A and B
Since A is T-pre compact set
Therefore, there exist a finite subcover { $U_{\alpha_{i1}}$, $U_{\alpha_{i2}}$, …, $U_{\alpha_{in}}$ } such that {T($U_{\alpha_{i1}}$), T($U_{\alpha_{i2}}$), …, T($U_{\alpha_{in}}$)} of W covers A

Hence $A \subseteq \bigcup_{i=1}^{n} T(U_{\alpha_i})$

Also, since B is T-pre compact set
Therefore, there exist a finite subcover { $U_{\alpha_{j1}}$, $U_{\alpha_{j2}}$, …, $U_{\alpha_{jm}}$ } such that {T($U_{\alpha_{j1}}$), T($U_{\alpha_{j2}}$), …, T($U_{\alpha_{jm}}$)} of W covers B

Hence $B \subseteq \bigcup_{j=1}^{m} T(U_{\alpha_j})$

Therefore, $A \cup B \subseteq \bigcup_{k=1}^{n+m} T(U_{\alpha_k})$

Thus, $A \cup B$ is T-pre compact set.

2.6 Corollary:
The finite union of pre-operator compact subsets of X is pre-operator compact.

2.7 Definition:
Let f : $(X,\Gamma,T) \longrightarrow (Y,\delta,L)$ be a function. The two pre-operators T and L are said to be good pre-operators if $f(T(f^{-1}(U))) \subseteq L(U)$, for all U is L-pre-open set in Y.

2.8 Proposition:
If T and L are good pre-operators, then the (T,L) pre-continuous image of T-pre compact space is T-compact.
Proof:
Suppose that f : $(X,\Gamma,T) \longrightarrow (Y,\delta,L)$ be (T,L) pre-continuous function and (X,Γ,T) be T-pre compact space.
Let $W = \{A_\alpha : \alpha \in \Omega\}$ be L-open cover of Y

Hence, $Y = \bigcup_{\alpha\in\Omega} A_\alpha$

Since f is (T,L) pre-continuous function
Therefore, $f^{-1}(W) = \{f^{-1}(A_\alpha) : \alpha \in \Omega\}$ is T-pre-open cover of X and since X is T-pre compact space
Therefore, there exist a finite subcover $\{f^{-1}(A_{\alpha 1}), f^{-1}(A_{\alpha 2}), \ldots, f^{-1}(A_{\alpha n})\}$, such that $\{T(f^{-1}(A_{\alpha 1})), T(f^{-1}(A_{\alpha 2})), \ldots, T(f^{-1}(A_{\alpha n}))\}$ covers X

Thus, $X = \bigcup_{i=1}^{n} T(f^{-1}(A_{\alpha i}))$

$$f(X) = f\left(\bigcup_{i=1}^{n} T(f^{-1}(A_{\alpha i}))\right)$$

$$Y = \bigcup_{i=1}^{n} f\left(T(f^{-1}(A_{\alpha i}))\right)$$

Since T and L are good pre-operators, then:

$$Y = \bigcup_{i=1}^{n} L(A_{\alpha i})$$

Hence Y is T- compact space.

2.9 Theorem :

If T and L are good pre-operators, then the T-pre compact space is (T,L) pre-irresolute topological property.

Proof:

Suppose that (X,Γ,T) be a pre-operator compact space and (Y,δ,L) be a pre-operator topological space
Let $f : (X,\Gamma,T) \longrightarrow (Y,\delta,L)$ be (T,L) pre-irresolute homeomorphism function and let $W = \{A_\alpha : \alpha \in \Omega\}$ be L-pre-open cover of Y

Hence, $Y = \bigcup_{\alpha\in\Omega} A_\alpha$

Since f is (T,L) pre-irresolute continuous function, therefore $f^{-1}(W) = \{f^{-1}(A_\alpha) : \alpha \in \Omega\}$ is T-pre-open cover of X

Thus, $X = \bigcup_{\alpha\in\Omega} f^{-1}(A_\alpha)$

Since X is T-pre compact space
Hence there exist a finite subcover $\{f^{-1}(A_{\alpha 1}), f^{-1}(A_{\alpha 2}), \ldots, f^{-1}(A_{\alpha n})\}$, such that $\{T(f^{-1}(A_{\alpha 1})), T(f^{-1}(A_{\alpha 2})), \ldots, T(f^{-1}(A_{\alpha n}))\}$ covers X

Thus, $X = \bigcup_{i=1}^{n} T(f^{-1}(A_{\alpha i}))$

$$f(X) = f\left(\bigcup_{i=1}^{n} T(f^{-1}(A_{\alpha i}))\right)$$

Since f is on to, hence f(X) = Y, and thus:

$$Y = \bigcup_{i=1}^{n} f\left(T(f^{-1}(A_{\alpha i}))\right)$$

Since T and L are good pre-operators
Hence, by definition (3.4.9) $f(T(f^{-1}(A_{\alpha i}))) \subseteq L(A_{\alpha i})$

Therefore, $Y = \bigcup_{i=1}^{n} L(A_{\alpha i})$

Hence, (Y,δ,L) is L-pre compact space.

III. T-pre-compact & T-pre-separation axioms

3.1 Theorem :

If T is a pre-regular pre-subadditive operator, then every T-pre-compact subset of T-pre-Hausdorff space is T-pre-closed.

Proof:

Suppose that (X,Γ,T) be a T-pre-Hausdorff space
Let F be T-pre-compact set in X and let $x \in F^c$
Since X is T-pre-Hausdorff space
Hence, for each $y \in F$, here exist disjoint T-pre-open sets U_x, V_y of the points x and y, respectively, such that $T(U_x) \cap T(V_y) = \varnothing$
The collection $\{V_y : y \in F\}$ is T-pre-open cover of F
Since F is T-pre-compact set
Hence there exist a finite subcover $\{V_{y1}, V_{y2}, \ldots, V_{yn}\}$ such that $\{T(V_{y1}), T(V_{y2}), \ldots, T(V_{yn})\}$ covers F

Thus, $F \subseteq \bigcup_{i=1}^{n} T(V_{yi})$

Let $V = \bigcup_{i=1}^{n} T(V_{yi})$ and $U = \bigcap_{i=1}^{n} T(U_{xi})$

Since $F \subseteq V$

Therefore, we have U is T-pre-open set, $x \in U$ and $U \subseteq F^c$

Hence, F^c is T-pre-open set

Thus, F is T-pre-closed set.

3.2 Theorem:

If T is a regular subadditive operator, then every T-pre-compact subset of T-Hausdorff space is T-closed.

Proof:

Suppose that (X,Γ,T) be an operator Hausdorff space

Let F be T-pre-compact set in X and let $x \in F^c$

Since X is T-Hausdorff space

Hence, for each $y \in F$, there exist disjoint T-open sets U_x, V_y of the points x and y, respectively, such that $T(U_x) \cap T(V_y) = \emptyset$

Since, every T-pre-compact space is T-compact space

Hence, F is T-compact set and the collection $\{V_y : y \in F\}$ is T-open cover of F

Since F is T-compact

Hence there exist a finite subcover $\{V_{y1}, V_{y2}, \ldots, V_{yn}\}$ such that $\{T(V_{y1}), T(V_{y2}), \ldots, T(V_{yn})\}$ covers F

Thus, $F \subseteq \bigcup_{i=1}^{n} T(V_{yi})$

Let $V = \bigcup_{i=1}^{n} T(V_{yi})$ and $U = \bigcap_{i=1}^{n} T(U_{xi})$

Since $F \subseteq V$, then we have U is T-open set, $x \in U$ and $U \subseteq F^c$

Hence, F^c is T-open set

Thus, F is T-closed set.

3.3 Proposition :

If T is a pre-regular pre-subadditive operator, then every T-pre-compact subset of T-Hausdorff space is T-pre-closed.

3.4 Theorem:

If T is pre-subadditive operator, then every T-pre-closed subset of T-pre-compact space is T-compact.

Proof:

Suppose that (X,Γ,T) be a T-pre-compact space and let F be T-pre-closed subset of X

Let the collection $\{A_\alpha : \alpha \in \Omega\}$ be T-open cover of F, that is, $F \subseteq \bigcup_{\alpha \in \Omega} A_\alpha$

Since F is T-pre-closed subset of X

Hence F^c is T-pre-open subset of X

Since, every T-open set is T-pre-open set

Hence, the collection $\{A_\alpha : \alpha \in \Omega\}$ is T-pre-open cover and $\{A_\alpha : \alpha \in \Omega\} \cup \{F^c\}$ s T-pre-open cover of X

Since X is T-pre-compact space

Therefore, there exist a finite subcover $\{A_{\alpha_1}, A_{\alpha_2}, \ldots, A_{\alpha_n}\}$ such that $\{T(A_{\alpha_1}), T(A_{\alpha_2}), \ldots, T(A_{\alpha_n})\} \cup \{F^c\}$ covers X

Hence, $X = \bigcup_{i=1}^{n} T(A_{\alpha i}) \cup F^c$

Since T is pre-subadditive operator

Therefore, $\bigcup_{i=1}^{n} T(A_{\alpha i}) \cup F^c$ is T-pre-open

But $F \subseteq X$

Hence, $F \subseteq \bigcup_{i=1}^{n} T(A_{\alpha i})$

Thus, F is T-compact.

3.5 Corollary :

If T is pre-subadditive operator, then a T-closed subset of T-pre-compact space is T-pre-compact.

3.6 Corollary :

If T is subadditive operator, then a T-closed subset of T-pre-compact space is T-compact.

3.7 Corollary :

If T is subadditive operator, then a T-pre-closed subset of T-pre-compact space is T-pre-compact.

3.8 Corollary :

Let (X,Γ,T) be a T-pre-Hausdorff space and T is a regular subadditive operator. If $Y \subseteq X$ is T-pre-compact, $x \in Y^c$, then there exist T-pre-open sets U and V with $x \in U$, $Y \subseteq V$, $x \notin T(V)$, $y \not\subseteq T(U)$ and $T(U) \cap T(V) = \varnothing$.

Proof:

Let y be any point in Y

Since (X,Γ,T) is a T-pre-Hausdorff space, therefore there exist two T-pre-open sets V_y, U_x, such that $T(U_x) \cap T(V_y) = \varnothing$

The collection $\{V_y : y \in Y\}$ is T-pre-open cover of Y

Now, since Y is T-pre-compact, therefore there exists a finite subcollection $\{V_{y1}, V_{y2}, \ldots, V_{yn}\}$ such that $\{T(V_{y1}), T(V_{y2}), \ldots, T(V_{yn})\}$ covers Y

Let $U = \bigcap_{i=1}^{n} (U_{xi})$, $V = \bigcup_{i=1}^{n} (V_{yi})$

Since $U \subseteq T(U_{xi})$, for every $i \in \{1, 2, \ldots, n\}$

Therefore, $T(U) \cap T(V_{yi}) = \varnothing$, for every $i \in \{1, 2, \ldots, n\}$

Hence, $T(U) \cap T(V) = \varnothing$.

In the following theorem, we present the relation between T-pre-compact and strongly T-pre-regular space.

3.9 Theorem:

If T is a regular subadditive operator, then every T-pre-compact and T-pre-Hausdorff space is strongly T-pre-regular space.

Proof:

Suppose that (X,Γ,T) be a T-pre-compact and T-pre-Hausdorff space

Let $x \in X$ and B be T-pre-closed subset of X, such that $x \notin B$

By corollary (3.4.20), B is T-pre-compact subset of T-pre-Hausdorff space and by theorem (3.4.14) B is T-pre-closed set

Hence, (X,Γ,T) is strongly T-pre-regular space.

3.10 Theorem:

If T is a regular subadditive operator, then every T-pre-compact and T-pre-Hausdorff space is T-pre-regular space.

Proof:

Suppose that (X,Γ,T) be a T-pre-compact and T-pre-Hausdorff space

Let F be T-closed subset of X and $x \in X$, such that $x \notin F$

Since X is T-pre-Hausdorff space

Hence, for each $y \in F$, there exist T-pre-open sets U_x, V_y, such that $x \in U_x$, $y \in V_y$ and $T(U_x) \cap T(V_y) = \varnothing$

The collection $\{V_y : y \in Y\}$ is T-open cover of F

By corollary (3.4.19)

Thus F is T-compact set

Therefore, there exists a finite subcover $\{V_{y1}, V_{y2}, \ldots, V_{yn}\}$ such that $\{T(V_{y1}), T(V_{y2}), \ldots, T(V_{yn})\}$ covers F and $F \subseteq \bigcup_{i=1}^{n} T(V_{yi})$

Let $V = \bigcup_{i=1}^{n} (V_{yi})$ and $U = \bigcap_{i=1}^{n} (U_{xi})$,

Then, $x \in U$ and U, V are disjoit T-pre-open sets, such that $x \in U$, $F \subseteq V$ and $T(U) \cap T(V) = \varnothing$

Hence, (X,Γ,T) is T-pre-regular space.

3.11 Theorem:

If T is a regular subadditive operator, then every T-pre-compact and T-pre-Hausdorff space is strongly T-pre-T_3 space.

3.12 Theorem:

If T is a regular subadditive operator, then every T-pre-compact and T-pre-Hausdorff space is T-pre-T_3 space.

3.13 Theorem:

If T is a regular subadditive operator, then every T-pre-compact and T-pre-Hausdorff space is T-pre-normal space.

Proof:

Suppose that (X,Γ,T) be a T-pre-compact and T-pre-Hausdorff space

Let E, F be a pair of disjoint T-closed subsets of X

Let $x \in F$ and by theorem (3.4.21), there exist two T-pre-open sets U_x, V_E, such that $x \in U_x$, $E \subseteq V_E$ and $T(U_x) \cap T(V_E) = \varnothing$

The collection $\{U_x : x \in F\}$ be T-pre-open cover of F

Since F is T-closed subset of T-pre-compact space and by corollary (3.4.19), F is T-compact

Hence, there exist a finite subcollection $\{U_{x1}, U_{x2}, \ldots, U_{xn}\}$ such that $\{T(U_{x1}), T(U_{x2}), \ldots, T(U_{xn})\}$ covers F and $F \subseteq \bigcup_{i=1}^{n} T(U_{xi})$

Let $U = \bigcup_{i=1}^{n} (U_{xi})$ and $V = \bigcap_{i=1}^{n} (V_{Ei})$

Then, U and V are disjoint T-pre-open sets, such that $F \subseteq U$, $E \subseteq V$ and $T(U) \cap T(V) = \varnothing$

Hence, X is T-pre-normal space.

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