IOSR Journal of Mathematics (IOSR-JM)
e-ISSN: 2278-5728, p-ISSN: 2319-765X. Volume 11, Issue 6 Ver. II (Nov. - Dec. 2015), PP 60-66
www.iosrjournals.org

Hybrid Block Method for the Solution of First Order Initial Value Problems of Ordinary Differential Equations

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Abstract: *Method of collocation of the differential system and interpolation of the approximate solution which is a combination of power series and exponential function at some selected grid and off-grid points to generate a linear multistep method which is implemented in block method is considered in this paper. The basic properties of the block method which include; consistency, convergence and stability interval is verified. The method is tested on some numerical experiments and found to have better stability condition and better approximation than the existing methods.*
Keywords: *Interpolation, Collocation, Consistent, Convergent, Block Method, Power Series.*

I. Introduction

This paper considers a new numerical block integrator for the solution of first order initial value problems of the form

$$y' = f(x, y),\ y(x_0) = y_0 \quad (1)$$

Where f is continuous and satisfies Lipschitz's conditions, x_0 is the initial point and y_0 is the solution at x_0 .Problems in the form (1) has wide application in engineering, physical sciences, medicine etc. The solution of (1) has been discussed by various scholars among them are Onumanyi et al. [1,2], Lambert [3], James and Adesanya [6], Sirisena [7,8]. Adoption of collocation and interpolation of power series approximate solution to developed block method for solution of initial value problems have been studied by many scholars, among them are James et al. [5], Fasasi et al. [16], Areo and Adeniyi [15], Adesanya et al. [12], Skwame et al. [18], Adesanya et al. [11,13]. These authors independently implemented their methods such that the solutions are simultaneously generated at different grid points within the interval of integration. It has been reported that block method is more efficient than the existing method in terms of time of development and execution. Moreover, block method gives better approximation than the predictor corrector method and enables the nature of the problem to be understood at the selected grid points Adesanya et al.,[11,13]. The introduction of hybrid method to circumvent the Dahlquist stability barrier has been studied by many scholars which include Anake et al. [14], Fasasi, et al. [16], Adesanya, et al. [12], Lambert [3]. This scholars reported that though hybrid method are difficult to develop but enables the reduction in the step length. These scholars equally reported that lower k step method gives better result than the higher k step method. Approximate solution of the form

$$y(x) = \sum_{j=0}^{n-1} a_j x^j + a_n e^{-nx} \quad (2)$$

Where n is the number of Interpolation and Collocation point has been studied by scholars, among them are: Sunday, et al. [9,10], Momoh, et al. [17]. These authors reported that this method possess a good stability condition which is good for stiff, oscillatory and nonlinear problems. In this paper, we combined the desire qualities of hybrid method, block method and the approximate solution which is the combination of power series and exponential function to derive a new method for the solution of first order ordinary differential equation. It should be noted that our approximate solution considered more exponential functions than the one proposed by the authors mentioned above.

II. Methodology

We consider an approximate solution of the form

$$y(x) = \sum_{n=0}^{1} a_n x^n + \sum_{n=2}^{5} a_n e^{-nx} \quad (3)$$

The first derivative of (3) is given by

$$y(x) = a_1 x - \sum_{n=2}^{5} na_n e^{-nx} \quad (4)$$

Substituting (4) into (1) gives

$$f(x, y) = a_1 x - \sum_{n=2}^{5} na_n e^{-nx} \quad (5)$$

We sought the motion of (1) on the partition $\prod_N : x_0 < x_1 < x_2 < \ldots < x_N$ over a constant stepsize $h = x_{n+1} - x_n$. Interpolating (3) at $x_{n+s}, s = 0$ and collocating (4) at points x_{n+r}, gives a system of nonlinear equation of the form;

$$XA = U \quad (6)$$

Where

$$A = [a_0, a_1, a_2, a_3, a_4, a_5]^T, U = \left[y_n, f_n, f_{n+\frac{1}{4}}, f_{n+\frac{1}{2}}, f_{n+\frac{3}{4}}, f_{n+1} \right]^T$$

and

$$X = \begin{bmatrix} 1 & x_n & e^{-2x_n} & e^{-3x} & e^{-4x_n} & e^{-5x_n} \\ 0 & 1 & -2e^{-2x_n} & -3e^{-3x_n} & -4e^{-4x_n} & -5e^{-5x_n} \\ 0 & 1 & -2e^{-2x_{n+\frac{1}{4}}} & -3e^{-3x_{n+\frac{1}{4}}} & -4e^{-4x_{n+\frac{1}{4}}} & -5e^{-5x_{n+\frac{1}{4}}} \\ 0 & 1 & -2e^{-2x_{n+\frac{1}{2}}} & -3e^{-3x_{n+\frac{1}{2}}} & -4e^{-4x_{n+\frac{1}{2}}} & -5e^{-5x_{n+\frac{1}{2}}} \\ 0 & 1 & -2e^{-2x_{n+\frac{3}{4}}} & -3e^{-3x_{n+\frac{3}{4}}} & -4e^{-4x_{n+\frac{3}{4}}} & -5e^{-5x_{n+\frac{3}{4}}} \\ 0 & 1 & -2e^{-2x_{n+1}} & -3e^{-3x_{n+1}} & -4e^{-4x_{n+1}} & -5e^{-5x_{n+1}} \end{bmatrix}$$

Solving (5) for the constants to be determined $a'_j\ s$,and substituting back into (3) gives a continuous linear multistep method of the form

$$y(t) = \alpha_0(t) y_n + h\left(\beta_0(t) f_n - \beta_{\frac{1}{4}}(t) f_{n+\frac{1}{4}} + \beta_{\frac{1}{2}}(t) f_{n+\frac{1}{2}} - \beta_{\frac{3}{4}}(t) f_{n+\frac{3}{4}} + \beta_1(t) f_{n+1} \right) \quad (7)$$

Where $t = \dfrac{x - x_n}{h}$, $f_{n+j} = f(x_n + jh, y(x_n + jh))$

$$\alpha_0 = 1$$

$$\beta_0 = \frac{h}{90}\left(192t^5 - 600t^4 + 700t^3 - 375t^2 + 90t\right)$$

$$\beta_{\frac{1}{4}} = \frac{8h}{45}\left(-48t^5 + 135t^4 - 130t^3 + 45t^2\right)$$

$$\beta_{\frac{1}{2}} = -\frac{2h}{15}\left(-96t^5 + 240t^4 - 190t^3 + 45t^2\right)$$

$$\beta_{\frac{3}{4}} = -\frac{8h}{45}\left(48t^5 - 105t^4 + 70t^3 - 15t^2\right)$$

$$\beta_1 = \frac{h}{90}\left(192t^5 - 360t^4 + 220t^3 - 45t^2\right)$$

Evaluating (6) at $t = 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1$ and writing in block form gives a discrete block formula in the form

$$A^{(0)} Y_m = Ey_n + hdf\ (y_n) + hbF\ (Y_m) \quad (8)$$

where

$$Y_m = \left[y_{n+\frac{1}{4}}, y_{n+\frac{1}{2}}, y_{n+\frac{3}{4}}, y_{n+1} \right]^T, y_n = \left[y_{n-\frac{1}{4}}, y_{n-\frac{1}{2}}, y_{n-\frac{3}{4}}, y_n \right]^T$$

$$F(Y_m) = \left[f_{n+\frac{1}{4}}, f_{n+\frac{1}{2}}, f_{n+\frac{3}{4}}, f_{n+1} \right]^T, f(y_n) = \left[f_{n-\frac{1}{4}}, f_{n-\frac{1}{2}}, f_{n-\frac{3}{4}}, f_n \right]$$

$$A^{(0)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, E = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$d = \begin{bmatrix} 0 & 0 & 0 & \frac{251}{2880} \\ 0 & 0 & 0 & \frac{29}{360} \\ 0 & 0 & 0 & \frac{27}{320} \\ 0 & 0 & 0 & \frac{7}{90} \end{bmatrix}, b = \begin{bmatrix} \frac{323}{1440} & -\frac{11}{120} & \frac{53}{1440} & -\frac{19}{2880} \\ \frac{31}{90} & \frac{1}{15} & \frac{1}{60} & -\frac{1}{360} \\ \frac{51}{160} & \frac{9}{40} & \frac{21}{160} & -\frac{3}{320} \\ \frac{16}{45} & \frac{2}{15} & \frac{16}{45} & \frac{7}{90} \end{bmatrix}$$

Analysis of Basic Properties of the Developed Method.

Order of the Block Method

Let the linear operator $L\{y(x):h\}$ associated with the block integrator (8) be defined as

$$L\{y(x):h\} = A^{(0)}Y_m - Ey_n - hdf(y_n) + hbF(Y_m) \quad (9)$$

Expanding using Taylor series and comparing the coefficients of h gives

$$\begin{aligned} L\{y(x):h\} &= C_0 y(x) + C_1 hy'(x) + C_2 hy''(x) + \dots \\ &+ C_p h^p y^p(x) + C_{p+1} h^{p+1} y^{p+1}(x) + C_{p+2} h^{p+2} y^{p+2}(x) + \dots \end{aligned} \quad (10)$$

Definition 1.1: Order of Block Method

The linear operator L and associated block method are said to be of order p if $c_0 = c_1 = c_2 = \dots = c_p = 0, c_{p+1} \neq 0 . c_{p+1}$ is called the error constant and implies that the truncation error is given by

$$t_{n+k} = C_{p+1} h^{p+1} y^{p+1}(x) + O(h^{p+2}) \quad (11)$$

For our method

$$L\{y(x):h\} = \left[\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_{n+\frac{1}{4}} \\ y_{n+\frac{1}{2}} \\ y_{n+\frac{3}{4}} \\ y_{n+1} \end{bmatrix} \right] - \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_{n-\frac{1}{4}} \\ y_{n-\frac{1}{2}} \\ y_{n-\frac{3}{4}} \\ y_n \end{bmatrix} - h \begin{bmatrix} \frac{251}{2880} & \frac{323}{1440} & -\frac{11}{120} & \frac{53}{1440} & -\frac{19}{2880} \\ \frac{29}{360} & \frac{31}{90} & \frac{1}{15} & \frac{1}{90} & -\frac{1}{360} \\ \frac{27}{320} & \frac{51}{160} & \frac{9}{40} & \frac{21}{160} & -\frac{3}{320} \\ \frac{7}{90} & \frac{16}{45} & \frac{2}{15} & \frac{16}{45} & \frac{7}{90} \end{bmatrix} \begin{bmatrix} f_n \\ f_{n+\frac{1}{4}} \\ f_{n+\frac{1}{2}} \\ f_{n+\frac{3}{4}} \\ f_{n+1} \end{bmatrix} \quad (12)$$

Expanding (12) in Taylor series, and comparing the coefficient of h , gives

$$\begin{bmatrix} \sum_{j=0}^{\infty}\frac{(\frac{1}{4}h)^j}{j!}y'_n - y_n - \frac{251h}{2880}y'_n - \sum_{j=0}^{\infty}\frac{h^{j+1}}{j!}y_n^{j+1}\left\{\frac{323}{1440}(\frac{1}{4})^j - \frac{11}{120}(\frac{1}{2})^j + \frac{53}{1440}(\frac{3}{4})^j - \frac{19}{2880}(1)^j\right\} \\ \sum_{j=0}^{\infty}\frac{(\frac{1}{2}h)^j}{j!}y'_n - y_n - \frac{29h}{360}y'_n - \sum_{j=0}^{\infty}\frac{h^{j+1}}{j!}y_n^{j+1}\left\{\frac{31}{90}(\frac{1}{4})^j + \frac{1}{15}(\frac{1}{2})^j + \frac{1}{90}(\frac{3}{4})^j - \frac{1}{360}(1)^j\right\} \\ \sum_{j=0}^{\infty}\frac{(\frac{3}{4}h)^j}{j!}y'_n - y_n - \frac{27h}{320}y'_n - \sum_{j=0}^{\infty}\frac{h^{j+1}}{j!}y_n^{j+1}\left\{\frac{51}{160}(\frac{1}{4})^j + \frac{9}{40}(\frac{1}{2})^j + \frac{21}{160}(\frac{3}{4})^j - \frac{3}{320}(1)^j\right\} \\ \sum_{j=0}^{\infty}\frac{(1)^j}{j!}y'_n - y_n - \frac{7h}{90}y'_n - \sum_{j=0}^{\infty}\frac{h^{j+1}}{j!}y_n^{j+1}\left\{\frac{16}{45}(\frac{1}{4})^j + \frac{2}{15}(\frac{1}{2})^j + \frac{16}{45}(\frac{3}{4})^j + \frac{7}{90}(1)^j\right\} \end{bmatrix} = \begin{bmatrix}0\\0\\0\\0\end{bmatrix} \quad (13)$$

Hence, $c_0=c_1=c_2=c_3=c_4=c_5=0.\ c_6 = [4.5776(-06), 2.7127(-06), 4.5776(-06), -5.1670(-07)]^T$

Therefore,our new hybrid block method is of order 5.

Zero-Stability

Definition 1.2: Zero-stability

The block method (8) is said to be zero stable, if the roots $r_s, s = 1,2,..., N$ of the first characteristics polynomial $\rho(r)$ defined by $\rho(r) = \det(rA^{(0)} - E)$ satisfies $|r_s| \le 1$ and if every root with modulus $|r_s| = 1$ have multiplicity not exceeding the order of the differential equation. Moreover as $h \to 0, \rho(r) = r^{z-\mu}(r-1)^{\mu}$, where μ is the order of the differential equation, r is the order of the matrices $A^{(0)}$ and E.

$$\rho(r) = \left| r\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix} - \begin{bmatrix}0&0&0&1\\0&0&0&1\\0&0&0&1\\0&0&0&1\end{bmatrix}\right| = 0$$

$\rho(r) = r^4 - r^3 = r^3(r-1) = 0 \Rightarrow r_1 = r_2 = r_3 = 0, r_4 = 1$. Hence our method is zero stable.

III. Consistency

A block method is said to be consistent, if it has order greater than one.From the above analysis,it is obvious that our method is consistent.

Convergence

Theorem 1

The necessary and sufficient conditions for a linear multistep method to be convergent are that it be consistent and zero stable.

IV. Region of Absolute Stability

Definition1.3: Region of Absolute Stability.

Region of absolute stability is a region in the complex z plane,where $z = \lambda h$ .It is defined as those values of z such that the numerical solution of $y' = -\lambda y$ satisfies $j_j \to 0\ 0$ as $j \to \infty$ for any initial condition.

To determine the absolute stability region of the new block method, we adopt the boundary locus method.This is achieved by substituting the test equation

$$y' = -\lambda y \quad (14)$$

into the block formula (8).This gives

$$A^{(0)}Y_m(w) = Ey_n(w) - h\lambda Dy_n(w) - h\lambda BY_m(w). \quad (15)$$

This gives,

$$\bar{h}(w) = -\left(\frac{A^{(0)}Y_m(w) - Ey_n(w)}{Dy_n(w) + BY_m(w)}\right) \quad (16)$$

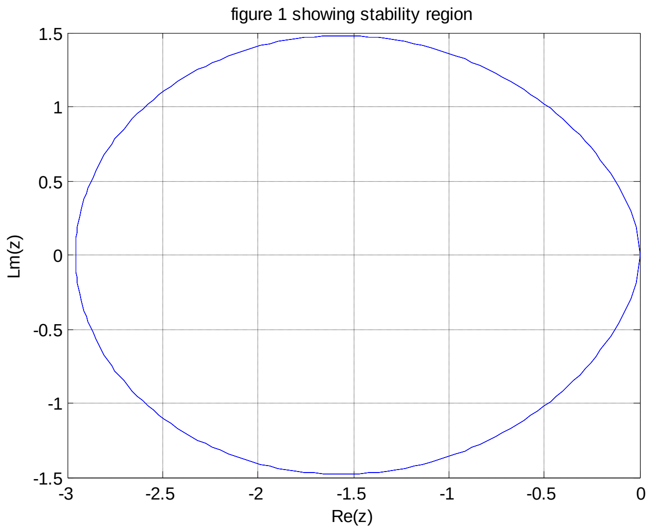
since $\bar{h}$ is given by $\bar{h} = \lambda h$ and $w = e^{i\theta}$.Equation (16) is called characteristic or stability polynomial.For our method, equation (16) is given by

$$\bar{h} = -h^4\left(\frac{1}{1280}w^3 - \frac{233}{172800}w^4\right) - h^3\left(\frac{1199}{64800}w^4 + \frac{5}{384}w^3\right)$$

h (w)=-h⁴((1/(1280))w³-below is the

$$-h^2\left(\frac{7}{64}w^3 - \frac{18431}{129600}w^4\right) - h\left(\frac{26}{45}w^4 + \frac{1}{2}w^3\right) + w^4 - w^3.$$

Region of Absolute Stability for the method.



Definition 1.4: A-stable: A numerical integrator is said to be A-stable if its region of absolute stability R incorporates the entire half of the complex plane denoted by C i.e.

$R = \{Z \in C \,/\, real\ (Z) < 0\}$.This shows that the method is A-stable.

Numerical Examples

We shall apply the method newly developed to solve some sample problems as shown below. The following notations shall be used in the tables below.

ENM-Error in our new method

EJM-Error in (James *et al.*, [5]).

Problem 1

We consider a linear first order ordinary differential equation:

$$y' = -y,\ y(0) = 1, 0 \le x \le 1, h = 0.1$$

Exact solution: $y(x) = e^x$.

This problem was solved by James *et al.*, [5].

Table 1: Results for Problem 1

X	Exact solution	Computed solution	ENM	EJM
0.1	1.1051709180756477	1.10517091807564868.8818(-16)		1.7444(-11)
0.2	1.22140275816017011.2214027581601712		1.1102(-15)	1.5783(-11)
0.3	1.3498588075760034	1.3498588075760041	6.6613(-16)	1.4281(-11)
0.4	1.4918246976412703	1.4918246976412712	4.4409(-16)	1.2925(-11)
0.5	1.6487212707001286	1.6487212707001289	2.2204(-16)	1.1696(-11)
0.6	1.8221188003905089	1.8221188003905107	8.8818(-16)	1.0580(-11)
0.7	2.0137527074704775	2.0137527074704775	0.0000(+00)	9.5701(-11)
0.8	2.2255409284924688	2.2255409284924688	0.0000(+00)	8.6612(-11)
0.9	2.4596031111569512	2.4596031111569534	2.2204(-15)	7.8371(-11)
1.0	2.7182818284590473	2.7182818284590504	3.1086(-15)	7.0927(-11)

Problem 2

$$y' = xy,\ y(0) = 1, 0 \le x \le 1, h = 0.1$$

Exact solution: $y(x) = e^{\frac{1}{2}x^2}$.

Table 2: Results for Problem

X	Exact solution	Computed solution	ENM	EJM
0.1	1.0050125208594012	1.0050125208594014	2.2204(-16)	1.6554(-11)
0.2	1.0202013400267558	1.0202013400267567	8.8818(-16)	4.3981(-11)
0.3	1.0460278599087169	1.0460278599087172	2.2204(-16)	7.8451(-11)
0.4	1.0832870676749586	1.0832870676749586	0.0000(+00)	1.2925(-10)
0.5	1.1331484530668265	1.1331484530668250	1.5543(-15)	1.9709(-10)
0.6	1.1972173631218104	1.1972173631218075	2.8866(-15)	3.0180(-10)
0.7	1.2776213132048870	1.2776213132048844	2.6645(-15)	4.5771(-10)
0.8	1.3771277643359578	1.3771277643359543	3.5527(-15)	6.8954(-10)
0.9	1.4993025000567677	1.4993025000567641	3.5527(-15)	1.0336(-09)
1.0	1.6487212707001293	1.6487212707001255	3.7748(-15)	1.5435(-09)

Problem 3

$y' = x - y, y(0) = 0, 0 \leq x \leq 1, h = 0.1$

Exact solution: $y(x) = x + e^{x} - 1$.

This problem was solved by (James *et al.*, [5])

Table 3: Results for Problem 3

X	Exact solution	Computed solution	ENM	EJM
0.1	0.0048374180359596	0.0048374180359596	2.9490(-17)	1.7443(-11)
0.2	0.0187307530779819	0.0187307530779819	2.0817(-17)	1.5786(-11)
0.3	0.0408182206817180	0.0408182206817179	1.4571(-16)	1.4283(-11)
0.4	0.0703200460356395	0.0703200460356394	4.1633(-17)	1.2924(-11)
0.5	0.1065306597126337	0.1065306597126336	9.7145(-17)	1.1694(-11)
0.6	0.1488116360940266	0.1488116360940267	8.3267(-17)	1.0581(-11)
0.7	0.1965853037914098	0.1965853037914098	2.7756(-17)	9.5739(-12)
0.8	0.2493289641172218	0.2493289641172220	1.9429(-16)	8.6613(-12)
0.9	0.3065696597405996	0.3065696597405995	1.1102(-16)	7.8396(-12)
1.0	0.3678794411714428	0.3678794411714428	0.0000(+00)	7.0906(-12)

V. Discussion of Result

We have considered three numerical examples in this paper to test the efficiency of our method. The three problems were earlier solved by (James *et al.*, [5]). In all the three examples our new method gave better approximation when compared to that of (James *et al*., [5]).

VI. Conclusion

In this paper, we have developed a new hybrid block method for the solution of first order initial value problems in ordinary differential equations. Our method was found to be zero stable,consistent and convergent. The numerical results show that our method is computationally reliable and gave better accuracy than the existing methods.

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