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Casson flow of blood through an arterial tube with overlapping stenosis

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Abstract: *The objective of the present analysis is to study the effect of overlapping stenosis on blood flow through an artery by taking the blood as Casson type non-Newtonian fluid. The expressions for flux and resistance to flow have been studied here by assuming the stenosis is to be mild. The results are shown graphically for different values of yield stress, stenosis length, stenosis height and discussed.*
Key Words: *Stenosis, Casson fluid, resistance to flow, flux.*

I. Introduction:

As human system, its growth function are very complicated in nature, actual blood flow model is unknown to us. Many biomedical researchers have tried to develop mathematical models to get insight into the blood flow characteristics through an arterial tube by considering the blood as Newtonian or non-Newtonian fluid. Nowadays healthcare problems are concerned by the people. Presently cardiovascular diseases have been noticed as one of the major cause of death in the industrialized world. Cardiovascular system consists of heart and blood vessels, which plays an important role in blood flow through an artery. The blood flow characteristics can be altered significantly by the arterial disease, such as stenosis and aneurysm. Stenosis is a serious cardiovascular disease. Stenosis is formed due to the deposition of fats or lipids in the inner wall along the lumen of the artery. Once arterial stenosis occurs, resistance to flow is increased and so normal blood flow is disturbed abruptly. Hence blood flow is insufficient to reach every cell and this resists nutrient supplement and as a result of this, several diseases like hypertension, heart attack, brain haemorrhage occur.

In view of this several authors (Young [1], Lee and Fung [2], Shukla et. al [3], Chaturani and Samy [4], Fry [5], Caro et. al [6], Texon [7], Richard et. al [8] and Radhakrishnamacharya et. al [9]) have presented mathematical models for blood flow through stenosed/constricted ducts. In all these studies blood has been considered as Newtonian fluid. However it may be noted that blood does not behave as Newtonian fluid under certain conditions. It is well known that whole blood behaves as a non-Newtonian fluid at low shear rate, as blood consists of cells in an aqueous solution.

Many mathematicians have studies some mathematical models by treating the blood as a non-Newtonian fluid (Charm and Kurland [10], Hershey et. al[11], Whitmore [12], Cokelet [13], Lih [14]). It has been pointed out that at low shear rate ($0.1 sec^{-1}$) the blood behaves like a Casson fluid model (Casson [15], Reiner and Scott Blair [16]).

However, all these investigations considered the effect of single stenosis, but the constrictions may develop in series (multiple stenosis) or may be of irregular shapes or overlapping. Chakrabortyet. al [17] have studied effect of overlapping stenosis on arterial flow.

In the present analysis I propose to discuss the effect of overlapping stenosis on blood flow through arterial tube by considering the blood as Casson type non-Newtonian fluid.

II. Mathematical Formulation:

Let us consider the steady flow of blood through an axially symmetric but radially non symmetric overlapping stenosed artery.

The geometry of Stenosis can be taken as (Maruthiprasad et. al [18], Layek et. al [19], Srivastava [20])

$$h = \frac{R(z)}{R_0}$$

$$=1-\frac{3\delta}{2R_0L_0^4}\left[11(z-d)L_0^3-47(z-d)^2L_0^2+72(z-d)^3L_0-36(z-d)^4\right],\ d\le z\le d+L_0$$

$$= 1, \text{ otherwise,} \qquad (1)$$

Where $R(z)$ is the radius of the tube in the stenotic region, R_0 is the radius of the tube outside the stenotic region, R_p is the radius in the plug flow region , L_0 is the length of the stenosis and d indicates its location, δ is the maximum height of the stenosis. Projection of stenosis at the two positions is denoted by z as z = $d+\frac{L_0}{6}$, z = $d+\frac{5L_0}{6}$. The critical height is taken as $\frac{3\delta}{4}$ at z = $d+\frac{L_0}{2}$ from the origin.

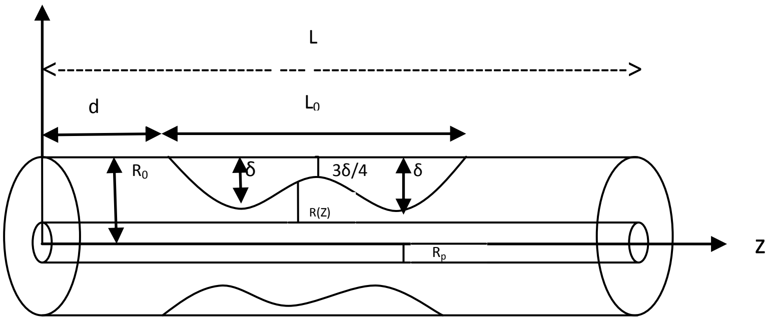


Fig.1: Geometry of a uniform tube of circular cross section with overlapping stenosis

The equation governing the flow is given by

$$-\frac{\partial p}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}), \qquad (2)$$

In which τ_{rz} represents the shear stress of blood considered as Casson fluid and p is the pressure gradient.

The relationship between shear stress and shear rate is given by

$$\tau_{rz}^{\frac{1}{2}} = (-\mu\frac{\partial w}{\partial r})^{\frac{1}{2}} + \tau_0^{\frac{1}{2}} \ , \tau_{rz} \geq \tau_0$$

$$\frac{\partial w}{\partial r} = 0\tau_{rz} < \tau_0, \qquad (3)$$

where$w$ stands for the axial velocity of blood; τ_0, the yield stress and μ , the coefficient of viscosity of blood.

The boundary conditions are

(i) τ_{rz} is finite at r = 0
(ii)w = 0 at r= $h(z)$
(iii)$\frac{\partial w}{\partial r} = 0atr$ = 0 if $\tau_{rz} < \tau_0$ (4)

Solution:

Integrating (2) and using the boundary condition (i) of (4) we get

$$\tau_{rz} = \frac{Pr}{2},$$

where P = $-\frac{\partial p}{\partial z}$

From (3)

$$(-\mu\frac{\partial w}{\partial r})^{\frac{1}{2}} = \sqrt{\frac{P}{2}}r^{\frac{1}{2}} - \tau_0^{\frac{1}{2}} \qquad (5)$$

Integrating (5) and then using the boundary condition (4) we get

$$w = \frac{Ph^2}{4\mu}[(1-\frac{r^2}{h^2}) + \frac{4\tau_0}{Ph}\left(1-\frac{r}{h}\right) - \sqrt{\frac{128\tau_0}{9ph}}(1-\frac{r^{3/2}}{h^{3/2}})] \qquad (6)$$

Since $\frac{\partial w}{\partial r} = 0$ at r= r_0, the upper limit of the plug flow region is obtained as

$$r_0 = \frac{2\tau_0}{P}$$

Thus we get

$$w = \frac{Ph^2}{4\mu}[(1-\frac{r^2}{h^2}) + \frac{2r_0}{h}\left(1-\frac{r}{h}\right) - \frac{8}{3}\sqrt{\frac{r_0}{h}}(1-\frac{r^{3/2}}{h^{3/2}})] \qquad (7)$$

The plug velocity w_p is given by

$$w_p = \frac{Ph^2}{4\mu}[1-\frac{1}{3}\frac{r_0^2}{h^2} + \frac{2r_0}{h} - \frac{8}{3}\sqrt{\frac{r_0}{h}}] \qquad (8)$$

The volumetric flow rate i.e, the flux is given by

$$Q = 2\,[\int_0^{r_0} w_p r dr + \int_{r_0}^{h} wrdr]$$

$$= \frac{Ph^4}{168\mu}[21+28\frac{r_0}{h} - 48\sqrt{\frac{r_0}{h}} - \frac{r_0^4}{h^4}] \qquad (9)$$

Thus $$\frac{\partial p}{\partial z} = -P = \frac{-168\mu Q}{h^4[21+28\frac{r_0}{h}-48\sqrt{\frac{r_0}{h}}-\frac{r_0^4}{h^4}]} \quad (10)$$

The pressure drop Δp across the stenosis between z = 0 to z = L is obtained as

$$\Delta p = \int_0^L \frac{\partial p}{\partial z} dz$$

$$= -168\mu \int_0^L \frac{Q}{h^4[21+28\frac{r_0}{h}-48\sqrt{\frac{r_0}{h}}-\frac{r_0^4}{h^4}]} dz \quad (11)$$

Introducing the following non-dimensional quantities

$$\bar{z} = \frac{z}{L}, \bar{\delta} = \frac{\delta}{R_0}, \bar{R}(z) = \frac{R(z)}{R_0}, \bar{Q} = \frac{Q}{\pi U R_0^2},$$

$$\overline{\tau_0} = \frac{\tau_0}{\mu U/R_0}, \overline{\tau_{rz}} = \frac{\tau_{rz}}{\mu U/R_0}, \bar{P} = \frac{P}{\mu UL/R_0^2}, \quad (12)$$

in equation (10) we finally get (after dropping the bars)

$$\Delta p = \int_0^1 \frac{-168\mu Q}{h^4[21+28\frac{r_0}{h}-48\sqrt{\frac{r_0}{h}}-\frac{r_0^4}{h^4}]} dz$$

The resistance to flow λ is defined as

$$\lambda = \frac{\Delta p}{Q} = \int_0^1 \frac{-168\mu}{h^4[21+28\frac{r_0}{h}-48\sqrt{\frac{r_0}{h}}-\frac{r_0^4}{h^4}]} dz \quad (13)$$

The pressure drop in the absence of stenosis (h =1) is denoted by Δp_N and is obtained from (13) as

$$\Delta p_N = \int_0^1 \frac{-168\mu Q}{(21+28r_0-48\sqrt{r_0}-r_0^4)} dz \quad (14)$$

The resistance to flow in the absence of stenosis as

$$\lambda_N = \frac{\Delta p_N}{Q} = \frac{-168\mu}{(21+28r_0-48\sqrt{r_0}-r_0^4)}$$

Hence the normalised resistance to flow $\bar{\lambda}$ is given by

$$\bar{\lambda} = \frac{\lambda}{\lambda_N} = (21+28r_0-48\sqrt{r_0}-r_0^4) \int_0^1 \frac{1}{(21h^4+28r_0h^3-48r_0h^{7/2}-r_0^4)} dz \quad (15)$$

III. Results and Discussions:

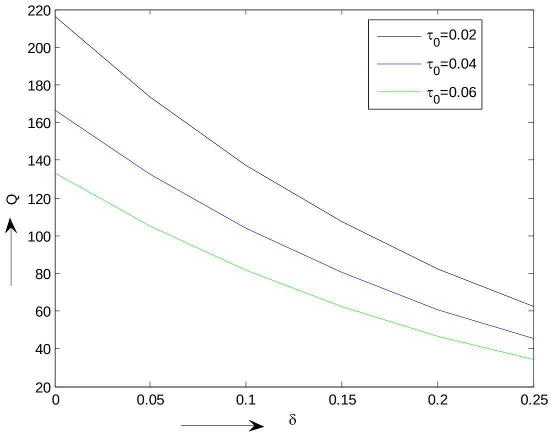
To illustrate the flow analysis the results are shown graphically with the help of MATLAB-7.6. The effect of various parameters on flux and resistance to floware computed numerically.

Figures 2, 3, 4 reveal that the variation of flux for different values of τ_0, L_0 and z with the variation of δ. It is clear from the figures that Q decreases as $\delta$and $\tau_0$increase, but the reverse effect occurs when L_0 and z increase.

Figures 5, 6 and 7 describe the effectsof τ_0, L_0 and d on resistance to flow. It is found that as δincreases $\bar{\lambda}$ increases with the increase of $\tau_0$and L_0, but it decreases when d increases for fixed values of $\tau_0$and L_0.

IV. Conclusions

Blood flow through an arterial tube mainly depends on the volumetric flow rate and resistance to flow. It is clear from the present analysis that as stenosis grows resistance to flow increases with in the stenotic region, for which several diseases occur, such as high blood pressure, stroke, brain haemorrhage etc. so the results are greatly influenced by the change of shape parameters. Hence the present analysis may be helpful for the diagnosis of various types of cardiovascular diseases.



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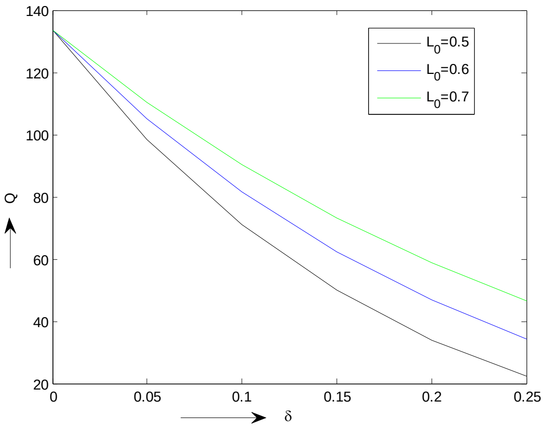

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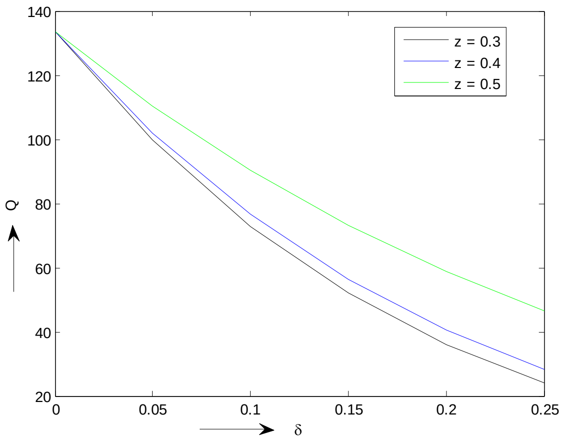

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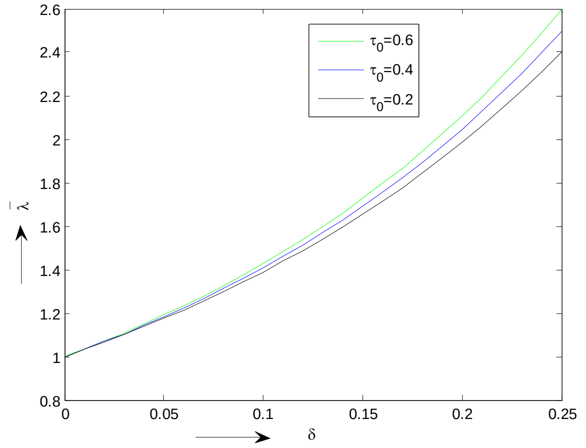

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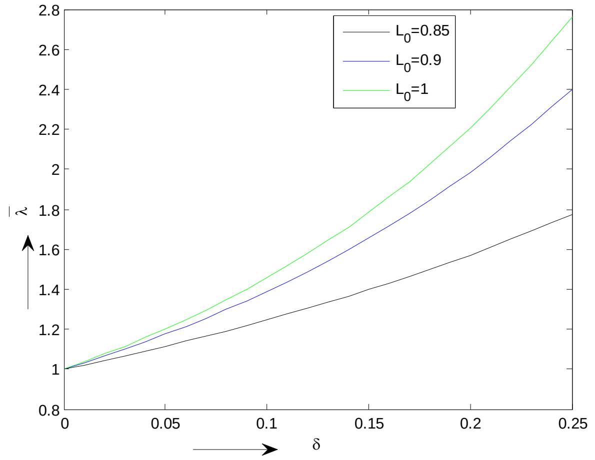

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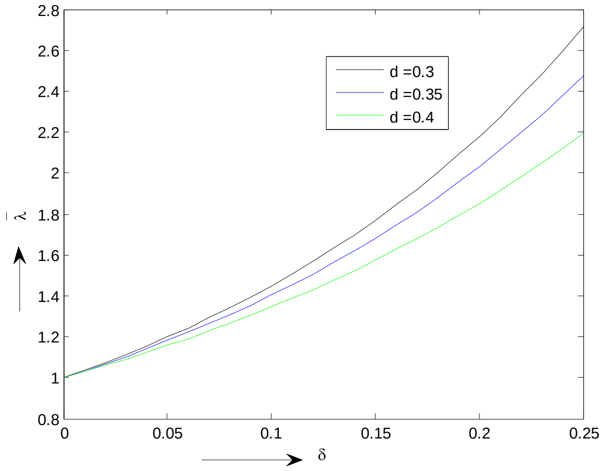

Figure -7

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