

MHD Free Convection three dimensional flow through a porous medium between two vertical plates

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Abstract : The effects of radiation and hall current on MHD free convection three dimensional flow in a vertical channel filled with a porous medium has been studied. We consider an incompressible viscous and electrically conducting incompressible viscous fluid in a parallel plate channel bounded by a loosely packed porous medium. The fluid is driven by a uniform pressure gradient parallel to the channel plates and the entire flow field is subjected to a uniform inclined magnetic field of strength H_0 inclined at an angle of inclination α with the normal to the boundaries in the transverse xy -plane. The temperature of one of the plates varies periodically and the temperature difference of the plates is high enough to induce radiative heat transfer. The effects of various parameters on the velocity profiles, the skin friction, temperature field, rate of heat transfer in terms of their amplitude and phase angles are shown graphically.

Keywords - Optically thin fluid, parallel plate channel, porous medium, radiative heat transfer, steady hydro magnetic flows, three dimensional flows

I. INTRODUCTION

The flow of fluids through porous media are encountered in a wide range of engineering and industrial applications such as in recovery or extraction of crude oil, geothermal systems, thermal insulation, heat exchangers, storage of nuclear wastes, packed bed catalytic reactors, atmospheric and oceanic circulations. Several scholars viz. Crammer and Pai [1], Ferraro and Plumpton [2], Shercliff [3] have studied such flows because of their varied importance. MHD channel or duct flows are important from its practical point of view. Chang and Lundgren [4] have studied a hydro magnetic flow in a duct. Yen and Chang [5] analysed the effect of wall electrical conductance on the magneto hydro dynamic Couette flow. From the technological point of view and due to practical applications, free convective flow and heat transfer problems are always important. This process of heat transfer is encountered in cooling of nuclear reactors, providing heat sinks in turbine blades and aeronautics. Ostrach [6] studied the combined effects of natural and forced convection laminar flow and heat transfer of fluids with and without heat sources in channels with linearly varying wall temperature. Jain and Gupta [7] studied three dimensional free convection Couette flow with transpiration cooling. There are numerous important engineering and geophysical applications of the channel flows through porous medium, for example in the fields of agricultural engineering for channel irrigation and to study the underground water resources, in petroleum technology to study the movement of natural gas, oil and water through the oil channels/reservoirs. Transient natural convection between two vertical walls with a porous material having variable porosity has been studied by Paul *et al.* [8]. Sahin [9] investigated the three-dimensional free convective channel flow through porous medium. In recent years, the effects of transversely applied magnetic field on the flows of electrically conducting viscous fluids have been discussed widely owing to their astrophysics, geophysical and engineering applications. Attia and Kotb [10] studied MHD flow between two parallel plates with heat transfer. When the strength of the magnetic field is strong, one cannot neglect the effects of Hall current. The rotating flow of an electrically conducting fluid in the presence of a magnetic field is encountered in geophysical and cosmical fluid dynamics. It is also important in the solar physics involved in the sunspot development. Soundalgekar [11] studied the Hall effects in MHD Couette flow with heat transfer. Mazumder *et al.* [12, 13] have studied the effects of Hall current on MHD Ekman layer flow and heat transfer over porous plate and on free and forced convective hydro magnetic flow through a channel. Hall effects on unsteady MHD free and forced convection flow in a porous rotating channel has been investigated by Siva Prasad *et al.* [14]. Singh and Kumar [15] studied the combined effects of Hall current and rotation on free convection MHD flow in a porous channel. Ghosh

et al.[16] studied the Hall effects on MHD flow in a rotating system with heat transfer characteristics. Radiative convective flows have gained attention of many researchers in recent years. Radiation plays a vital role in many engineering, environment and industrial processes e.g. heating and cooling chambers, fossil fuel combustion energy processes astrophysical flows and space vehicle re-entry. Raptis [17] studied the radiation and free convection flow through a porous medium. Alagoa *et al.*[18] analysed the effects of radiation on free convective MHD flow through a porous medium between infinite parallel plates in the presence of time-dependent suction. Mebine [19] studied the radiation effects on MHD Couette flow with heat transfer between two parallel plates. Singh and Kumar [20] have studied radiation effects on the exact solution of free convective oscillatory flow through porous medium in a rotating vertical porous channel. Venkata Ramana [23] studied Hall current effect on magneto hydro dynamics free-convection flow past a semi infinite vertical porous plate with mass transfer. The effects of Hall current and rotation on MHD free convection flow in a vertical rotating channel filled with porous medium have been studied by Singh and Reena pathak [24]. M.V.Krishna and Irfan [26] investigated the unsteady MHD flow of Maxwell fluid through a porous medium in Rotating parallel plate channel and then extended taking hall current into account by M.V.Krishna and Irfan [25]. M.V.Krishna [27] discussed Hall currents on MHD flow of a couple stress fluid in a parallel plate channel bounded by a porous bed on the lower half in presence of inclined magnetic field. Syamala Sarojini [28] discussed the effects of hall currents on MHD flow of a couple stress fluid through a porous medium in a parallel plate channel in presence of effect of inclined magnetic field. Raju [29] studied the hall current effects on unsteady MHD three dimensional flow of a couple stress fluid through a porous medium in parallel plate channel. Recently, M.V.Krishna and J.Prakash [30] discussed the hall current effects on Unsteady MHD flow in a Rotating parallel plate channel bounded by Porous bed on the Lower half. Motivated from the above studies, in this paper, we discussed the MHD free convective flow with porous medium in a vertical channel filled. The transverse magnetic field applied is strong enough so that the hall currents are induced. The temperature difference between the walls of the channel is sufficiently high to radiate the heat. The fluid is driven by a uniform pressure gradient parallel to the channel plates and the entire flow field is subjected to a uniform inclined magnetic field of strength H_o inclined at an angle of inclination α with the normal to the boundaries in the transverse xy -plane.

1.1. BASIC EQUATIONS

The equations governing the unsteady free convective flow of an incompressible, viscous and electrically conducting fluid in a vertical channel filled with porous medium in the presence of magnetic field are

Equation of Continuity:

$$\nabla \cdot \bar{V} = 0 \tag{1.1.1}$$

Momentum Equation:

$$\rho \left(\frac{\partial \bar{V}}{\partial t} + (\bar{V} \cdot \nabla) \bar{V} \right) = -\nabla p + \bar{J} \times \bar{B} + \mu \nabla^2 \bar{V} - \frac{\mu}{k} \bar{V} + g \beta T \tag{1.1.2}$$

Energy Equation:

$$\rho C_p \left(\frac{\partial T}{\partial t} + (\bar{V} \cdot \nabla) T \right) = K \nabla^2 T - \nabla q \tag{1.1.3}$$

Kirchhoff's First Law:

$$\text{div} \bar{J} = 0 \tag{1.1.4}$$

General Ohm's Law:

$$\bar{J} + \frac{\omega_e \tau_e}{B_0} (\bar{J} \times \bar{B}) = \sigma \left[\bar{E} + \bar{V} \times \bar{B} + \frac{1}{en_e} \nabla p_e \right] \tag{1.1.5}$$

Gauss's Law of Magnetism:

$$\text{div} \bar{B} = 0 \tag{1.1.6}$$

Where, \bar{V} is the velocity vector, p the pressure, ρ the density, B is the magnetic induction vector, \bar{J} the current density, μ the coefficient of viscosity, t the time, g the acceleration due to gravity,

β the coefficient of volume expansion, k is the permeability of the porous medium, C_p the specific heat at constant pressure, T the temperature, T_0 the reference temperature that of the left plate, K the thermal conductivity, q the radiative heat, σ the electrical conductivity, B_0 the strength of the applied magnetic field, e the electron charge, ω_e the electron frequency, τ_e the electron collision time, p_e the electron pressure, E the electric field and η_e is the number density of electron.

II. FORMULATION AND SOLUTION OF THE PROBLEM

Consider an unsteady MHD free convective flow of an electrically conducting, viscous, incompressible fluid through a porous medium bounded between two insulated infinite vertical plates in the presence of Hall current and thermal radiation. The plates are at a distance d apart. A Cartesian coordinate system with x -axis oriented vertically upward along the centre line of the channel is introduced. The z -axis is taken perpendicular to the planes of the plates as shown in Fig. 1.

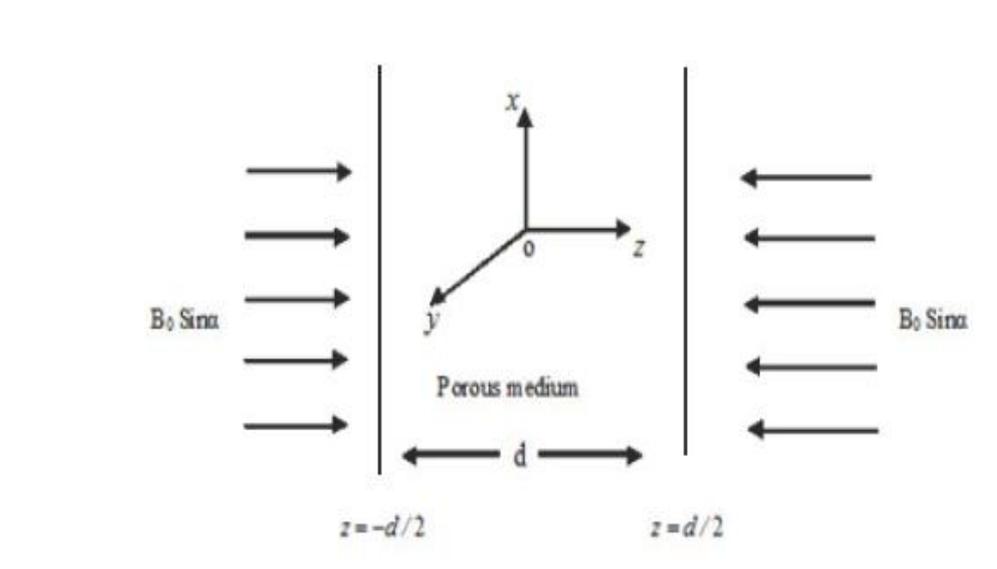


Fig 1. Physical Configuration of the Problem

We choose a Cartesian system $O(x, y, z)$ such that the boundary walls are at $z=0$ and $z=d$ are assumed to be parallel to xy -plane. The steady flow through porous medium is governed by Brinkman's equations. At the interface the fluid satisfies the continuity condition of velocity and stress. The boundary plates are assumed to be parallel to xy -plane and the magnetic field of strength H_0 inclined at an angle of inclination α to the z -axis in the transverse xz -plane. The component along z -direction induces a secondary flow in that direction while its x -components changes perturbation to the axial flow. The steady hydro magnetic equations governing the incompressible fluid under the influence of a uniform inclined magnetic field of strength H_0 inclined at an angle of inclination α with reference to a frame are

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial z^2} - \frac{\mu_e J_z H_0 \text{Sin} \alpha}{\rho} - \frac{\nu}{k} u + g \beta T \tag{2.1}$$

$$\frac{\partial w}{\partial t} = \nu \frac{\partial^2 w}{\partial z^2} + \frac{\mu_e J_x H_0 \text{Sin} \alpha}{\rho} - \frac{\nu}{k} w \tag{2.2}$$

Where, all the physical quantities in the above equation have their usual meaning. (u, w) are the velocity components along $O(x, z)$ directions respectively. ρ is the density of the fluid, μ_e is the magnetic permeability, ν is the coefficient of kinematic viscosity, k is the permeability of the medium, H_0 is the applied magnetic field.

When the strength of the magnetic field is very large, the generalized Ohm's law is modified to include the Hall current, so that

$$J + \frac{\omega_e \tau_e}{H_0} J \times H = \sigma (E + \mu_e q \times H) \tag{2.3}$$

Where, \mathbf{q} is the velocity vector, H is the magnetic field intensity vector, E is the electric field, J is the current density vector, ω_e is the cyclotron frequency, τ_e is the electron collision time, σ is the fluid conductivity and μ_e is the magnetic permeability. In equation (2.3) the electron pressure gradient, the ion-slip and thermo-electric effects are neglected. We also assume that the electric field $E=0$ under assumptions reduces to

$$J_x - m J_z \text{Sin}\alpha = -\sigma\mu_e H_0 w \text{Sin}\alpha \tag{2.4}$$

$$J_z + m J_x \text{Sin}\alpha = -\sigma\mu_e H_0 u \text{Sin}\alpha \tag{2.5}$$

Where $m = \omega_e \tau_e$ is the hall parameter.

On solving equations (2.3) and (2.4) we obtain

$$J_x = \frac{\sigma\mu_e H_0 \text{Sin}\alpha}{1+m^2 \text{Sin}^2\alpha} (um \text{Sin}\alpha - w) \tag{2.6}$$

$$J_z = \frac{\sigma\mu_e H_0 \text{Sin}\alpha}{1+m^2 \text{Sin}^2\alpha} (u + wm \text{Sin}\alpha) \tag{2.7}$$

Using the equations (2.6.) and (2.7), the equations of the motion with reference to frame are given by

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial z^2} - \frac{\sigma\mu_e^2 H_0^2 \text{Sin}\alpha}{\rho(1+m^2 \text{Sin}^2\alpha)} (u + wm \text{Sin}\alpha) - \frac{\nu}{k} u + g\beta T \tag{2.8}$$

$$\frac{\partial w}{\partial t} = \nu \frac{\partial^2 w}{\partial z^2} + \frac{\sigma\mu_e^2 H_0^2 \text{Sin}\alpha}{\rho(1+m^2 \text{Sin}^2\alpha)} (um \text{Sin}\alpha - w) - \frac{\nu}{k} w \tag{2.9}$$

$$\rho C_p \frac{\partial T}{\partial t} = K \frac{\partial^2 T}{\partial z^2} - \frac{\partial q}{\partial z} \tag{2.10}$$

The boundary conditions for the problem are

$$u = w = T = 0, \quad z = -\frac{d}{2} \tag{2.11}$$

$$u = w = 0, T = T_w \text{Cos } \omega t, \quad z = \frac{d}{2} \tag{2.12}$$

Where, T_w is the mean temperature of the plate at $z = d/2$ and ω is the frequency of oscillations. Following Cogley et.al [22], the last term in the energy equation (2.10),

$$\frac{\partial q}{\partial z} = 4\alpha^2 (T - T_0) \tag{2.13}$$

Stands for radiative heat flux modifies to

$$\frac{\partial q}{\partial z} = 4\alpha^2 T \tag{2.14}$$

In view of the reference temperature $T_0 = 0$, where α is mean radiation absorption co-efficient.

We introduce the following non-dimensional variables and parameters.

$$z^* = \frac{z}{d}, x = \frac{x}{d}, u = \frac{u^*}{U}, v = \frac{v^*}{U}, q = \frac{q^*}{U}, t^* = \frac{tU}{d}, \omega^* = \frac{\omega d}{U}, p^* = \frac{p}{\rho U^2}, T^* = \frac{T}{T_w}$$

Where, U is the mean axial velocity.

Making use of non-dimensional variables, the governing equations reduces to (dropping asterisks),

$$\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{1}{Re} \frac{\partial^2 u}{\partial z^2} - \frac{M^2 \text{Sin}^2\alpha (u + wm \text{Sin}\alpha)}{Re(1+m^2 \text{Sin}^2\alpha)} - \frac{D^{-1}}{Re} u - Gr T \tag{2.15}$$

$$\frac{\partial w}{\partial t} = \frac{1}{Re} \frac{\partial^2 w}{\partial z^2} + \frac{M^2 \text{Sin}^2\alpha (um \text{Sin}\alpha - w)}{Re(1+m^2 \text{Sin}^2\alpha)} - \frac{D^{-1}}{Re} w \tag{2.16}$$

$$\frac{\partial T}{\partial t} = \frac{1}{Pe} \frac{\partial^2 T}{\partial z^2} - \frac{R^2}{Pe} \frac{\partial q}{\partial z} \tag{2.17}$$

Where, $Re = \frac{Ud}{\nu}$ is the Reynolds number, $M^2 = \frac{\sigma\mu_e^2 H_0^2 d^2}{\rho U}$ is the Hartmann number, $D = \frac{K}{d^2}$ is the

permeability parameter (Darcy parameter), $Gr = \frac{g\beta d^2 T}{\nu U}$ is the Grashof number, $Pe = \frac{\rho C_p d U}{\nu U}$ is the

Peclet number, $R = \frac{2\alpha d}{\sqrt{K}}$ is the radiation parameter.

The corresponding transformed boundary conditions are

$$u = w = T = 0, \quad z = -\frac{1}{2} \tag{2.18}$$

$$u = w = 0, T = \text{Cos } \omega t, \quad z = \frac{1}{2} \tag{2.19}$$

For the oscillatory internal flow, we shall assume that the fluid flows only under the influence of a non-dimensional pressure gradient oscillating in the in the direction of x -axis only which is of the form,

$$-\frac{\partial p}{\partial x} = P \text{Cos } \omega t \tag{2.20}$$

In order to combine equations (2.15) and (2.16) into single equation, we introduce a complex function $q = u + iw$, we obtain

$$Re \frac{\partial q}{\partial t} = -P \text{Cos } \omega t + \frac{\partial^2 q}{\partial z^2} - \left(\frac{M^2 \text{Sin}^2 \alpha}{(1-im\text{Sin}\alpha)} + i\omega Re + D^{-1} \right) q - Gr T \tag{2.21}$$

The boundary conditions in complex form are

$$q = T = 0, \quad z = -\frac{1}{2} \tag{2.22}$$

$$q = 0, T = e^{i\omega t}, \quad z = \frac{1}{2} \tag{2.23}$$

In order to solve the equations (2.17) and (2.21) making use of boundary conditions (2.22) and (2.23), we assume in the complex form the solution of the problem as

$$q(z, t) = q_0(z) e^{i\omega t}, T(z, t) = \theta_0(z) e^{i\omega t}, -\frac{\partial p}{\partial x} = P e^{i\omega t} \tag{2.24}$$

Substituting equations (2.24) in equations (2.17) and (2.21), we get

$$\frac{d^2 q_0}{dz^2} - \lambda^2 q_0 = -P Re - Gr \theta_0 \tag{2.25}$$

$$\frac{d^2 q_0}{dz^2} - \xi^2 \theta_0 = 0 \tag{2.26}$$

$$\text{Where, } \lambda^2 = \frac{M^2 \text{Sin}^2 \alpha}{(1-im\text{Sin}\alpha)} + i\omega Re + D^{-1} \text{ and } \xi^2 = i\omega Pe + R^2$$

The boundary conditions given in equations (2.22) and (2.23) become

$$q_0 = \theta_0 = 0, \quad z = -\frac{1}{2} \tag{2.27}$$

$$q_0 = 0, \theta_0 = 1, \quad z = \frac{1}{2} \tag{2.28}$$

The ordinary differential equations (2.24) and (2.25) are solved under the boundary conditions given in equations (2.27) and (2.28) for the velocity and temperature fields. The solution of the problem is obtained as

$$q(z,t) = \left\{ \frac{PRe}{\lambda^2} \left(1 - \frac{\text{Cosh}\lambda z}{\text{Cosh}\frac{\lambda}{2}} \right) + \frac{Gr}{\lambda^2 - \xi^2} \left(\frac{\text{Sinh}\lambda \left(z + \frac{1}{2} \right)}{\text{Sinh}\lambda} - \frac{\text{Sinh}\xi \left(z + \frac{1}{2} \right)}{\text{Sinh}\xi} \right) \right\} e^{i\omega t} \quad (2.29)$$

$$T(z,t) = \frac{\text{Sinh}\xi \left(z + \frac{1}{2} \right)}{\text{Sinh}\xi} e^{i\omega t} \quad (2.30)$$

Now from the velocity field, we can obtain the skin-friction at the left plate in terms of its amplitude and phase angle as

$$\tau_L = \left(\frac{\partial q}{\partial z} \right)_{z=-1/2} = \left(\frac{\partial q_o}{\partial z} \right)_{z=-1/2} e^{i\omega t} = |q| \text{Cos}(\omega t + \phi) \quad (2.31)$$

Where, $|q| = \sqrt{(\text{Re } q)^2 + (\text{Im } q)^2}$ and $\phi = \text{Tan}^{-1} \left(\frac{\text{Re } q}{\text{Im } q} \right)$

$$\text{Re } q + i \text{Im } q = \frac{PRe}{\lambda^2} \text{Tanh} \left(\frac{\lambda}{2} \right) + \frac{Gr}{\lambda^2 - \xi^2} \left[\frac{\lambda}{\text{Sinh}\lambda} - \frac{\xi}{\text{Sinh}\xi} \right]$$

From the temperature field, the rate of heat transfer Nu (Nusselt number) at the left plate in terms of its amplitude and phase angle is obtained

$$Nu = \left(\frac{\partial T}{\partial z} \right)_{z=-1/2} = \left(\frac{\partial \theta_o}{\partial z} \right)_{z=-1/2} e^{i\omega t} = |H| \text{Cos}(\omega t + \psi) \quad (2.32)$$

Where, $|H| = \sqrt{(\text{Re } H)^2 + (\text{Im } H)^2}$, $\psi = \text{Tan}^{-1} \left(\frac{\text{Re } H}{\text{Im } H} \right)$ and $\text{Re } H + i \text{Im } H = \frac{\xi}{\text{Sinh}\xi}$.

III. RESULTS AND DISCUSSION

We consider an incompressible viscous and electrically conducting fluid in a parallel plate channel bounded by a loosely packed porous medium. The fluid is driven by a uniform pressure gradient parallel to the channel plates and the entire flow field is subjected to a uniform inclined magnetic field of strength H_o inclined at an angle of inclination α with the normal to the boundaries in the transverse xy -plane. The temperature of one of the plates varies periodically and the temperature difference of the plates is high enough to induce radiative heat transfer. The complete expressions for the velocity, $q(z)$ and temperature, $T(z)$ profiles as well as the skin friction, τ and the heat transfer rate, Nu are given in equations (2.28)-(2.31). In order to understand the physical situation of the problem and hence the manifestations of the effects of the material parameters entering into the solution of problem, To study the effects of these different parameters appearing in the governing flow problem, we have carried out computational and numerical calculations for the velocity field, skin-friction, temperature field and temperature in terms of its amplitude and the phase. The computational results are presented in Figures (2-19) for the velocity profiles (fixing $\alpha = \pi/3$), Figures (20-22) for temperature profiles and Figures (24-26) for amplitude and phase angle of rate of heat transfer with respect to different governing parameters and also tables (1-2) for shear stresses and rate of heat transfer at $z = -1/2$. We noticed that, from Figures (2 & 3) shows the variation of velocity profiles under the influence of the Reynolds parameter Re . The magnitude of the velocity u increases and w decreases with increase in Reynolds number R . It is evident from that increasing value of Re leads to the increase of resultant velocity. It is interesting to note that from figures (4 & 5) both the magnitude of velocity components u and w decreases with the increase of intensity of the magnetic field (Hartmann number M). This is because of the reason that effect of a inclined magnetic field on an electrically conducting fluid gives rise to a resistive type force (called Lorentz force) similar to drag force and upon increasing the values of M increases the drag force which has tendency to slow down the motion of the fluid. The resultant velocity also reduces with increase in the intensity of the magnetic field. The magnitudes of the velocity components u and w increase with the increase in permeability of the porous medium (D) is observed from Figures (6 & 7). Lower the permeability of the porous medium lesser the fluid speed is in the entire fluid region. It is expected physically also because the resistance posed by the porous medium to the decelerated flow due to inclined magnetic field reduces with decreasing permeability D which leads to decrease in the velocity. The resultant velocity also increases with increase

in D . The variation of the velocity profiles with hall parameter m is shown in Figures (8 & 9). The magnitudes of the velocity components u , w and the resultant velocity increases with the increase of hall parameter m throughout the channel and there is no significant effect of hall parameter m on both the velocity components with the effect of inclined magnetic field. The variations of the velocity profiles with the Grashof number Gr are shown in Figures (10 & 11). The magnitude of the velocity components u enhances & w decreases with the increasing Grashof number Gr . The maximum of the velocity profiles shifts towards right half of the channel due to the greater buoyancy force in this part of the channel due to the presence of hotter plate. In the right half there lies hot plate at $z = 1/2$ and heat is transferred from the hot plate to the fluid and consequently buoyancy force enhances the flow velocity further. In the left half of the channel, the transfer of heat takes place from the fluid to the cooler plate at $z = -1/2$. Thus, the effect of Grashof number on the resultant velocity is reversed i.e. velocity decreases with increasing Gr . The velocity profiles with the Peclet number Pe are shown in Figures (12 & 13). The magnitude of the velocity components u enhances & w decreases with the increasing Peclet number Pe . We noticed that with increasing Peclet number Pe the resultant velocity decreases. The variation of velocity profile with radiation parameter R is shown in Figures (14 & 15). The magnitude of velocity components u and w decrease with increasing in Radiation parameter R . In the left half of the channel, the effect of R on velocity is insignificant while in the right half of the channel velocity decreases with increase of R . It is evident from the Figures (16 & 17) that, the velocity components u enhances w reduces with increase in pressure gradient P . The increasing pressure gradient P leads to the increase of resultant velocity. The velocity profiles with the frequency of oscillation ω are shown in Figures (18 & 19). The magnitude of the velocity component u enhances firstly and gradually decreases then experienced enhancement as observed with increase in the frequency of oscillation ω . Likewise the behaviour of the velocity component w experiences enhancement and then gradually decreases throughout the fluid region with increase in the frequency of oscillation ω . The resultant velocity decreases with increasing the frequency of oscillations ω .

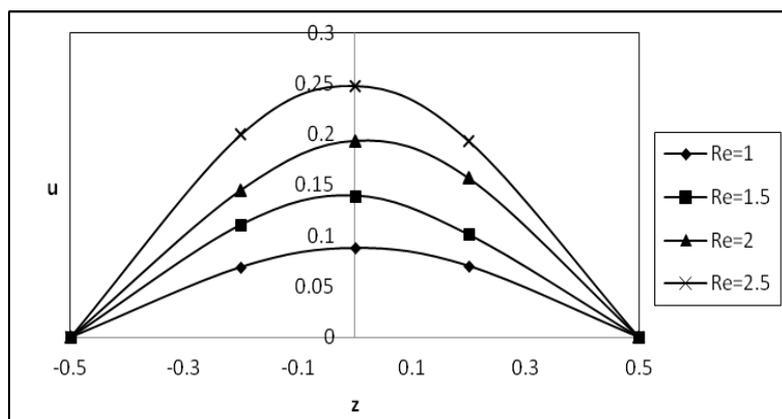


Fig. 2: The velocity Profile for u against Re with $P = 5, Pe = 0.7, Gr = 1, D = 1, R = 1, M = 5, m = 1, \omega = 5, t = 1$

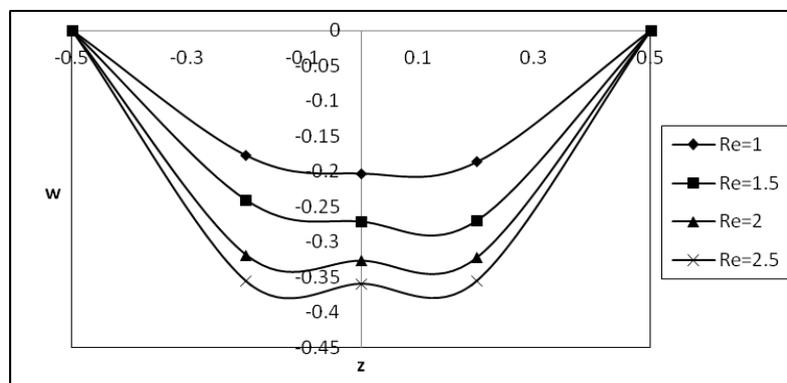


Fig. 3: The velocity Profile for w against Re with $P = 5, Pe = 0.7, Gr = 1, D = 1, R = 1, M = 5, m = 1, \omega = 5, t = 1$

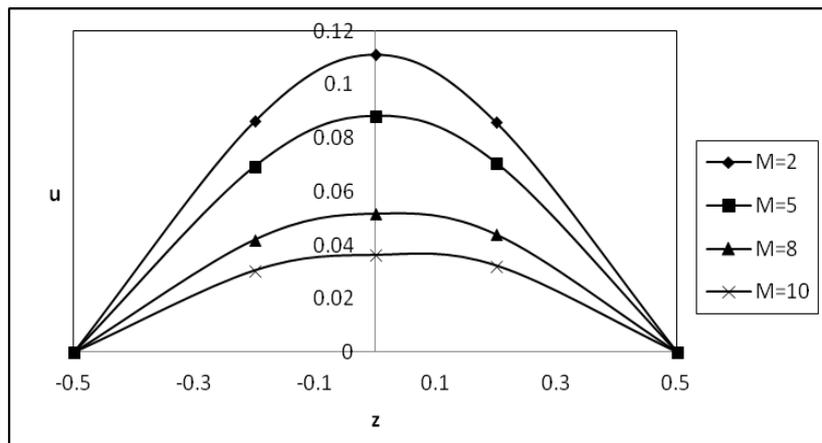


Fig. 4: The velocity Profile for u against M with $P = 5, Pe = 0.7, Re = 1, Gr = 1, D = 1, R = 1, m = 1, \omega = 5, t = 1$

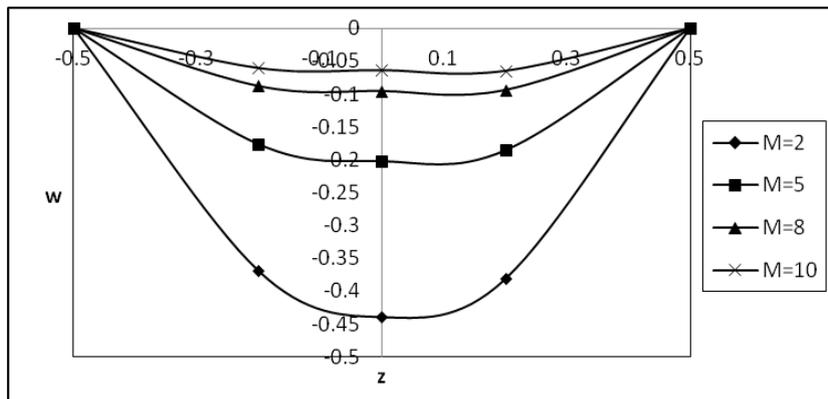


Fig. 5: The velocity Profile for w against M with $P = 5, Pe = 0.7, Re = 1, Gr = 1, D = 1, R = 1, m = 1, \omega = 5, t = 1$

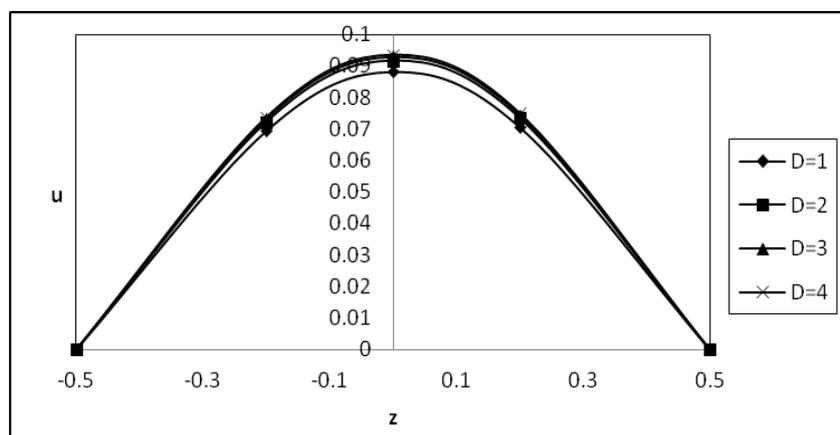


Fig. 6: The velocity Profile for u against D with $P = 5, Pe = 0.7, Re = 1, Gr = 1, R = 1, M = 5, m = 1, \omega = 5, t = 1$

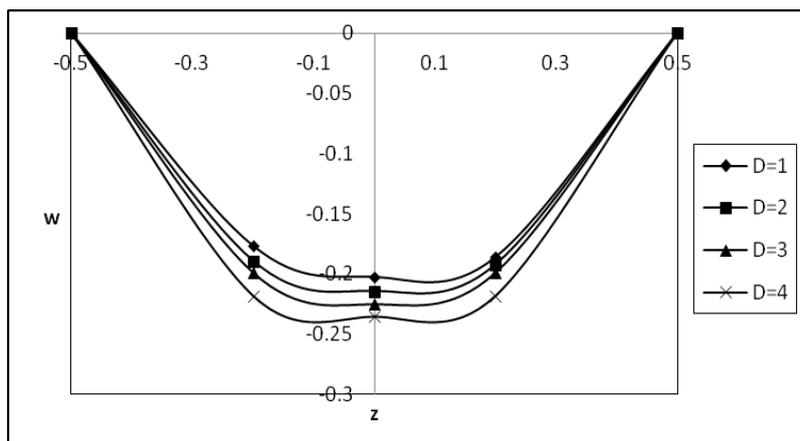


Fig. 7: The velocity Profile for w against D with $P = 5, Pe = 0.7, Re = 1, Gr = 1, R = 1, M = 5, m = 1, \omega = 5, t = 1$

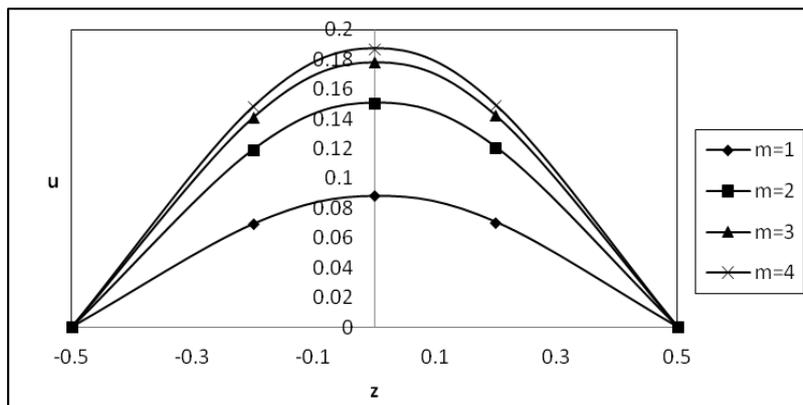


Fig. 8: The velocity Profile for u against m with $P = 5, Pe = 0.7, Re = 1, Gr = 1, D = 1, R = 1, M = 5, \omega = 5, t = 1$

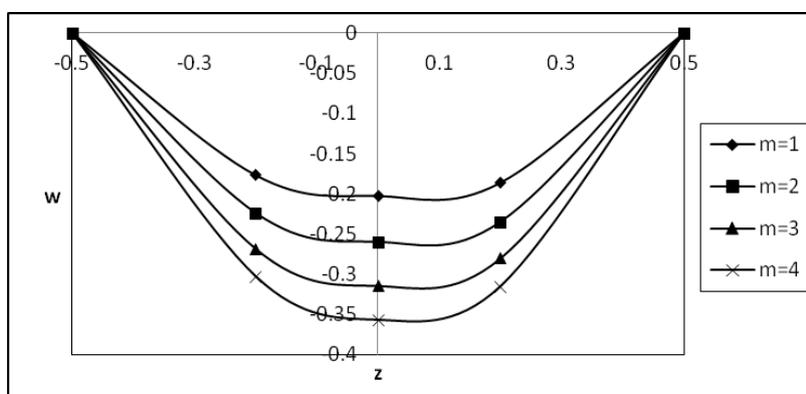


Fig. 9: The velocity Profile for w against m with $P = 5, Pe = 0.7, Re = 1, Gr = 1, D = 1, R = 1, M = 5, \omega = 5, t = 1$

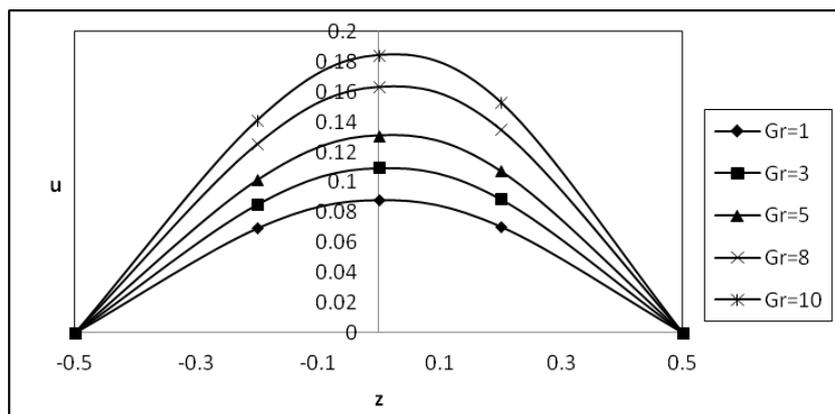


Fig. 10: The velocity Profile for u against Gr with $P = 5, Pe = 0.7, Re = 1, D = 1, R = 1, M = 5, m = 1, \omega = 5, t = 1$

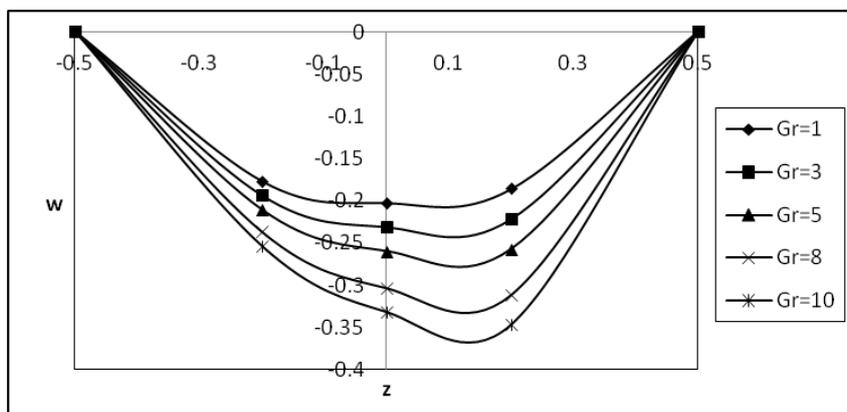


Fig. 11: The velocity Profile for w against Gr with $P = 5, Pe = 0.7, Re = 1, D = 1, R = 1, M = 5, m = 1, \omega = 5, t = 1$

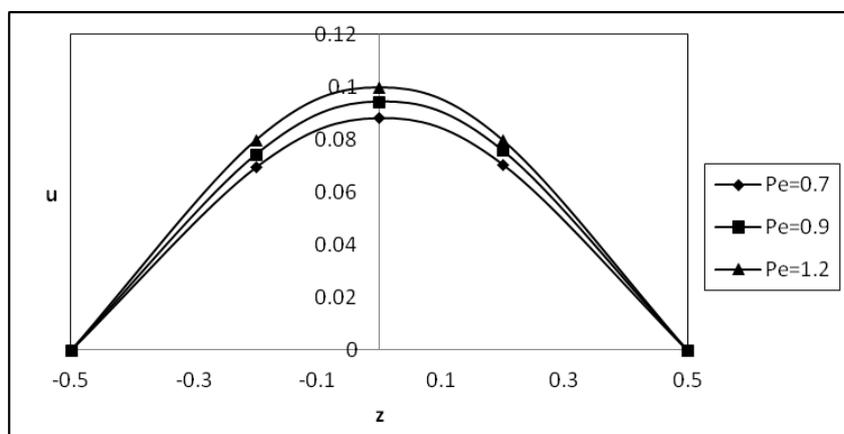


Fig. 12: The velocity Profile for u against Pe with $P = 5, Re = 1, Gr = 1, D = 1, R = 1, M = 5, m = 1, \omega = 5, t = 1$

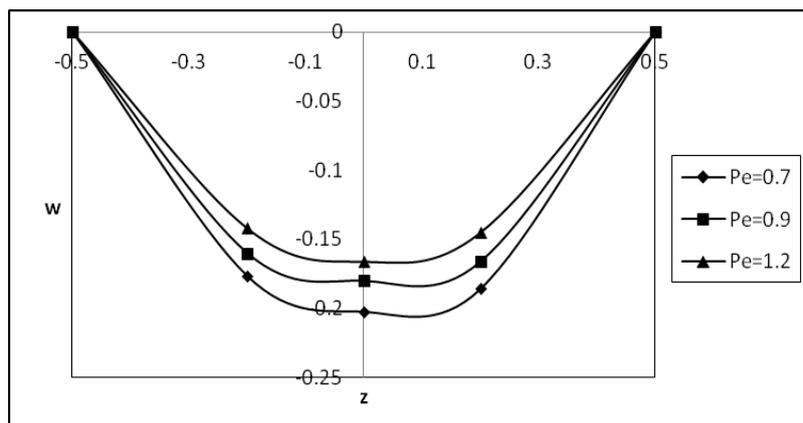


Fig. 13: The velocity Profile for w against Pe with $P = 5, Re = 1, Gr = 1, D = 1, R = 1, M = 5, m = 1, \omega = 5, t = 1$

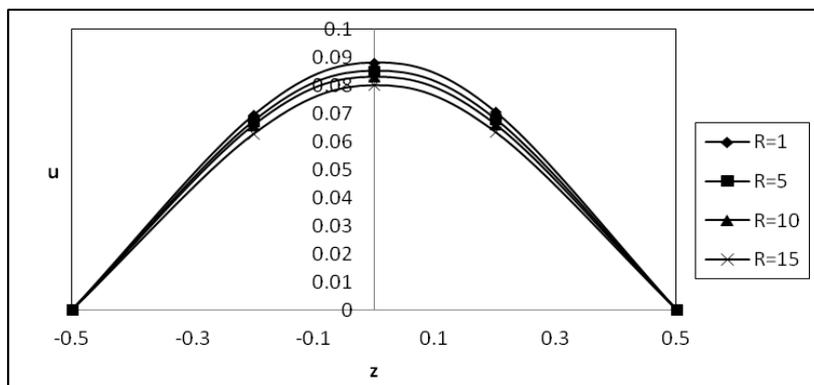


Fig. 14: The velocity Profile for u against R with $P = 5, Pe = 0.7, Re = 1, Gr = 1, D = 1, M = 5, m = 1, \omega = 5, t = 1$

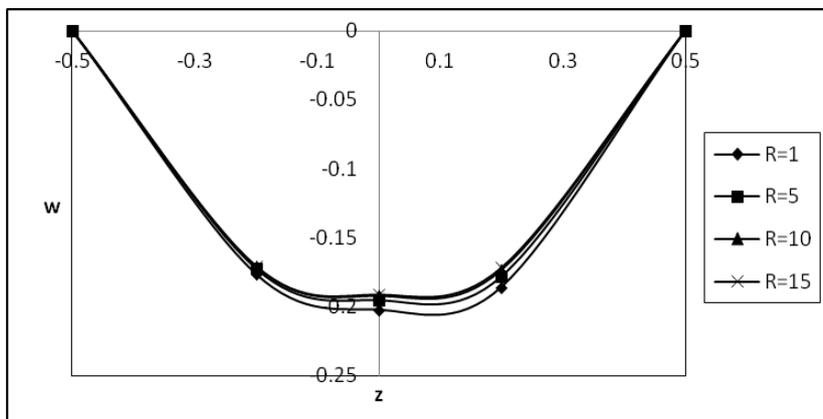


Fig. 15: The velocity Profile for w against R with $P = 5, Pe = 0.7, Re = 1, Gr = 1, D = 1, M = 5, m = 1, \omega = 5, t = 1$

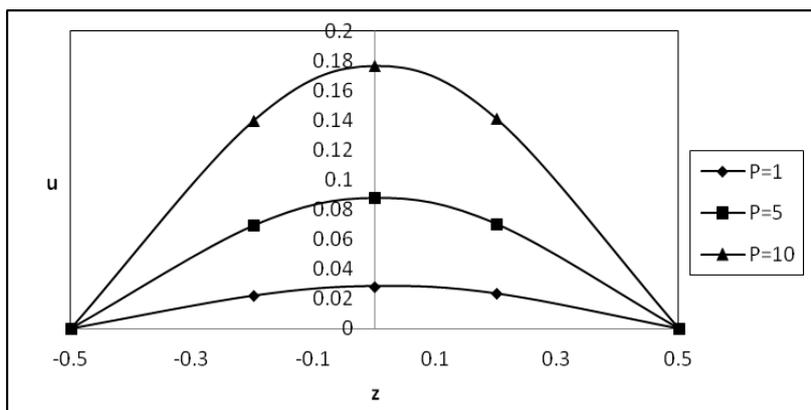


Fig. 16: The velocity Profile for u against P with $Pe = 0.7, Re = 1, Gr = 1, D = 1, R = 1, M = 5, m = 1, \omega = 5, t = 1$

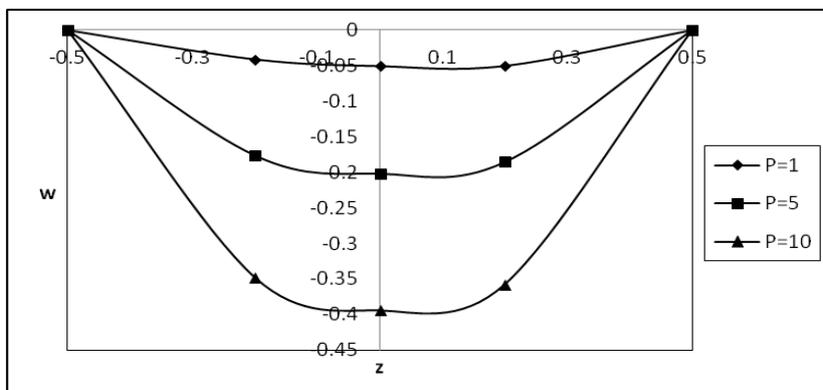


Fig. 17: The velocity Profile for w against P with $Pe = 0.7, Re = 1, Gr = 1, D = 1, R = 1, M = 5, m = 1, \omega = 5, t = 1$

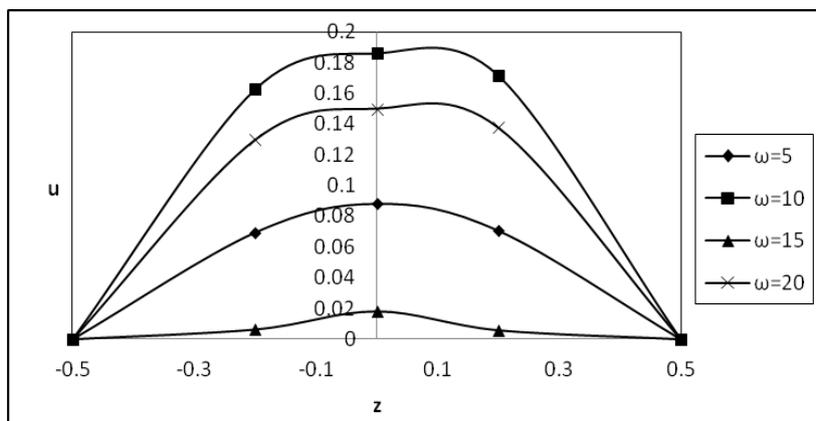


Fig. 18: The velocity Profile for u against ω with $Pe = 0.7, Re = 1, Gr = 1, D = 1, R = 1, M = 5, m = 1, P = 5, t = 1$

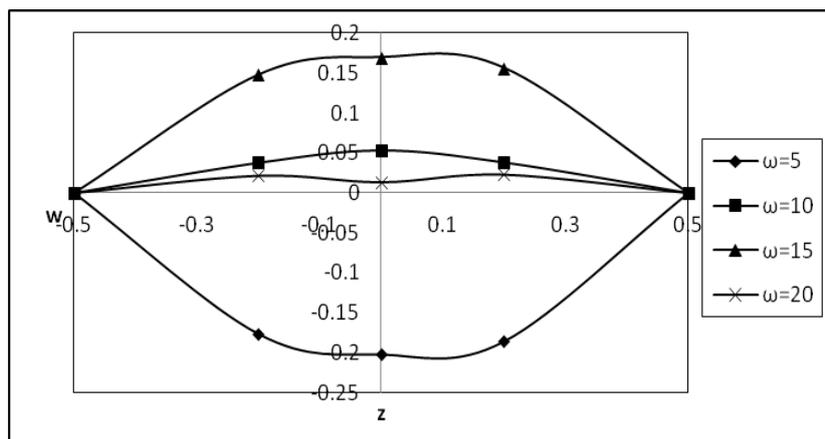


Fig. 19: The velocity Profile for w against ω with $Pe = 0.7, Re = 1, Gr = 1, D = 1, R = 1, M = 5, m = 1, P = 5, t = 1$

The temperature profiles are shown in Figure (20-22). The temperature decreases with the increase of radiation parameter R , the Peclet number Pe (Figures 20-21). The temperature enhances initially and then gradually decreases with increase in the frequency of oscillations ω (Figure 22). We notice that the flow of heat transfer is reversed with the increase in Peclet number Pe . The amplitude of the rate of heat transfer is shown in (Fig. 23 & 24) which shows that the value of amplitude of the rate of heat transfer decreases with the increase of R and Pe . The phase angle ψ of the rate of heat transfer is shown in (Fig. 25 & 26). It is noticed that phase angle ψ decreases with the increase in Pe and increases with increase in R . there is only phase log for the values of the frequency of oscillations ω .

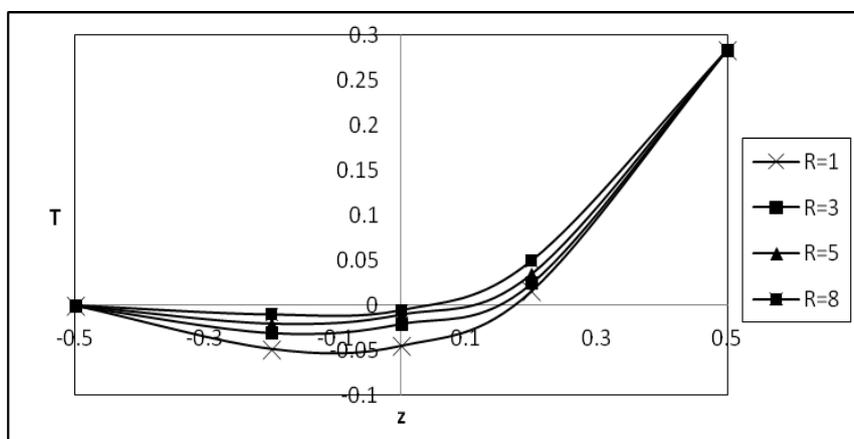


Fig. 20: The Temperature Profile for T against R with $Pe = 0.7, \omega = 5, t = 1$

The skin-friction at the plate $z = -1/2$ is obtained in terms of its amplitude $|q|$ and the phase angle ϕ . The amplitude $|q|$ is presented in Table 1. The amplitude $|q|$ increases with increase in Reynolds number Re , pressure gradient P , Grashof number Gr and the Hall parameter m . The amplitude $|q|$ increases with increase of permeability of the porous medium D for small values of ω ($\omega \leq 5$) but decreases for large values of ω ($\omega > 5$). However, the effect of D is insignificant for large values of frequency of oscillations ω . The amplitude $|q|$ decreases with increase in the intensity of the magnetic field (Hartmann number M). The amplitude $|q|$ increases with increase in Peclet number Pe or Radiation parameter R for the values of $\omega \leq 15$ but decreases for large values of $\omega > 15$. A decrease in $|q|$ is noticed with increasing frequency of oscillations ω .

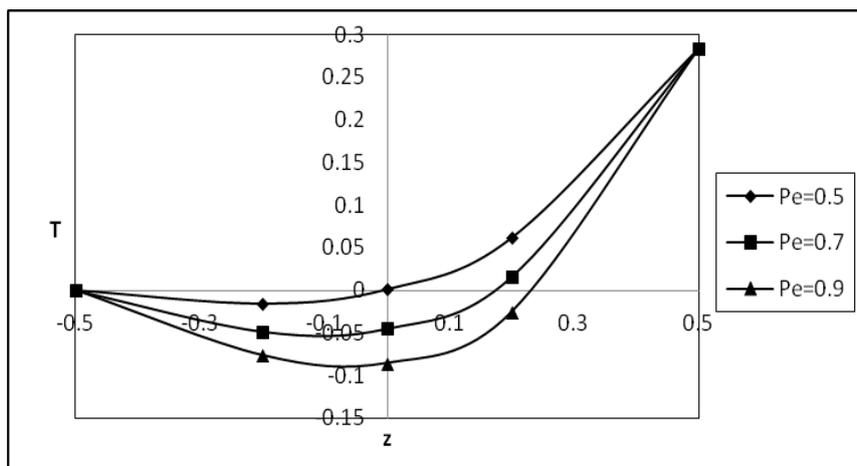


Fig. 21: The Temperature Profile for T against Pe with $R = 1, \omega = 5, t = 1$

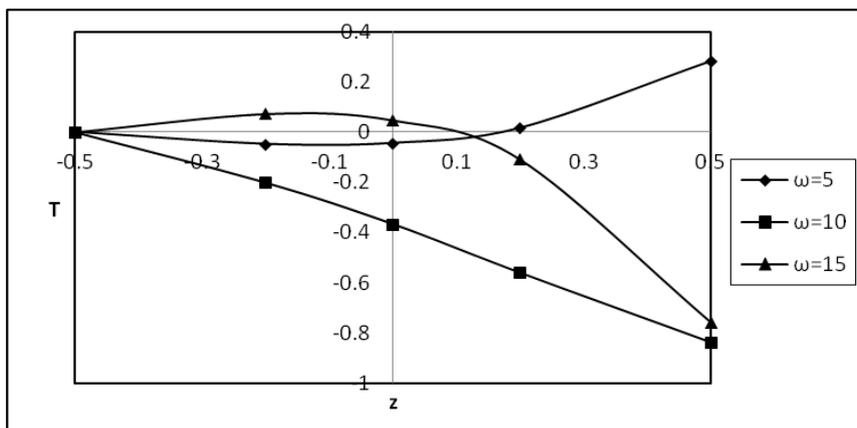


Fig.22: The Temperature Profile for T against ω with $Pe = 0.7, R = 1, t = 1$

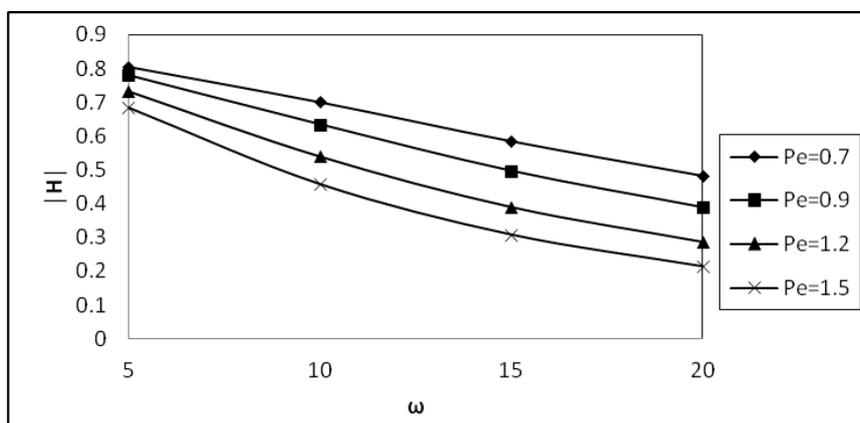


Fig. 23: Amplitude $(|H|)$ of Rate of Heat transfer profile against Pe with $R=1$

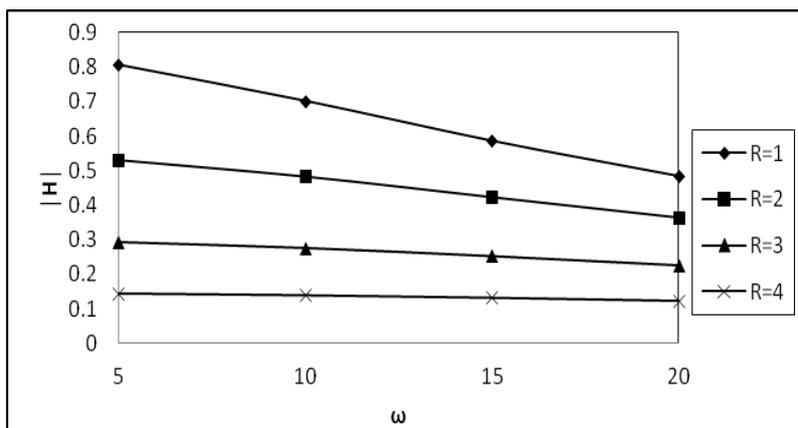


Fig. 24: Amplitude ($|H|$) of Rate of Heat transfer profile against R with $Pe=0.7$

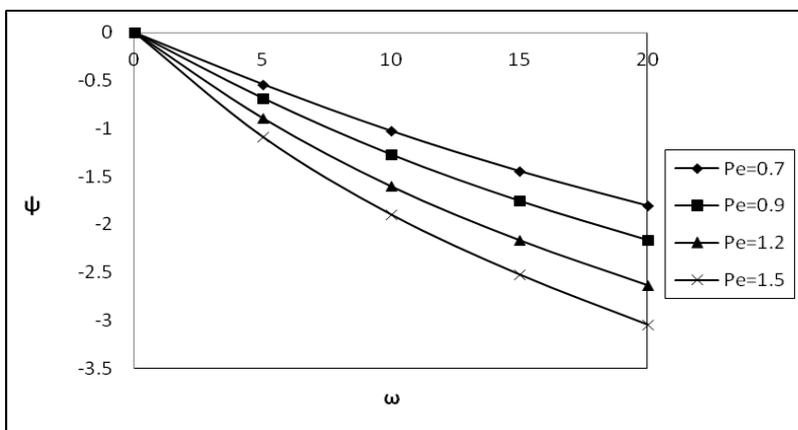


Fig. 25: Phase angle (ψ) of Rate of Heat transfer profile against Pe with $R=1$

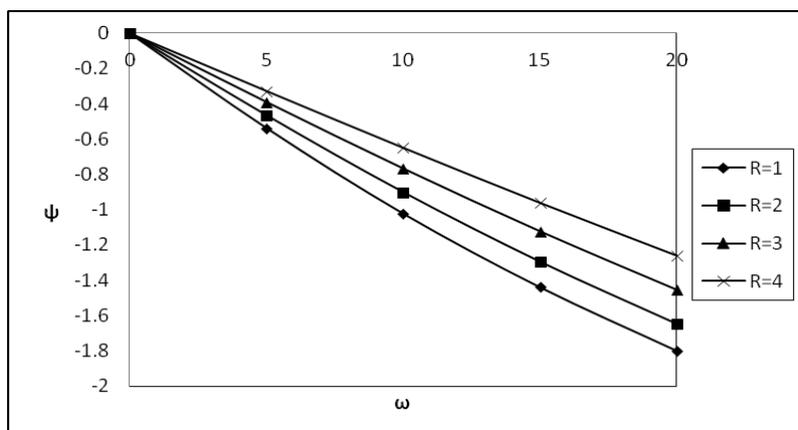


Fig. 26: Phase angle (ψ) of Rate of Heat transfer profile against R with $Pe=0.7$

Table 1: Amplitude ($|q|$) of Skin friction (τ_L) at lower plate

| Re | M | m | D | Gr | Pe | R | P | $\omega=5$ | $\omega=10$ | $\omega=15$ | $\omega=20$ |
|------------|----------|----------|----------|----------|------------|-----------|-----------|------------|-------------|-------------|-------------|
| 1 | 2 | 1 | 1 | 1 | 0.7 | 1 | 10 | 0.74175 | 0.799029 | 0.622992 | 0.513032 |
| 1.5 | 2 | 1 | 1 | 1 | 0.7 | 1 | 10 | 1.59843 | 0.879376 | 0.643293 | 0.502725 |
| 2 | 2 | 1 | 1 | 1 | 0.7 | 1 | 10 | 1.75104 | 0.90803 | 0.645668 | 0.494522 |
| 1 | 5 | 1 | 1 | 1 | 0.7 | 1 | 10 | 0.45904 | 0.356544 | 0.339405 | 0.306986 |
| 1 | 8 | 1 | 1 | 1 | 0.7 | 1 | 10 | 0.19977 | 0.173817 | 0.179968 | 0.171313 |
| 1 | 2 | 2 | 1 | 1 | 0.7 | 1 | 10 | 1.50641 | 0.859331 | 0.651048 | 0.530602 |
| 1 | 2 | 3 | 1 | 1 | 0.7 | 1 | 10 | 1.61843 | 0.900872 | 0.671557 | 0.543758 |
| 1 | 2 | 1 | 2 | 1 | 0.7 | 1 | 10 | 1.23184 | 0.770207 | 0.611035 | 0.506134 |
| 1 | 2 | 1 | 3 | 1 | 0.7 | 1 | 10 | 1.13851 | 0.739593 | 0.597566 | 0.498285 |
| 1 | 2 | 1 | 1 | 3 | 0.7 | 1 | 10 | 1.40054 | 0.863886 | 0.633323 | 0.58511 |
| 1 | 2 | 1 | 1 | 5 | 0.7 | 1 | 10 | 1.49245 | 0.930019 | 0.65862 | 0.65719 |
| 1 | 2 | 1 | 1 | 1 | 0.9 | 1 | 10 | 1.31844 | 0.803141 | 0.631482 | 0.507125 |
| 1 | 2 | 1 | 1 | 1 | 1.2 | 1 | 10 | 1.33554 | 0.812094 | 0.638265 | 0.498909 |
| 1 | 2 | 1 | 1 | 1 | 0.7 | 5 | 10 | 1.30839 | 0.850364 | 0.638895 | 0.487542 |
| 1 | 2 | 1 | 1 | 1 | 0.7 | 10 | 10 | 1.35168 | 0.862647 | 0.642625 | 0.479671 |
| 1 | 2 | 1 | 1 | 1 | 0.7 | 1 | 1 | 0.17854 | 0.021697 | 0.077568 | 0.083739 |
| 1 | 2 | 1 | 1 | 1 | 0.7 | 1 | 5 | 0.68079 | 0.365671 | 0.313108 | 0.274536 |

Table 2: Phase angle (ϕ) of Skin friction (τ_L) at lower plate

| Re | M | m | D | Gr | Pe | R | P | $\omega=5$ | $\omega=10$ | $\omega=15$ | $\omega=20$ |
|------------|----------|----------|----------|----------|------------|-----------|-----------|------------|-------------|-------------|-------------|
| 1 | 2 | 1 | 1 | 1 | 0.7 | 1 | 10 | -0.968351 | -1.23578 | -1.278810 | -1.43403 |
| 1.5 | 2 | 1 | 1 | 1 | 0.7 | 1 | 10 | -1.165925 | -1.36156 | -1.398440 | -1.50892 |
| 2 | 2 | 1 | 1 | 1 | 0.7 | 1 | 10 | -1.264428 | -1.42965 | -1.453360 | -1.53187 |
| 1 | 5 | 1 | 1 | 1 | 0.7 | 1 | 10 | -0.955126 | -1.00185 | -1.038550 | -1.19757 |
| 1 | 8 | 1 | 1 | 1 | 0.7 | 1 | 10 | -0.928135 | -0.87620 | -0.898628 | -1.03272 |
| 1 | 2 | 2 | 1 | 1 | 0.7 | 1 | 10 | -1.119362 | -1.31771 | -1.342920 | -1.48771 |
| 1 | 2 | 3 | 1 | 1 | 0.7 | 1 | 10 | -1.132782 | -1.33859 | -1.361470 | -1.50357 |
| 1 | 2 | 1 | 2 | 1 | 0.7 | 1 | 10 | -0.949917 | -1.16214 | -1.222660 | -1.38714 |
| 1 | 2 | 1 | 3 | 1 | 0.7 | 1 | 10 | -0.873865 | -1.09131 | -1.167210 | -1.34025 |
| 1 | 2 | 1 | 1 | 3 | 0.7 | 1 | 10 | -1.157523 | -1.27146 | -1.321500 | -1.45603 |
| 1 | 2 | 1 | 1 | 5 | 0.7 | 1 | 10 | -1.267641 | -1.32527 | -1.472756 | -1.49762 |
| 1 | 2 | 1 | 1 | 1 | 0.9 | 1 | 10 | -1.034021 | -1.22182 | -1.289411 | -1.44721 |
| 1 | 2 | 1 | 1 | 1 | 1.2 | 1 | 10 | -1.232442 | -1.32076 | -1.305121 | -1.45667 |
| 1 | 2 | 1 | 1 | 1 | 0.7 | 5 | 10 | -0.982762 | -1.25016 | -1.337091 | -1.45547 |
| 1 | 2 | 1 | 1 | 1 | 0.7 | 10 | 10 | -0.991132 | -1.27391 | -1.353022 | -1.47119 |
| 1 | 2 | 1 | 1 | 1 | 0.7 | 1 | 1 | -1.484871 | -1.74928 | -1.865452 | -1.69033 |
| 1 | 2 | 1 | 1 | 1 | 0.7 | 1 | 5 | -1.097251 | -1.25197 | -1.359513 | -1.53511 |

The phase angle ϕ of the skin-friction is presented in Table 2. Since all the values presented in Table 2 are negative, therefore, there is always a phase lag. The phase angle ϕ increases with increasing Reynolds number Re , hall parameter m , Peclet number Pe , radiation parameter R and Grashof number Gr , while it decreases with increase in permeability of the porous medium D , Hartmann number M and pressure gradient P .

IV. CONCLUSIONS

The effects of radiation and hall current on MHD free convection three dimensional flow in a vertical channel filled with porous medium has been studied. The conclusions are made as the following.

1. The velocity component for primary flow enhances with increasing in Re , D , m , Gr , Pe and P ; and reduces with increasing in the intensity of the magnetic field M (Hartmann number) and Radiation parameter.
2. The velocity component for secondary flow enhances with increasing in D and m ; and reduces with increasing in Re , M , Gr , Pe , P and Radiation parameter R .
3. The resultant velocity enhances with increasing in Re , D , m and P ; and reduces with increasing in M , Gr , Pe , R and the frequency of oscillation ω .
4. Temperature reduces with increase in R or Pe while it enhances initially and then gradually reduces with increase in frequency of oscillation ω .
5. The amplitude of rate of heat transfer decreases with the increase of R and Pe .
6. Phase angle ψ decreases with the increase of Pe and increases with increase in R . there is only phase lag for the values of the frequency of oscillations ω .
7. The amplitude $|q|$ increases with increase in Reynolds number Re , pressure gradient P , Grashof number Gr and the Hall parameter m .
8. The amplitude $|q|$ increases with increase of permeability of the porous medium D for small values of ω but decreases for large values of ω . However, the effect of D is insignificant for large values of frequency of oscillations ω .
9. The amplitude $|q|$ decreases with increase in M . The amplitude $|q|$ increases with increase in Peclet number Pe or Radiation parameter R for the values of ω but decreases for large values of ω .
10. The phase angle ϕ increases with increase in Re , m , Pe , R and Gr , while it decreases with increase in D , M and P .

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