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Deformation of a Monoclinic Half-Space with Free Boundary Caused by a Long Inclined Strike-Slip Fault of Finite Width

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Abstract: *Closed-form analytic expression for displacement and stresses at any point in a homogenous monoclinic half-space are developed. The fault is of infinite length in the strike direction and is of finite width located at an arbitrary point of a homogenous, monoclinic, perfectly elastic half-space. Deformation of monoclinic half-space with free boundary caused by a long inclined strike-slip fault is studied analytically and numerically.*

Keywords: *Inclined Strike-slip Fault, Monoclinic, Free Boundary, Finite Width*

I. Introduction

The mathematical analysis of static deformation of the earth's crust being caused by an earthquake was presented by Steketee(1958) and Maruyama(1964). The method taken by both these authors is based on the representation of a fault as a displacement dislocation in a three dimensional elastic half-space. On the other hand, in connection with the mathematical description of the earth's crust static deformations, a method of two dimensional models also has been developed. Maruyama (1966) derived the formulas for the displacement and stress field of a two dimensional dislocation in a semi-infinite isotropic medium. The extension of these results for a very long strike-slip fault in a isotropic half-space with a vertical and horizontal discontinuity was developed by Rybicki(1971). Ting (1995) derived the green's function for the antiplane deformation of a monoclinic elastic medium consisting of a single half-space or two half-spaces in welded contact. Kumar et al (2003) studied static deformation of monoclinic half-spaces due to a long inclined strike-slip fault. Malik and Singh (2013) obtained the displacement field for a long inclined strike-slip fault of finite width located at an arbitrary point of a homogenous, isotropic, perfectly elastic half-space of rigid boundary by integration over the width of the fault. Sahrawat, Godara, Singh(2014) have obtained the displacement and stresses for a uniform half-space with rigid boundary caused by a long strike-slip fault of finite width.

In the present paper, we have obtained closed-form analytical expression for the displacement and stresses in a monoclinic half-space caused by a long inclined strike-slip fault of finite width.

II. Theory

Let the Cartesian Co-ordinates be with x_2 vertically upward. Consider a homogenous, perfectly elastic monoclinic half-space with free boundary. The half-space is satisfy the stress-strain relation

$$\tau_{ij} = c_{ijks} e_{ks} \quad (i, j = 1, 2, 3) \qquad (1)$$

where c_{ijks} are the elastic stiffness coefficients satisfying the symmetry relations

$$c_{ijks} = c_{jiks} = c_{ksij} \qquad (2)$$

For monoclinic `half-spaces, these coefficients are 13 in number and e_{ks} is a strain tensor. If (u_1, u_2, u_3) denote the components of the displacement vector, the strain-displacement relation are

$$e_{ks} = \frac{1}{2}(u_{k,s} + u_{s,k}) \qquad (3)$$

From (1), (2) and (3), we obtain the stress-displacement relation

$$\tau_{ij} = c_{ijks} u_{k,s} \qquad (4)$$

Consider an antiplane strain problem for which displacement components are in the form

$$u_1 = 0 = u_2, \quad u_3 = u_3(x_1, x_2) \qquad (5)$$

In equation (4), we have used the contracted Voigt notation for the stiffnesses c_{ijks} according to the scheme

$11 \to 1, 22 \to 2, 33 \to 3, 23 \to 4, 13 \to 5, 12 \to 6$

For monoclinic material $x_3 = 0$,

$$c_{14} = c_{15} = c_{24} = c_{25} = c_{46} = c_{56} = c_{34} = c_{35} = 0 \qquad (6)$$

For zero body force, the equilibrium relation reduces to

$$\tau_{31,1} + \tau_{32,2} = 0 \qquad (7)$$

Using equation (6), equation (7) become

$$c_{55}u_{3,11} + 2c_{45}u_{3,12} + c_{44}u_{3,22} = 0 \qquad (8)$$

From equation (4), the non-zero stress components are

$$\tau_{31} = c_{55}u_{3,1} + c_{45}u_{3,2}$$

$$\tau_{32} = c_{45}u_{3,1} + c_{44}u_{3,2} \qquad (9)$$

$$\tau_{33} = c_{33}u_{3,1} + c_{34}u_{3,2}$$

The displacement field due to a long inclined strike-slip line dislocation can be expressed in terms of a displacement due to a long inclined strike-slip dislocation given by Kumar, Singh and Singh (2003) in the form

$$u_3(x_1, x_2) = \frac{1}{2\Pi m} \int_L b \left[\begin{array}{l} (n_1 c_{55} + n_2 c_{45}) \{ (x_1 - \xi_1) + p_r (x_2 - \xi_2) \} \left(\frac{1}{R^2} + \frac{1}{S^2} \right) + \\ (n_1 c_{45} + n_2 c_{44}) \left\{ p_r \{ (x_1 - \xi_1) + (x_2 - \xi_2) \} \left(\frac{1}{R^2} + \frac{1}{S^2} \right) + p_i^2 \left(\frac{x_2 - \xi_2}{R^2} + \frac{x_2 + \xi_2}{S^2} \right) \right\} \end{array} \right]$$

(10)

where

b = displacement discontinuity,

ds = width of the source,

δ = dip angle,

(x_1, x_2) = receiver location,

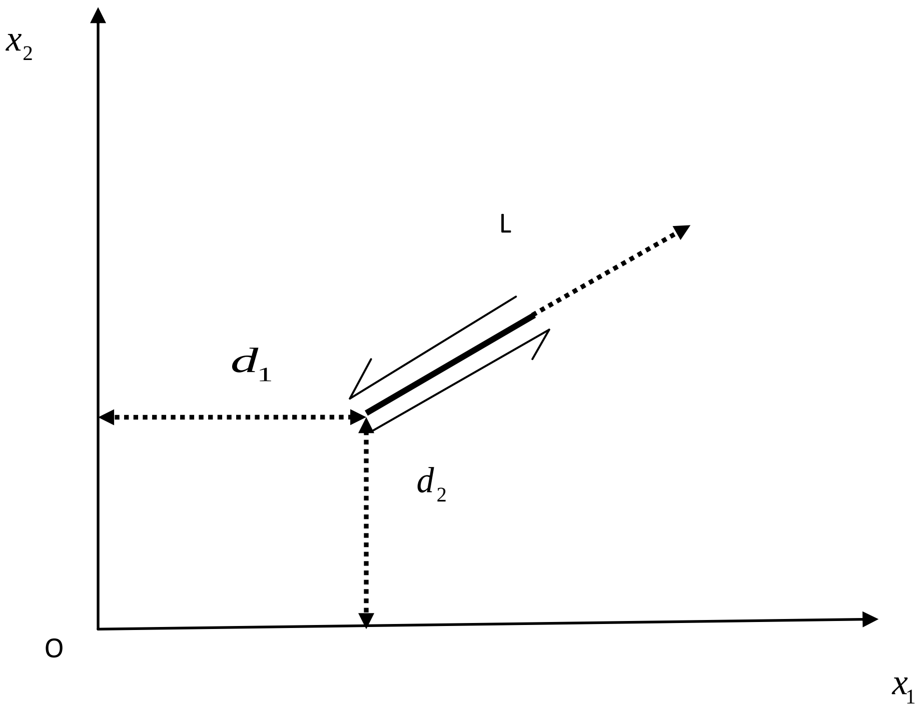
$$m = \sqrt{c_{44}c_{55} - c_{45}{}^2}, \; p_r = \frac{-c_{45}}{c_{44}}, \quad p_i = \sqrt{\frac{c_{55}}{c_{44}} - \left(\frac{c_{45}}{c_{44}} \right)^2},$$

$$R^2 = \left[(x_1 - \xi_1) + p_r (x_2 - \xi_2) \right]^2 + p_i{}^2 (x_2 - \xi_2)^2,$$

$$S^2 = \left[(x_1 - \xi_1) + p_r (x_2 - \xi_2) \right]^2 + p_i{}^2 (x_2 + \xi_2)^2 \qquad (11)$$

`

Figure 1



Geometry of Strike-slip fault of width L at a distance d_1 from x_2 -axis and d_2 from x_1-axis for the uniform monoclinic half-space

Using the polar co-ordinates (s, δ) at any point (ξ_1, ξ_2) on the fault, we obtain

$$\xi_1 = d_1 + s\cos\delta, \qquad \xi_2 = d_2 + s\sin\delta$$
$$n_1 = -\sin\delta, \qquad n_2 = \cos\delta \qquad (12)$$

Using these values in (10), we get

$$u_3\,(x_1,x_2)=$$

$$\frac{1}{2\Pi\left(c_{44}c_{55}+c_{45}{}^2\right)^{\frac{1}{2}}}\int_L b\left[\begin{array}{l}\left(-c_{55}\sin\delta+c_{45}\cos\delta\right)\left\{\begin{array}{l}\left(x_1-d_1-s\cos\delta\right)\\-\frac{c_{45}}{c_{44}}\left(x_2-d_2-s\sin\delta\right)\end{array}\right\}\left(\frac{1}{R^2}+\frac{1}{S^2}\right)+\\ \left(-c_{45}\sin\delta+c_{44}\cos\delta\right)\left\{\begin{array}{l}-\frac{c_{45}}{c_{44}}\left\{\begin{array}{l}\left(x_1-d_1-s\cos\delta\right)+\\\left(x_2-d_2-s\sin\delta\right)\end{array}\right\}\left(\frac{1}{R^2}+\frac{1}{S^2}\right)+\\ \left(\frac{c_{55}}{c_{44}}-\frac{c_{45}{}^2}{c_{44}{}^2}\right)\left(\begin{array}{l}\frac{x_2-d_2-s\sin\delta}{R^2}+\\\frac{x_2+d_2+s\sin\delta}{S^2}\end{array}\right)\end{array}\right\}\end{array}\right]ds$$

$$u_3\,(x_1,x_2)=\frac{1}{2\Pi m}\int_L b\left[\begin{array}{l}\left(-c_{55}\sin\delta+c_{45}\cos\delta\right)\left\{\left(x_1-d_1-s\cos\delta\right)+p_r\left(x_2-d_2-s\sin\delta\right)\right\}\left(\frac{1}{R^2}+\frac{1}{S^2}\right)+\\ \left(-c_{45}\sin\delta+c_{44}\cos\delta\right)\left\{\begin{array}{l}p_r\left\{\left(x_1-d_1-s\cos\delta\right)+\left(x_2-d_2-s\sin\delta\right)\right\}\left(\frac{1}{R^2}+\frac{1}{S^2}\right)+\\ p_i{}^2\left(\frac{x_2-d_2-s\sin\delta}{R^2}+\frac{x_2+d_2+s\sin\delta}{S^2}\right)\end{array}\right\}\end{array}\right]ds$$

(13)

Equation (11) and (13) yield

$$u_3\,(x_1,x_2)=\frac{\alpha}{2\Pi}\int_0^L b\left(\frac{T_1}{R^2}-\frac{T_2}{S^2}\right)ds \qquad (14)$$

where

$$R^2=\left(A+\varepsilon\sin 2\delta\right)s^2-2\left[C\left(x_1-d_1\right)+B\left(x_2-d_2\right)\right]s+\left(x_1-d_1\right)^2+\gamma\left(x_2-d_2\right)^2+2\varepsilon\left(x_1-d_1\right)\left(x_2-d_2\right)$$

$$=\frac{1}{\left(A+\varepsilon\sin 2\delta\right)}\left[\left\{\left(A+\varepsilon\sin 2\delta\right)s-C\left(x_1-d_1\right)+B\left(x_2-d_2\right)\right\}^2+\alpha^2T_1^2\right]$$

$$S^2=\left(A+\varepsilon\sin 2\delta\right)s^2-2\left[C\{\left(x_1-d_1\right)+2\varepsilon x_2\}-B\left(x_2+d_2\right)\right]s+\left(x_1-d_1\right)^2+\gamma\left(x_2+d_2\right)^2+$$
$$2\varepsilon\left(x_1-d_1\right)\left(x_2-d_2\right)-4\varepsilon^2x_2d_2$$

$$=\frac{1}{\left(A+\varepsilon\sin 2\delta\right)}\left[\left\{\left(A+\varepsilon\sin 2\delta\right)s-C\{\left(x_1-d_1\right)+2\varepsilon x_2\}+B\left(x_2+d_2\right)\right\}^2+\alpha^2T_2{}^2\right]$$

where

$$T_1=-\left(x_1-d_1\right)\sin\delta+\left(x_2-d_2\right)\cos\delta,$$
$$T_2=\left\{\left(x_1-d_1\right)+2\in x_2\right\}\sin\delta+\left(x_2-d_2\right)\cos\delta,$$
$$\varepsilon=p_r=-\frac{c_{45}}{c_{44}},\ \gamma=\frac{c_{55}}{c_{44}},\ \alpha=p_i=\left(\gamma-\varepsilon^2\right)^{\frac{1}{2}},$$

$$A=\cos^2\delta+\gamma\sin^2\delta,$$
$$B=\varepsilon\cos\delta+\gamma\sin\delta, \qquad (15)$$
$$C=\cos\delta+\varepsilon\sin\delta$$

Assuming b to be constant over L and performing the integration in equation (14), we obtain

$$u_3 = \frac{b}{2\Pi}\left| \begin{array}{l} \tan^{-1}\dfrac{(A+\varepsilon\sin 2\delta)s - C(x_1-d_1) - B(x_2-d_2)}{\alpha\{-(x_1-d_1)\sin\delta + (x_2-d_2)\cos\delta\}} - \\ \tan^{-1}\dfrac{(A+\varepsilon\sin 2\delta)s - C\{(x_1-d_1)+2\varepsilon x_2\} + B(x_2+d_2)}{\alpha\{\{(x_1-d_1)+2\varepsilon x_2\}\sin\delta + (x_2+d_2)\cos\delta\}} \end{array} \right|_0^L \qquad (16)$$

$$u_3 = \frac{b}{2\Pi}\left[\begin{array}{l} \tan^{-1}\dfrac{(A+\varepsilon\sin 2\delta)L - C(x_1-d_1) - B(x_2-d_2)}{\alpha\{-(x_1-d_1)\sin\delta + (x_2-d_2)\cos\delta\}} + \tan^{-1}\dfrac{C(x_1-d_1)+B(x_2-d_2)}{\alpha\{-(x_1-d_1)\sin\delta + (x_2-d_2)\cos\delta\}} \\ -\tan^{-1}\dfrac{(A+\varepsilon\sin 2\delta)L - C\{(x_1-d_1)+2\varepsilon x_2\} + B(x_2+d_2)}{\alpha\{\{(x_1-d_1)+2\varepsilon x_2\}\sin\delta + (x_2+d_2)\cos\delta\}} \\ -\tan^{-1}\dfrac{C\{(x_1-d_1)+2\varepsilon x_2\} - B(x_2+d_2)}{\alpha\{\{(x_1-d_1)+2\varepsilon x_2\}\sin\delta + (x_2+d_2)\cos\delta\}} \end{array} \right] \qquad (17)$$

Corresponding to the displacement given by (13), the strains are

$$2e_{31} = \frac{\partial u_3}{\partial x_1} = \frac{\alpha b}{2\Pi}\left[\begin{array}{l} \left\{\dfrac{(A+\varepsilon\sin 2\delta)L\sin\delta - (x_2-d_2)(C\cos\delta + B\sin\delta)}{(A+\varepsilon\sin 2\delta)R_1^2}\right\} + \\ \dfrac{(x_2-d_2)(C\cos\delta + B\sin\delta)}{\{C(x_1-d_1)+B(x_2-d_2)\}^2 + \alpha^2 T_1^2} + \\ \dfrac{(A+\varepsilon\sin 2\delta)L\sin\delta + (x_2+d_2)(C\cos\delta + B\sin\delta)}{(A+\varepsilon\sin 2\delta)S_1^2} - \\ \dfrac{(x_2+d_2)(C\cos\delta + B\sin\delta)}{\{C\{(x_1-d_1)+2\varepsilon x_2\} - B(x_2+d_2)\}^2 + \alpha^2 T_2^2} \end{array} \right] \qquad (18)$$

$$2e_{32} = \frac{\partial u_3}{\partial x_2} = -\frac{\alpha b}{2\Pi}\left[\begin{array}{l} \left\{\dfrac{(A+\varepsilon\sin 2\delta)L\cos\delta - (x_1-d_1)(C\cos\delta + B\sin\delta)}{(A+\varepsilon\sin 2\delta)R_1^2}\right\} + \\ \dfrac{(x_1-d_1)(C\cos\delta + B\sin\delta)}{\{C(x_1-d_1)+B(x_2-d_2)\}^2 + \alpha^2 T_1^2} + \\ \dfrac{-(A+\varepsilon\sin 2\delta)L(2\varepsilon\sin\delta + \cos\delta) + \{(x_1-d_1) - 2\varepsilon d\}(C\cos\delta + B\sin\delta)}{(A+\varepsilon\sin 2\delta)S_1^2} - \\ \dfrac{\{-(x_1-d_1) + 2\varepsilon d\}(C\cos\delta + B\sin\delta)}{\{C\{(x_1-d_1)+2\varepsilon x_2\} - B(x_2+d_2)\}^2 + \alpha^2 T_2^2} \end{array} \right] \qquad (19)$$

The stresses are

$$\tau_{31} = \frac{bm}{2\Pi}\left[\gamma u'_{3,1} + \varepsilon u'_{3,2}\right] \qquad (20)$$

$$\tau_{32} = \frac{bm}{2\Pi}\left[-\varepsilon u'_{3,1} + u'_{3,2}\right] \qquad (21)$$

where

$$u'_{3,1} = \frac{2\Pi}{\alpha b} u_{3,1}, \qquad u'_{3,2} = -\frac{2\Pi}{\alpha b} u_{3,2} \qquad (22)$$

For an orthotropic medium, $\varepsilon = 0$, Equation (17) reduces to,

$$u_3 = \frac{b}{2\Pi}\left[\begin{array}{l} \tan^{-1}\dfrac{AL-\cos\delta(x_1-d_1)-\gamma\sin\delta(x_2-d_2)}{\gamma^{1/2}\left\{-(x_1-d_1)\sin\delta+(x_2-d_2)\cos\delta\right\}}+\tan^{-1}\dfrac{\cos\delta(x_1-d_1)+\gamma\sin\delta(x_2-d_2)}{\gamma^{1/2}\left\{-(x_1-d_1)\sin\delta+(x_2-d_2)\cos\delta\right\}} \\ -\tan^{-1}\dfrac{AL-\cos\delta(x_1-d_1)+\gamma\sin\delta(x_2+d_2)}{\gamma^{1/2}\left\{(x_1-d_1)\sin\delta+(x_2+d_2)\cos\delta\right\}} \\ -\tan^{-1}\dfrac{\cos\delta(x_1-d_1)-\gamma\sin\delta(x_2+d_2)}{\gamma^{1/2}\left\{(x_1-d_1)\sin\delta+(x_2+d_2)\cos\delta\right\}} \end{array}\right] \qquad (23)$$

For an isotropic medium, $\varepsilon = 0, \gamma = 1$, and therefore equation (17) reduces to

$$u_3 = \frac{b}{2\Pi}\left[\begin{array}{l} \tan^{-1}\dfrac{L-\cos\delta(x_1-d_1)-\sin\delta(x_2-d_2)}{\left\{-(x_1-d_1)\sin\delta+(x_2-d_2)\cos\delta\right\}}+\tan^{-1}\dfrac{\cos\delta(x_1-d_1)+\sin\delta(x_2-d_2)}{\left\{-(x_1-d_1)\sin\delta+(x_2-d_2)\cos\delta\right\}} \\ -\tan^{-1}\dfrac{L-\cos\delta(x_1-d_1)+\sin\delta(x_2+d_2)}{\left\{(x_1-d_1)\sin\delta+(x_2+d_2)\cos\delta\right\}} \\ -\tan^{-1}\dfrac{\cos\delta(x_1-d_1)-\sin\delta(x_2+d_2)}{\left\{(x_1-d_1)\sin\delta+(x_2+d_2)\cos\delta\right\}} \end{array}\right] \qquad (24)$$

III. Numerical Calculations

Now, we will study the behavior of the parallel displacement u_3 at the interface $x_2 = 0$ and $x_2 = L$ for a long inclined strike-slip fault of dip angle δ and width L and also at different distances of the fault from the interface and x_2 − axis.

Figure 2a-c display the variation of displacement $(2\Pi u_3 / b)$ with the distance from the fault (x_1 / b) for different values of dip angle $\delta = 30^o, 45^o, 60^o, 90^o$ for $x_2 = L$ due to a long strike-slip fault $d_1 = L$ from x_1-axis and $d_2 = L$ from x_2-axis.

Figure(2a)

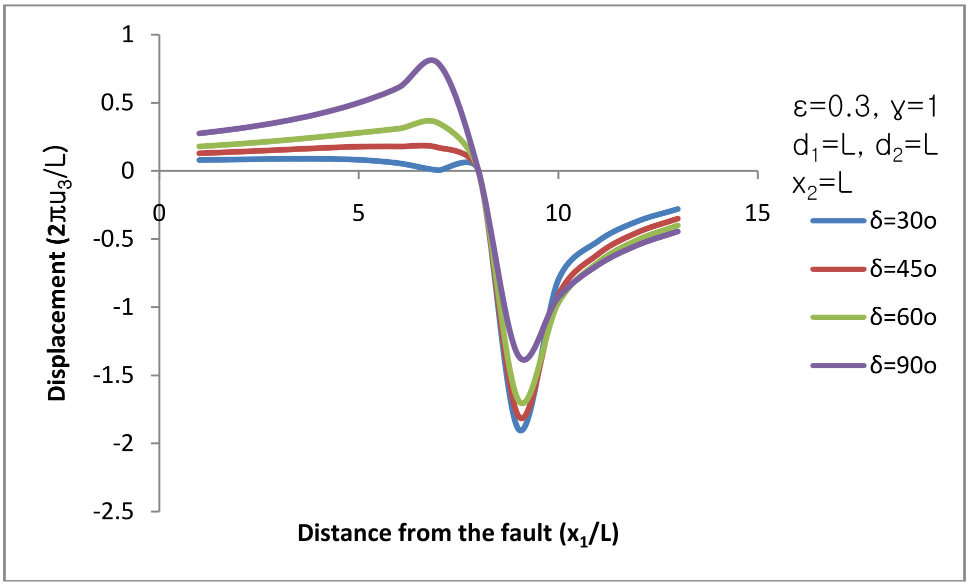


Figure (2b)

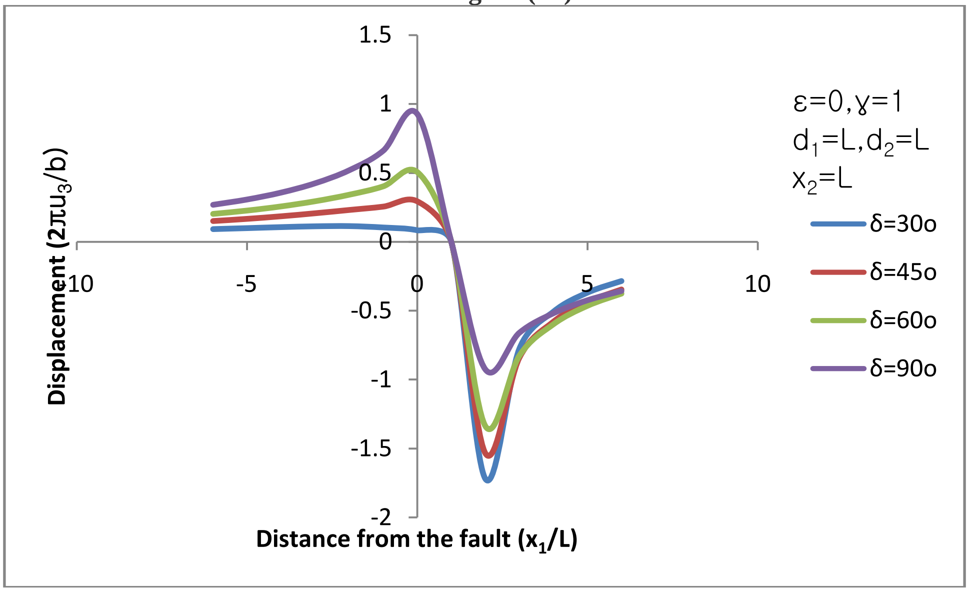


Figure 3a-c display the variation of the dimensionless displacement $\Pi u_3 / b$ with the dimensionless horizontal displacement (x_1 / L) from the fault-trace for the different values of $\varepsilon = -0.3, 0, 0.3$ and $\delta = 90^o$ for $x_2 = 0$ due to a long strike-slip fault located at a distance (a) at origin of the interface, (b) $d_1 = L$ from the $x_1 - \text{axis}$, (c) $d_2 = L$ from $x_2 - axis$, (d) $d_1 = L$ from $x_1 - \text{axis}$ and $d_2 = L$ from $x_2 - \text{axis}$.

Figure (3a)

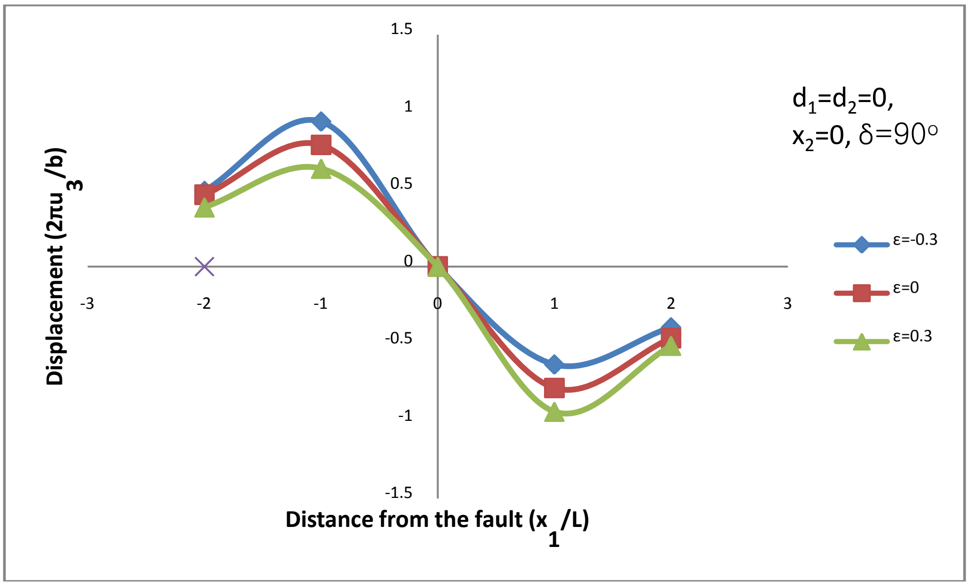

$d_1=d_2=0,$
$x_2=0, \delta=90°$
ε=-0.3
ε=0
ε=0.3
Displacement (2πu3/b)
Distance from the fault (x1/L)

Figure (3b)

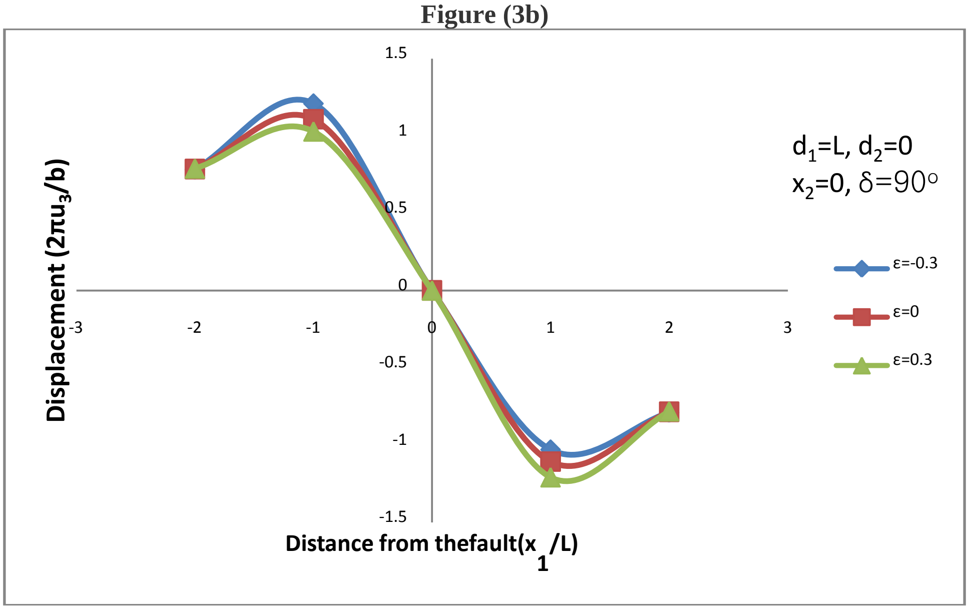

$d_1=L, d_2=0$
$x_2=0, \delta=90°$
ε=-0.3
ε=0
ε=0.3
Displacement (2πu3/b)
Distance from thefault(x1/L)

Figure (3c)

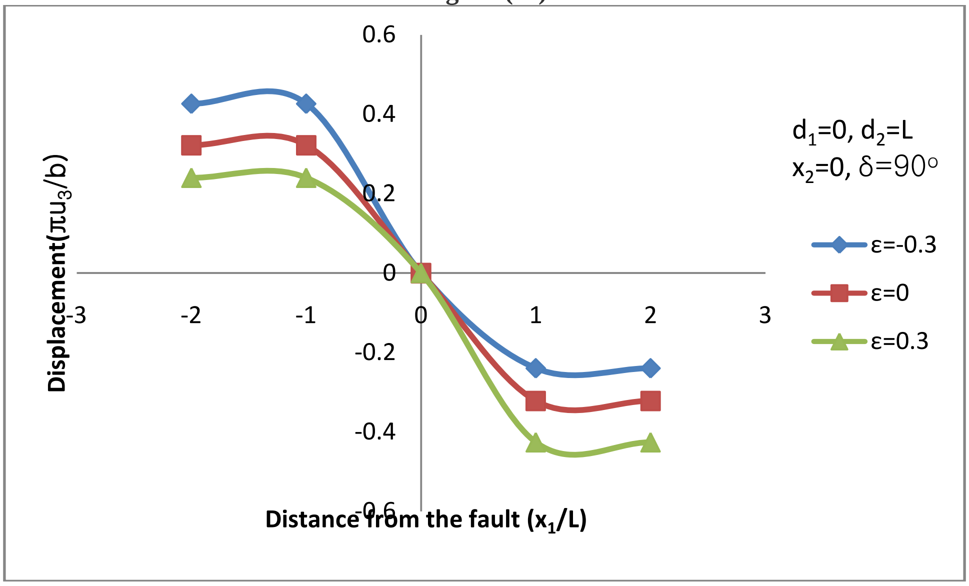
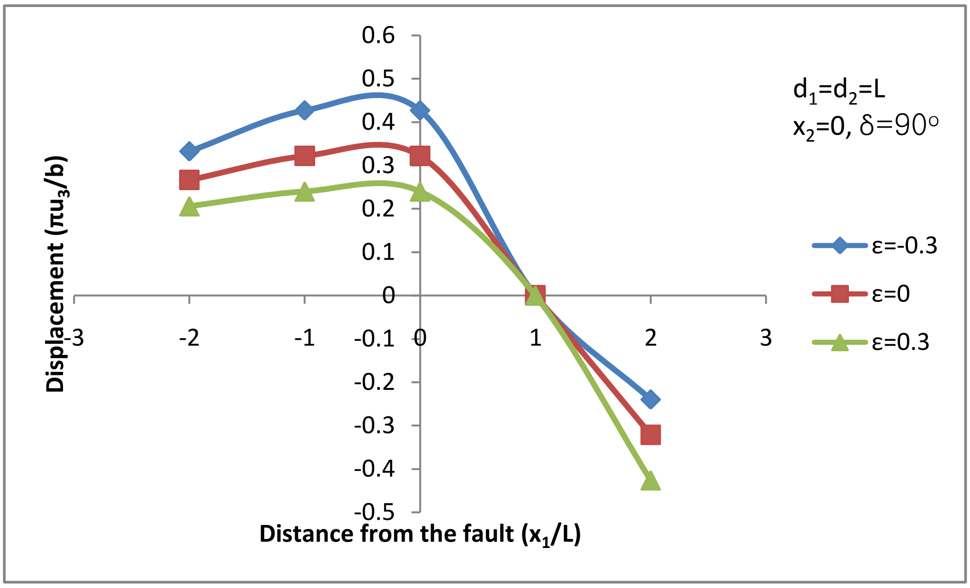


Figure (3d)



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