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A Balked M/D/1 Feedback Queueing Models

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Abstract: *In this paper, we present an analysis for an M/D/1 queueing system with balking. It is assumed that arriving customers may balk with a probability according to an exponential distribution. We develop the equations of the steady state probabilities and solution of the steady-state probabilities. The steady-state solution of the models is derived. Some useful measures of performance are derived. Some queueing models are obtained as particular cases of the model are discussed.*
Keywords: *Balking; Reneging; Steady-state Solution.*

I. Introduction

In real life, many queueing situations arise in which there may be a tendency for customers to be discouraged by a long queue. As a result, the customers either decide not to join the queue (i.e. balk) or depart after joining the queue without getting service due to impatience (i.e. renege). A queue, or a waiting line, involves arriving items that wait to be served at the facility which provides the service they seek. Queuing theory is concerned with the statistical description of the behavior of the queues with result. Balking and reneging are not only common phenomena in queues arising in daily activities, but also in various machine repair models. For related literature may refer to [17], [3], and references therein. An attractive example of the occurrence of balking and reneging is given in Ancker and Gafarian [2]. Queueing systems with balking, reneging, or both have been studied by many researchers. Haight [13] first considered an M/M/1 queue with balking. An M/M/1 queue with customers reneging was also proposed by Haight [14].

In this paper we have discussed about a steady state solution of the ordered queuing problem with reneging.

A customer may enter the queue, but after a time lose patience and decide to leave. In this case the customer is said to have reneged. A unit reneges (i.e., becomes impatient and leaves without having been served) after joining the queue if it is decided that the wait will be longer than can be tolerated. Here the waiting line is of Poisson balking probability which depend not only on the number of customers in the system, but also on the rate of service in the system. A queuing situation with the following characteristics has been considered. A customer receives the service immediately, when the system is empty. But a joining customer that has to wait for service due to impatience may leave i.e., the customer may renege. Service is performed on the customer at the head of the line. One service has commenced on a customer, it remains until the completion of service.

Barrer [10] has studied the problem of a unit leaving a queue after having waited longer than an acceptable time. O. Brien [12] has also found the solution of some queuing problem .Miller [22] and Konigsberg [11] have also studied about balk queue and queuing with special service. In this paper we have attempted to find out a steady state solution of queue, when the Poisson probabilities depend not only on the number of the customers in the system.

Customers are the backbone of any business, because without customers there will be no reason for a business to operate. Customer impatience leads to loss of potential customers. It has become a highly challenging problem in the current era of cut-throat competition. Queueing with customer impatience has special significance for the business world as it has a very negative effect on the revenue generation of a firm. Therefore, the concept of customer retention assumes a tremendous importance for the business management. Customer retention is the key issue in the organizations facing the problem of customer impatience. Firms are employing a number of customer retention strategies to maintain their businesses. An impatient customer (due to reneging) may be convinced to stay in service system for his service by utilizing certain persuasive mechanisms. Such customers are termed as retained customers.

When a customer gets impatient, he may leave the queue with some probability, say p_2 and may remain in the queue for service with some complementary probability q_2 ($=1-p_2$). Recently, Kumar and Sharma [20] study the retention of reneged customers in an M/M/1/N queuing model and perform sensitivity analysis of the model. Kumar and Sharma [21] study M/M/1/N queuing system with retention of reneged customers and balking. They extend the work of Kumar and Sharma [20] by taking balking aspects in their model to study the effect of probability of retaining the reneged customers on expected system size. They perform the sensitivity analysis of the model. We assume that after the completion of service, each customer may rejoin the system as a feedback customer for receiving another regular service with probability p_1 and may not join with complementary probability 1- p_1.

Sharma and Kumar [25] study single-server finite capacity, Markovian queue with feedback and retention of reneged customers. They perform steady-state analysis of the model.

Sharma and Kumar [25] further study M/M/1/N feedback queuing model with balking and retention of reneged customers in the same year. They obtain steady-state solution of the model. They derive important performance measures of the model. Some queueing models are derived as special cases of the model. This paper discussed the queueing model for deterministic service time. Some useful measures of performances are derived. Some queuing models are obtained as particular cases of the model. literature survey is also presented. Here , also present the steady-state analysis of a single-server Markovian feedback queuing system with retention of reneged customers. M/D/1 queuing model with retention of reneged customers considered. In the case of balking, immediately on arrival a unit decides not to join the queue, perhaps because of the length of the queue or because of other information on the length of service.

II. Literature Survey

The earlier work on feedback is found in Takacs [26]. He studies queue with feedback to determine the stationary process for the queue size and the first two moments of the distribution function of the total time spent in the system by a customer. Davignon and Disney [8] study single server queues with state dependent feedback. Santhakumaran and Thangaraj [24] consider a single server feedback queue with impatient and feedback customers. They study M/M/1 queueing model for queue length at arrival and obtain result for stationary distribution, mean and variance of queue length. The combined effects of balking and reneging in an M/M/1/N queue have been investigated by Ancker and Gafarian [1]. Wang and Chang [18] extended this work to study an M/M/c/N queue with balking, reneging and server breakdowns.

Thangaraj and Vanitha [27] obtain transient solution of M/M/1 feedback queue with catastrophes using continued fractions. The steady-state solution, moments under steady state and busy period analysis are calculated. Ayyappan [4] study M/M/1 retrial queueing system with loss and feedback under non-pre-emptive priority service by matrix geometric method. Customer retention in a single-server, finite capacity, Markovian queueing model is studied by Kumar and Sharma [20], the related work on customer retention and its application is in [25].

Several excellent surveys on these vacation models have been done by Doshi [9] and Takagi [15]. However, there are only a few works that take into consideration balking and reneging phenomena involving server vacations. The readers may refer to Zhang et al. [28] where an M/M/1/N queueing system with balking, reneging and server vacations was considered.

The concept of customer impatience appears in queueing theory in the work of Haight [13]. Haight [14] studies queuing with reneging. Ancker and Gafarian [1] study M/M/1/N queuing system with balking and reneging and derive its steady-state solution. Al-seedy et al. [5] study M/M/c queue with balking and reneging and derived its transient solution by using the probability generating function technique and the properties of Bessel function. Choudhury and Medhi [6] study customer impatience in multi-server queues. They consider both balking and reneging as functions of system state by taking into consideration the situations where the customer is aware of its position in the system. Kapodistria [19] study a single server Markovian queue with impatient customers and consider the situations where customers abandon the system simultaneously. He considers two abandonment scenarios. In the first one, all present customers become impatient and performed synchronized abandonments, while in the second scenario; the customer in service is excluded from the abandonment procedure. He extends this analysis to the M/M/c queue under the second abandonment scenario also.

In this paper, infinite capacity, single-server deterministic service feedback queueing model with customer retention and balking is studied. The steady-state analysis is performed. The steady state probabilities are obtained iteratively in case of reneging and retention of reneged customers. The performance measures are derived and some queuing models are discussed as particular cases of the model.

III. A Balked M/D/1 Feedback Queuing Model

It is necessary to know the reaction of the customer while entering in the system. A customer may decide to wait no matter how long the queue becomes, or there may be a case , if the queue is too long , the customer may decide to not to enter in the system. If a customer decides not to enter the queue upon arrival, then the customer is said to have balked. In case of feedback, after the completion of service, each customer may rejoin the system as a feedback customer for receiving another regular service with probability p_1 and may not join with complementary probability 1- p_1. We consider an M/D/1 feedback queuing model with reneging. The reneging times are assumed to exponentially distributed with parameter $\boldsymbol{\xi}$. It is visualized that a reneged customer may be convinced by applying certain convincing method to stay in the system for his service. Thus, there is a probability say, q_2 that a reneged customer may be retained in the system and may not be retained with some complementary probability say, p_2=(1−q_2). The model in this section is extension of the model M/D/1 Feedback Queueing Models with Retention of Reneged Customers by including the aspect of balking in the system.

On arrival, the customer may balk with certain balking probability. Let the probability of balking is q_3 or the customer may join the queue with probability p_3 such that $p_3+q_3 = 1$.

The differential-difference equations of the model are:

$$\frac{d}{dt}P_0(t) = -\lambda P_0(t) + \mu q_1 P_1(t)$$

Where $P_n(t)$ is the probability of n in the system at time 't'.
Here, we use deterministic service time model. The approach we use is similar to to be found in saaty [23] and which is essentially originally due to Crommelin [7]. In the M/D/1 queueing model the arrival rate is λ and the constant service time (say b=1/μ).
Now we rescale our parameter as **λ= λ/b** and **μ=1**[16] so that the traffic intensity 'ρ' $\left(=\frac{\lambda}{\square}\right)$ remains unaffected.
The differential-difference equation of the model according to above condition becomes:

$$\frac{d}{dt}P_0(t) = -\frac{\lambda}{b}P_0(t) + q_1P_1(t) \qquad \text{……………..……. (1)}$$

$$\frac{d}{dt}P_1(t) = -\left[\frac{\lambda}{b}p_3 + q_1\right]P_1(t) + (q_1+\xi p_2)P_2(t) + \frac{\lambda}{b}p_3P_0(t) \qquad \text{……. (2)}$$

$$\frac{d}{dt}P_n(t) = -\left[\frac{\lambda}{b}p_3 + q_1+(n-1)\xi p_2\right]P_n(t) + (q_1+n\xi p_2)P_{n+1}(t) + \frac{\lambda}{b}p_3P_{n-1}(t)$$

n ≥ ……. (3)

In steady state , $\lim_{t\to\infty} P_n = P_n$ and therefore $\frac{d}{dt}P_n(t) = 0 \; as \; t \to \infty$ and hence the equation (7) to (9) gives the difference equations as

$$0 = -\frac{\lambda}{b}P_0(t) + q_1P_1(t) \qquad \text{…….(4)}$$

$$0 = -\left[\frac{\lambda}{b}p_3 + q_1\right]P_1(t) + (q_1+\xi p_2)P_2(t) + \frac{\lambda}{b}p_3P_0(t) \qquad \text{……. (5)}$$

$$0 = -\left[\frac{\lambda}{b}p_3 + q_1+(n-1)\xi p_2\right]P_n(t) + (q_1+n\xi p_2)P_{n+1}(t) + \frac{\lambda}{b}p_3P_{n-1}(t)$$

n ≥ 2 ……. (6)

On solving iteratively, the steady state probabilities of system size are given by

$$P_n = \frac{\lambda}{bq1}\left[\frac{\left(\frac{\lambda}{b}p_3\right)^{n-1}}{\prod_{k=1}^{n-1} q_1+k\xi p_2}\right]P_0; \quad n \geq 1 \qquad \text{……(7)}$$

With $$P_0 = \frac{1}{\left(1+\frac{\lambda}{bq1}+\sum_{n=2}^{\infty}\frac{\lambda}{bq1}\left[\frac{\left(\frac{\lambda}{b}p_3\right)^{n-1}}{\prod_{k=1}^{n-1} q_1+k\xi p_2}\right]\right)} \qquad \text{…….(8)}$$

The steady state probabilities exist if

$$\left(1+\frac{\lambda}{bq1}+\sum_{n=2}^{\infty}\frac{\lambda}{bq1}\left[\frac{\left(\frac{\lambda}{b}p_3\right)^{n-1}}{\prod_{k=1}^{n-1} q_1+k\xi p_2}\right]\right) < \infty$$

IV. Measures Of Performance:

4.1 The Expected System size, $\mathbf{L_s}$

$$L_s = \sum_{n=0}^{\infty} nP_n$$

$$L_s = \sum_{n=1}^{\infty} n\left(\frac{\lambda}{bq1}\left[\frac{\left(\frac{\lambda}{b}p_3\right)^{n-1}}{\prod_{k=1}^{n-1} q_1 + k\xi p_2}\right]\right)P_0$$

4.2 The Expected Queue Length $\mathbf{L_q}$

$$L_q = \sum_{n=0}^{\infty} nP_n - \frac{\lambda}{bq_1}$$

$$L_q = \sum_{n=1}^{\infty} n\frac{\lambda}{bq1}\left[\frac{\left(\frac{\lambda}{b}p_3\right)^{n-1}}{\prod_{k=1}^{n-1} q_1 + k\xi p_2}\right]P_0 - \frac{\lambda}{bq_1}$$

4.3 The Expected waiting time $\mathbf{W_s}$

$$W_s = \frac{b}{\lambda}\sum_{n=1}^{\infty} n\ \frac{\lambda}{bq1}\left[\frac{\left(\frac{\lambda}{b}p_3\right)^{n-1}}{\prod_{k=1}^{n-1} q_1 + k\xi p_2}\right] P_0$$

4.4 The Expected waiting time in the queue $\mathbf{W_q}$

$$W_q = \frac{b}{\lambda}\sum_{n=1}^{\infty} n\ \frac{\lambda}{bq1}\left[\frac{\left(\frac{\lambda}{b}p_3\right)^{n-1}}{\prod_{k=1}^{n-1} q_1 + k\xi p_2}\right] P_0 - \frac{1}{q_1}$$

4.5 The Expected number of customers served, **($\mathbf{E_{cs}}$)**

The expected number of customer served is given by

$$E_{cs} = q_1\sum_{n=1}^{\infty} P_n \qquad as\ \ \mu = 1$$

$$E_{cs} = q_1\sum_{n=1}^{\infty} \frac{\lambda}{bq1}\left[\frac{\left(\frac{\lambda}{b}p_3\right)^{n-1}}{\prod_{k=1}^{n-1} q_1 + k\xi p_2}\right] P_0$$

4.6 Rate of Abandonment, $\mathbf{R_{aband}}$

$$R_{aband} = \frac{\lambda}{b}\sum_{n=0}^{\infty} P_n - E_{cs}$$

$$R_{aband} = \frac{\lambda}{b} - q_1\sum_{n=1}^{\infty} \frac{\lambda}{bq1}\left[\frac{\left(\frac{\lambda}{b}p_3\right)^{n-1}}{\prod_{k=1}^{n-1} q_1 + k\xi p_2}\right] P_0$$

4.7 Expected number of waiting customers actually waits, $\mathbf{E_{cw}}$

$$E_{cw} = \frac{\sum_{n=2}^{\infty}\ (n-1)P_n}{\sum_{n=2}^{\infty} P_n}$$

$$E_{cw} = \frac{\sum_{n=2}^{\infty}\ (n-1)\frac{\lambda}{bq1}\left[\frac{\left(\frac{\lambda}{b}p_3\right)^{n-1}}{\prod_{k=1}^{n-1} q_1 + k\xi p_2}\right] P_0}{\sum_{n=2}^{\infty} \frac{\lambda}{bq1}\left[\frac{\left(\frac{\lambda}{b}p_3\right)^{n-1}}{\prod_{k=1}^{n-1} q_1 + k\xi p_2}\right] P_0}$$

Where P_0 is given in eq. (8).

4.8 PARTICULAR CASES OF THE MODEL :

Here we introduce M/D/1 feedback queueing model with retention of reneged customer and balking. In the absence of retention of reneged customer and balking. When the probability of retention of Reneged customer is zero. i.e., q_2=0. We get

$$P_n = \frac{\lambda}{bq1}\left[\frac{\left(\frac{\lambda}{b}p_3\right)^{n-1}}{\prod_{k=1}^{n-1} q_1 + k\xi}\right] P_0; \qquad n \geq 1$$

With $$P_0 = \frac{1}{\left(1+\frac{\lambda}{bq\,1}+\sum_{n=2}^{N}\frac{\lambda}{bq\,1}\left[\frac{\left(\frac{\lambda}{b}p_3\right)^{n-1}}{\prod_{k=1}^{n-1}q_1+k\xi}\right]\right)}$$

When the capacity of the system is finite i.e., N.

$$P_n = \frac{\lambda}{bq\,1}\left[\frac{\left(\frac{\lambda}{b}p_3\right)^{n-1}}{\prod_{k=1}^{n-1}q_1+k\xi p_2}\right] P_0; \quad 1\leq n \leq N$$

With $$P_0 = \frac{1}{1+\frac{\lambda}{bq\,1}+\sum_{n=2}^{N}\frac{\lambda}{bq\,1}\left[\frac{\left(\frac{\lambda}{b}p_3\right)^{n-1}}{\prod_{k=1}^{n-1}q_1+k\xi p_2}\right]}$$

In the absence of balking, when p_3=1 the model reduce to M/D/1 feedback queueing model with retention of reneged customers.

V. Conclusion

This theory has been developed in an attempt to predict the fluctuating demands of queue length and units in the system and to enable an enterprise to provide adequate service for its customers and poisson balking probabilities. In this paper, we have studied customer retention in an M/D/1 feedback queuing model reneging and balking. The steady-state solution is obtained for the queuing model. Some important measures of performance are derived. Some important queuing models are obtained as particular cases of the model.

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