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Symmetric Skew 4-Reverse Derivations on Semi Prime Rings

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Abstract: *In this paper we introduce the notation of symmetric skew 4-reverse derivation of semiprime ring and we consider R be a non-commutative 2,3-torsion free semiprime ring, I be a non zero two sided ideal of R, α be an anti-automorphism of R, and $D: R^4 \to R$ be a symmetric skew 4-reverse derivation associated with theanti-automorphismα. Suppose that the trace function f is commuting on I and $[f(y), \alpha(y)] \in Z$, for all $y \in I$, then $[f(y), \alpha(y)] = 0$, for all $y \in I$.*

Keywords: *Semi prime ring, Derivation, Bi derivation,Reverse derivation,Symmetric Skew 3-derivation,Symmetric Skew 4-derivation,Symmetric skew 4-Reverse derivation and Anti-automorphism.*

I. Introduction

Bresar and Vukman [2] have introduced the notation of a reverse derivations and Samman and Alyamani [6] have studied some properties of semi prime rings with reverse derivations.The study of centralizing and commuting mappings onprime rings was initiated by the result of Posner [5] which states that the existence of a nonzero centralizing derivation on a prime ring implies that the ring has to be commutative.Vukman [7, 8] investigated symmetric bi derivation on prime and semi prime rings in connection with centralizing mappings.A.Fosner [1] have studied some results in symmetric skew 3-derivations with prime rings and semiprimerings.RecentlyFaizaShujat,Abuzaid Ansari[3] studied some results in symmetric skew 4-derivations inprime rings.C.Jaya Subba Reddy [4] have studied some results insymmetric skew 3-reverse derivations with semiprimerings.Motivated by the above work, in this paper we proved that under certain conditions of a semiprime ring with a nonzero symmetric skew 4-reverse derivations has to be commutative.

II. Preliminaries

Throughout the paper, R will represent a ring with a center Z and α ananti - automorphism of R.Let $n \geq 2$ be an integer. A ring R is said to be n-torsion free if for $x \in R, nx = 0$ implies $x = 0$.For all $x, y \in R$ the symbol $[x, y]$ will denote the commutator$xy - yx$.we make extensive use of basic commutator identities$[xy, z] = [x, z]y + x[y, z]and[x, yz] = [x, y]z + y[x, z]$.Recall that a ring R is semi prime if $xRx = 0$ implies that $x = 0$.An additive map $d: R \to R$ is called derivation if $d(xy) = d(x)y + xd(y)$, for all $x, y \in R$, and it is called a skew derivation (α -derivation) of R associated with the anti-automorphismα if $d(xy) = d(x)y + \alpha(x)d(y)$,for all $x, y \in R$.An additive map $d: R \to R$ is called reverse derivation if $d(xy) = xd(y) + yd(x)$,for all $x, y \in R$,and it is called a skew reverse derivation of R associated with anti- automorphismα if $d(xy) = xd(y) + \alpha(y)d(x)$, for all $x, y \in R$.
Before starting our main theorem, let us gives some basic definations and well known results which we will need in our further investigation.
Let D be a symmetric 4-additive map of R, then obviously
$D(-p, q, r, s) = -D(p, q, r, s)$,forall$p, q, r, s \in R$.(1)
Namely,for all$y, z \in R$,the map $D(.,.,y, z): R \to R$ is endo morphissm of the additive group of R.
The map $f: R \to R$ definedby $f(x) = D(x, x, x, x)$, $x \in R$ is called trace of D.
Note that f is not additive on R. But for all $x, y \in R$, we have

$$f(x + y) = [f(x) + 4D(x, x, x, y) + 6D(x, x, y, y) + 4D(x, y, y, y) + f(y)]$$

Recall that by (1),f is even function
More precisely, for all $p, q, r, s, u, v, w, x \in R$, we have

$$D(pu, q, r, s) = pD(u, q, r, s) + \alpha(u)D(p, q, r, s),$$
$$D(p, qv, r, s) = qD(p, v, r, s) + \alpha(v)D(p, q, r, s),$$
$$D(p, q, rw, s) = rD(p, q, w, s) + \alpha(w)D(p, q, r, s),$$
$$D(p, q, r, sx) = sD(p, q, r, x) + \alpha(x)D(p, q, r, s).$$

Of course, if D is symmetric, then the above four relations are equivalent to each other.

Lemma 1: Let$R$ be a prime ring and $a, b \in R$. If $a[x, b] = 0$ for all $x \in R$,then either $a = 0$ or $b \in Z$.
Proof: Note that
$0 = a[xy, b] = ax[y, b] + a[x, b]y = ax[y, b]$for all $x, y \in R$.
Thus $aR[y, b] = 0$,$y \in R$, and, since R is prime, either $a = 0$or $b \in Z$.

Theorem 1: Let R be a $2,3-$torsion free non commutative semiprime ring and I be a nonzero ideal of R. Suppose α is an anti automorphism of R and $D: R^4 \to R$ is a symmetric skew 4-reverse derivation associated with α. Suppose that the trace function f is commuting on I and $[f(y), \alpha(y)] \in Z$, for all $y \in I$, then $[f(y), \alpha(y)] = 0$, for all $y \in I$.

Proof: Let $[f(y), \alpha(y)] \in Z$, for all $y \in I$. (2)

Linearization of (2) yields that

$[f(x+y), \alpha(x+y)] \in Z$, for all $x, y \in I$.

$$[f(x+y), \alpha(x) + \alpha(y)] \in Z$$

By skew 4- derivation, we have

$(f(x+y), \alpha(x) + \alpha(y)) = [f(x) + 4D(x,x,x,y) + 6D(x,x,y,y) + 4D(x,y,y,y) + f(y), \alpha(x) + \alpha(y)]$, for all $x, y \in I$.

$[f(x), \alpha(x)] + 4[D(x,x,x,y), \alpha(x)] + 6[D(x,x,y,y), \alpha(x)] + 4[D(x,y,y,y), \alpha(x)] + [f(y), \alpha(x)] + [f(x), \alpha(y)] + 4[D(x,x,x,y), \alpha(y)] + 6[D(x,x,y,y), \alpha(y)] + 4[D(x,y,y,y), \alpha(y)] + [f(y), \alpha(y)] \in Z$, for all $x, y \in I$. (3)

From (2) & (3), we get

$4[D(x,x,x,y), \alpha(x)] + 6[D(x,x,y,y), \alpha(x)] + 4[D(x,y,y,y), \alpha(x)] + [f(y), \alpha(x)] + [f(x), \alpha(y)] + 4[D(x,x,x,y), \alpha(y)] + 6[D(x,x,y,y), \alpha(y)] + 4[D(x,y,y,y), \alpha(y)] \in Z$,

for all $x, y \in I$. (4)

Replacing y by $-y$ in (4) we have

$$-4[D(x,x,x,y), \alpha(x)] + 6[D(x,x,y,y), \alpha(x)] - 4[D(x,y,y,y), \alpha(x)] + [f(y), \alpha(x)] - [f(x), \alpha(y)] + 4[D(x,x,x,y), \alpha(y)] - 6[D(x,x,y,y), \alpha(y)] + 4[D(x,y,y,y), \alpha(y)] \in Z,$$

for all $x, y \in I$.(5) Comparing (4) and (5) and using 2-torsion freeness of R, we get

$4[D(x,x,x,y), \alpha(x)] + 4[D(x,y,y,y), \alpha(x)] + [f(x), \alpha(y)] + 6[D(x,x,y,y), \alpha(y)] \in Z$, for all $x, y \in I$. (6)

Substitute $y + z$ for y in (6) and using (6), we get

$4[D(x,x,x,y+z), \alpha(x)] + 4[D(x,y+z,y+z,y+z), \alpha(x)] + [f(x), \alpha(y+z)] + 6[D(x,x,y+z,y+z,\alpha y+z\in Z$, for all $x,y,z \in I$.

$$4[D(x,x,x,y), \alpha(x)] + 4[D(x,x,x,z), \alpha(x)] + 4[D(x,y,y,y), \alpha(x)] + 4[D(x,y,y,z), \alpha(x)] + 4[D(x,y,z,y), \alpha(x)] + 4[D(x,y,z,z), \alpha(x)] + 4[D(x,z,y,y), \alpha(x)] + 4[D(x,z,y,z), \alpha(x)] + 4[D(x,z,z,y), \alpha(x)] + 4[D(x,z,z,z), \alpha(x)] + [f(x), \alpha(y)] + [f(x), \alpha(z)] + 6[D(x,x,y,y), \alpha(y)] + 6[D(x,x,y,z), \alpha(y)] + 6[D(x,x,z,y), \alpha(y)] + 6[D(x,x,z,z), \alpha(y)] + 6[D(x,x,y,y), \alpha(z)] + 6[D(x,x,y,z), \alpha(z)] + 6[D(x,x,z,y), \alpha(z)] + 6[D(x,x,z,z), \alpha(z)] \in Z.$$

$$12[D(x,y,y,z), \alpha(x)] + 12[D(x,y,z,z), \alpha(x)] + 12[D(x,x,y,z), \alpha(y)] + 6[D(x,x,z,z), \alpha(y)] + 6[D(x,x,y,y), \alpha(z)] + 12[D(x,x,y,z), \alpha(z)] \in Z,$$

for all $x, y, z \in I$. (7)

Replacing z by $-z$ in (7) and compare with (7), we obtain

$$-12[D(x,y,y,z), \alpha(x)] + 12[D(x,y,z,z), \alpha(x)] - 12[D(x,x,y,z), \alpha(y)] + 6[D(x,x,z,z), \alpha(y)] - 6[D(x,x,y,y), \alpha(z)] + 12[D(x,x,y,z), \alpha(z)] \in Z$$

$2(12[D(x,z,y,y), \alpha(x)] + 12[D(x,x,y,z), \alpha(y)] + 6[D(x,x,y,y), \alpha(z)]) \in Z$,

for all $x, y, z \in I$.

Using of 2- torsion free ring, we have

$$12[D(x,z,y,y), \alpha(x)] + 12[D(x,x,y,z), \alpha(y)] + 6[D(x,x,y,y), \alpha(z)] \in Z,$$

for all $x, y, z \in I$. (8)

Substitute $y + u$ for y in (8) and use (8) we get

$12[D(x,z,y+u,y+u), \alpha(x)] + 12[D(x,x,y+u,z), \alpha(y+u)] + 6[D(x,x,y+u,y+u), \alpha(z)] \in Z$, for all $x, y, z, u \in I$.

$12[D(x,z,y,y), \alpha(x)] + 12[D(x,z,y,u), \alpha(x)] + 12[D(x,z,u,y), \alpha(x)] + 12[D(x,z,u,u), \alpha(x)] + 12[D(x,x,y,z), \alpha(y)] + 12[D(x,x,u,z), \alpha(y)] + 12[D(x,x,y,z), \alpha(u)] + 12[D(x,x,u,z), \alpha(u)] + 6[D(x,x,y,y), \alpha(z)] + 6[D(x,x,y,u), \alpha(z)] + 6[D(x,x,u,y), \alpha(z)] + 6[D(x,x,u,u), \alpha(z)] \in Z$, for all $x, y, z, u \in I$.

$24[D(x,z,y,u), \alpha(x)] + 12[D(x,x,y,z), \alpha(u)] + 12[D(x,x,u,z), \alpha(y)] + 12[D(x,x,y,u), \alpha(z)] \in Z$, for all $x, y, z, u \in I$. (9)

Since R is a 2 and 3-torsion free and replacing y, u by x in (9), we have

$24[D(x,z,x,x), \alpha(x)] + 12[D(x,x,x,z), \alpha(x)] + 12[D(x,x,x,z), \alpha(x)] + 12[D(x,x,x,x), \alpha(z)] \in Z$, for all $x, z \in I$.

$48[D(x,x,x,z), \alpha(x)] + 12[D(x,x,x,x), \alpha(z)] \in Z$, for all $x, z \in I$.

$4[D(x,x,x,z),\alpha(x)] + [f(x),\alpha(z)] \in Z$,for all $x,z \in I$. (10)
Again replaced z by zx in (10) and using (10) we obtain
$4[D(x,x,x,zx),\alpha(x)] + [f(x),\alpha(zx)] \in Z$,for all$x,z \in I$.
$4[D(x,x,x,zx),\alpha(x)] + [f(x),\alpha(x)\alpha(z)] \in Z$,for all $x,z \in I$.
$4[zf(x) + \alpha(x)D(x,x,x,z),\alpha(x)] + [f(x),\alpha(x)]\alpha(z) + \alpha(x)[f(x),\alpha(z)] \in Z$,
for all $x,z \in I$.
$4[z,\alpha(x)]f(x) + 4z[f(x),\alpha(x)] + 4\alpha(x)[D(x,x,x,z),\alpha(x)] + [f(x),\alpha(x)]\alpha(z) + \alpha(x)[f(x),\alpha(z)] \in Z$,for all $x,z \in I$.
$\alpha(x)([f(x),\alpha(z)] + 4[D(x,x,x,z),\alpha(x)]) + (\alpha(z) + 4z)[f(x),\alpha(x)] + 4[z,\alpha(x)]f(x) \in Z$,for all $x,z \in I$.
(11)
Therefore, from (11), we get
$[\alpha(x)([f(x),\alpha(z)] + 4[D(x,x,x,z),\alpha(x)]),\alpha(x)] + [(\alpha(z) + 4z)[f(x),\alpha(x)],\alpha(x)] +$
$4\big[[z,\alpha(x)]f(x),\alpha(x)\big] = 0$,for all $x,z \in I$ (12)
$\alpha(x)[([f(x),\alpha(z)] + 4[D(x,x,x,z),\alpha(x)]),\alpha(x)] + (\alpha(z) + 4z)[[f(x),\alpha(x)],\alpha(x)] +[\alpha(z) +$
$4z,\alpha(x)][f(x),\alpha(x)] + 4\big[[z,\alpha(x)],\alpha(x)\big]f(x) + 4[z,\alpha(x)][f(x),\alpha(x)] = 0$,for all $x,z \in I$.
$\alpha(x)\big[[f(x),\alpha(z)],\alpha(x)\big] + 4\alpha(x)\big[[D(x,x,x,z),\alpha(x)],\alpha(x)\big] + (\alpha(z) + 4z)\big[[f(x),\alpha(x)],\alpha(x)\big] +$
$[\alpha(z),\alpha(x)][f(x),\alpha(x)] + 4[z,\alpha(x)][f(x),\alpha(x)] + 4\big[[z,\alpha(x)],\alpha(x)\big]f(x) + 4[z,\alpha(x)][f(x),\alpha(x)] = 0$, for all $x,z \in I$.
$\alpha(x)\big[[f(x),\alpha(z)],\alpha(x)\big] + [\alpha(z),\alpha(x)][f(x),\alpha(x)] + [4z,\alpha(x)][f(x),\alpha(x)] + 4\big[[z,\alpha(x)],\alpha(x)\big]f(x) +$
$[4z,\alpha(x)][f(x),\alpha(x)] = 0$, for all $x,z \in I$.
$[(\alpha(z) + 8z),\alpha(x)][f(x),\alpha(x)] + 4\big[[z,\alpha(x)],\alpha(x)\big]f(x) = 0$, for all $x,z \in I$.(13)
Replacing z by $[f(x),\alpha(x)]f(x)$ in (13),we get
$\big[(\alpha\big([f(x),\alpha(x)]f(x)\big) + 8[f(x),\alpha(x)]f(x),\alpha(x)\big][f(x),\alpha(x)] + 4\Big[\big[[f(x),\alpha(x)]f(x),\alpha(x)\big],\alpha(x)\Big]f(x) =$
0, for all $x \in I$.
$[(\alpha(f(x))\alpha([f(x),\alpha(x)]),\alpha(x)][f(x),\alpha(x)] + 8[[f(x),\alpha(x)]f(x),\alpha(x)][f(x),\alpha(x)] +$
$4\Big[[f(x),\alpha(x)][f(x),\alpha(x)] + \big[[f(x),\alpha(x)],\alpha(x)\big]f(x),\alpha(x)\Big]f(x) = 0$,for all $x \in I$.
$\alpha\big(f(x)\big)[\alpha([f(x),\alpha(x)]),\alpha(x)][f(x),\alpha(x)] + \big[\alpha\big(f(x)\big),\alpha(x)\big]\alpha([f(x),\alpha(x)])[f(x),\alpha(x)] +$
$8\big[[f(x),\alpha(x)],\alpha(x)\big]f(x)[f(x),\alpha(x)] + 8[f(x),\alpha(x)][f(x),\alpha(x)][f(x),\alpha(x)] +$
$4\big[[f(x),\alpha(x)][f(x),\alpha(x)],\alpha(x)\big]f(x) = 0$, for all $x \in I$.
$\big[\alpha\big(f(x)\big),\alpha(x)\big]\alpha([f(x),\alpha(x)])[f(x),\alpha(x)] + 8[f(x),\alpha(x)][f(x),\alpha(x)][f(x),\alpha(x)] +$
$4[f(x),\alpha(x)]\big[[f(x),\alpha(x)],\alpha(x)\big]f(x) + 4\big[[f(x),\alpha(x)],\alpha(x)\big][f(x),\alpha(x)]f(x) = 0$,for all $x \in I$.
$\big[\alpha\big(f(x)\big),\alpha(x)\big]\alpha([f(x),\alpha(x)])[f(x),\alpha(x)] + 8[f(x),\alpha(x)][f(x),\alpha(x)][f(x),\alpha(x)] = 0$,for all $x \in I$.
$\big[\alpha\big(f(x)\big),\alpha(x)\big]\alpha[f(x),\alpha(x)][f(x),\alpha(x)] + 8[f(x),\alpha(x)]^3 = 0$,for all$x \in I$.
Since f is commutieson I,and we have 2, 3- torsion freeness of R,we have

$$2[f(x),\alpha(x)]^3 = 0$$

It follows that $(2[f(x),\alpha(x)]^2)\ R\ 2([f(x),\alpha(x)]^2) = 0$.
Since R is semiprime, we have
$2[f(x),\alpha(x)]^2 = 0$,for all$x \in I$.(14)
On the other hand, taking $z = x^2$ in equation (10), we get
$4[D(x,x,x,x^2),\alpha(x)] + [f(x),\alpha(x^2)] \in Z$, for all $x \in I$.
$4[xD(x,x,x,x) + \alpha(x)D(x,x,x,x),\alpha(x)] + [f(x),\alpha(x)\alpha(x)] \in Z$,for all $x \in I$.
$4[xf(x) + \alpha(x)f(x),\alpha(x)] + \alpha(x)[f(x),\alpha(x)] + [f(x),\alpha(x)]\alpha(x) \in Z$,
for all $x \in I$.
$4[xf(x),\alpha(x)] + 4[\alpha(x)f(x),\alpha(x)] + 2\alpha(x)[f(x),\alpha(x)] \in Z$,for all $x \in I$.
$4[x,\alpha(x)]f(x) + 4x[f(x),\alpha(x)] + 4\alpha(x)[f(x),\alpha(x)] + 4[\alpha(x),\alpha(x)]f(x) + 2\alpha(x)[f(x),\alpha(x)] \in Z$,for all $x \in I$.
$6\alpha(x)[f(x),\alpha(x)] + 4x[f(x),\alpha(x)] + 4[x,\alpha(x)]f(x) \in Z$,for all $x \in I$. (15)
Therefore, from equation (15), we get
$[f(x),6\alpha(x)[f(x),\alpha(x)] + 4x[f(x),\alpha(x)] + 4[x,\alpha(x)]f(x)] = 0$,for all $x \in I$.

$$\big[f(x),6\alpha(x)[f(x),\alpha(x)]\big] + \big[f(x),4x[f(x),\alpha(x)]\big] + [f(x),4[x,\alpha(x)]f(x)] = 0$$

$$6\alpha(x)[f(x),[f(x),\alpha(x)] + 6[f(x),\alpha(x)][f(x),\alpha(x)] + 4x\big[f(x),[f(x),\alpha(x)]\big] + 4[f(x),x][f(x),\alpha(x)] + 4\big[f(x),[x,\alpha(x)]\big]f(x) + 4[x,\alpha(x)][f(x),f(x)] = 0$$

$6[f(x),\alpha(x)]^2 + 4\big[f(x),[x,\alpha(x)]\big]f(x) = 0$,for all $x \in I$.
$6[f(x),\alpha(x)]^2 + 4[[f(x),x],\alpha(x)]f(x) = 0$,for all $x \in I$.
Since f is commuting on I and using equation (16), we get

$6[f(x), \alpha(x)]^2 = 0$,for all $x \in I$.
We have 2- torsion freeness, we get
$3[f(x), \alpha(x)]^2 = 0$,for all$x \in I$. (17)
Comparing (14) and (17), we get
$[f(x), \alpha(x)]^2 = 0$, for all $x \in I$.
Note that zero is the only nilpotent element in the center of semiprime ring.
Thus,$[f(x), \alpha(x)] = 0$,for all$x \in I$.
$[f(y), \alpha(y)] = 0$,for all $y \in I$.
This completes the proof.

Corollary 1: Let R be a 3!-torsion free prime ring, Ibe a non-zero ideal of R and α be an anti-automorphism of R.Suppose that there exists a non-zero symmetric skew 4-derivation$D: R^4 \rightarrow R$ associated with the anti-automorphism α such that the trace function$f$ is commuting on I and $(f(x), \alpha(x)) \in Z$, for all $x \in I$, then $D = 0$.

Proof: From theorem1,$[f(x), \alpha(x)] \in Z$, for all$x \in I$,then we have $[f(x), \alpha(x)] = 0$, for all$x \in I$. From[4,Theorem 1] states that $[f(x), \alpha(x)] = 0$, for all$x \in I$, then $D = 0$. Observing above two relations we concludes that $[f(x), \alpha(x)] \in Z$, for all$x \in I$, then $D = 0$.

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