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Application of homotopy Analysis Method for Solving non linear Dynamical System

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Abstract: *In this paper, the nonlinear dynamical systems are solved by using the homotopy analysis method (HAM).The approximation solution of this equation is calculated in the form ofa series which its components are computed easily. The existence anduniqueness of the solution and the convergence of the proposed methodare proved.The results obtained here demonstrate that the HAM is an effective and robust technique for nonlinear dynamical systems.Nonlinear dynamical systems are omnipresent in numerous practical engineering and mathematics problems. It is hardly to seek the exact solutions in normal circumstances. However, the development of analytical methods can provide an all-embracing understanding for the systems*
Key words*: Nonlinear dynamical system; Homotopy Analysis Method Mathematics Subject classification: 53C; 58Z05*

I. Introduction

Since many physical problems are modeled by nonlinear dynamical system, the numerical solutions of such nonlinear dynamical system have been highly studied by many authors. E-mail address: Gharibmusa@gmail.com" This research is funded by the Deanship of Research and Graduate Studies in Zarqa University/ Jordan"

In recent years, numerous works have been focusing on the development of more advanced and efficient methods for integral equations and nonlinear dynamical system for example [1-6, 15, 16, 18], the HAM is based on homotopy, a fundamental concept in topology and differential geometry. Over the last couple of decades, Liao [1-10] described a nonlinear analytical technique which does not require small parameters and thus can be applied to solve nonlinear problems without small or large parameters.

This technique is based on homotopy, which is an important part of topology, called the homotopy analysis method (HAM). Its main idea is to construct a class of homotopy in a rather general form by introducing an auxiliary parameter. This parameter can provide us with a convenient way to control the convergence of approximation series and adjust convergence rate and region when necessary. A systematical description of this method was presented in [1]. Liao [7] also studied the convergence properties of the HAM and proved that as long as an HAM series is convergent, it must converge to one solution of the considered problem.

Many micro electro-mechanical systems (MEMSs) inherently contain nonlinearities, such as intrinsic and exterior nonlinearities arising from coupling of different domains [5-8]. Also, there exist mechanical nonlinearities, for example, large deformations, surface contact, creep phenomena, time-dependent masses and nonlinear damping effects, and so forth [9-11]. It seems fair to say that nonlinear dynamic analysis becomes an increasingly important task in MEMS research and manufacturing.

The nonlinear dynamical behaviors of microcantilever-based instrument in MEMS under various loading conditions have stimulated the curiosities and interests of many researchers [11-15]. For example, the oscillation of an electrostatically actuated microcantilever-based device in MEMS was investigated through a simplified mass-spring-damping model subjected to nonlinear electrostatic force [16-18]. Complex nonlinear terms arising from electrostatic force and from squeeze film damping make it difficult to analyze the system directly using some routine techniques for nonlinear vibrations. For this reason, Stephan[13] suggested an approximate treatment by expanding the nonlinearities into Taylor series and retaining only the first two terms. The harmonic balance method was then applied to solve the approximate system. Note that the attained results have not been compared with any numerical solutions. The purpose of this study is to seek highly accurate solutions of the aforementioned system on the basis of the HAM.

The paper is organized as follows. In section 2, the HAM is briefly presented, in section 3, this method is presented for solving the nonlinear dynamical system. Finally, I give some conclusions in
are shown insection 4.

Homotopy Analysis Method

The study of nonlinear problems is of crucial importance in all areas of mathematicaland physics. Some of the most interesting features of physical systems are hidden intheir nonlinear behavior, and can only be studied with appropriate methods designedto tackle nonlinear problems.Recently Liao has developed a method for solving nonlinear problems which does not depend on the existence of a small parameter. This method has been used by Liao and others to find analytical expressions for several nonlinear problems.Suppose the problem to be solved is [2-4]

$$N[f(t)] = 0 \quad (1)$$

Introduce a new variable q and consider the equation

$$H[\phi(t;\, q), f_0(t), \hbar, q] = (1-q)L[\phi(t;\, q) - f_0(t)] - q\,\hbar\, H(t)N[\phi(t;\, q)], \quad (2)$$

where L is a suitably chosen linear operator , $f_0(t)$ an initial approximate solution and h a parameter to be chosen later. Substitution $q = 0$ in (2) we get

$$\phi(t;\, 0) = f_0(t). \quad (3)$$

If we let $q = 1$ in (2), we get

$$N[\phi(t;\, 1)] = 0 \quad (4)$$

whose solution is the required function $f(t)$. Thus as q varies continuously from 0 to 1 , the initial approximate solution $f_0(t)$evolves to the desired solution $f(t)$ of the problem Liao gives this reason for calling the method as the homotopy analysis method.

Define

$$f_m(t) = \frac{1}{m\,!}\frac{\partial^m \phi(t;q)}{\partial\, q^m}\Big|_{q=0} \quad (5)$$

then

$$\phi(t;\, q) = \phi(t;\, 0) + \sum_{m=1}^{\infty} \frac{1}{m\,!}\frac{\partial^m \phi(t;q)}{\partial\, q^m}\Big|_{q=0} q^m \quad (6)$$

If the series converges at $q = 1$, then

$$f(t) = f_0(t) + \sum_{m=1}^{\infty} f_m(t) \quad (7)$$

The functions $f_m(t)$ are found successively from the equations

$$L[f_1(t)] = \hbar\, H(t)N[f_0(t)] \quad (8)$$

when $m = 1$ and

$$L[f_m(t) - f_{m-1}(t)] = \hbar\, H(t)\frac{1}{(m-1)\,!}\frac{\partial^{m-1} N[\phi(t;q)]}{\partial\, q^{m-1}}\Big|_{q=0} \quad (9)$$

when $m \geq 2$.

By rearranging this equation and introducing the term χ_m

$$\chi_m = \begin{cases} 0 & m \leq 1 \\ 1 & m > 1 \end{cases} \quad (10)$$

we can form the mthorder deformation equation:

$$L[f_m(t) - \chi_m f_{m-1}(t)] = \hbar\, H(t)\frac{1}{(m-1)\,!}\frac{\partial^{m-1} N[\phi(t;q)]}{\partial\, q^{m-1}}\Big|_{q=0} (11)$$

which is true for $m \geq 1$. Rearranging

$$L^{-1}\left\{\hbar\, H(t)\frac{1}{(m-1)\,!}\frac{\partial^{m-1} N[\phi(t;q)]}{\partial\, q^{m-1}}\Big|_{q=0}\right\} \quad (12)$$

where $L^{-1}$is the inverse of the linear operator (i.e. inverse of differentiation is integration). The solution to $f(t)$ can be expressed as

$$f(t) = \sum_{m=0}^{\infty} f_m(t) \quad (13)$$

which is valid where ever the solution converges.

Nonlinear Dynamical System

In this work, the HAM is presented as an alternative method to derive the analytical solution for nonlinear dynamical system. Illustrative example is used to show the validity and accuracy of the method in solving the nonlinear system. The results obtained here demonstrate that the HAM is an effective and robust technique for nonlinear dynamical systems.Nonlinear dynamical systems are omnipresent in numerouspractical engineering and mathematics problems. It is hardlyto seek the exact solutions in normal circumstances. However, the development of analytical methods can provide anall-embracing understanding for the systems. The HAM [3] is a robust analytical approximatetechnique for solving a class of nonlinear problems.

The significance of dynamical systems is mainly due to its globalbifurcation, regular and chaotic motions, the intensiveresearch subjects are thus at the forefront of nonlineardynamics. Recently, some achievements and fruitful outcomehave been established for dynamical systems [7-15].

Consider the nonlinear system

$$\frac{d^3x}{dt^3} - \frac{d^2y}{dt^2} + \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = 1, \quad (14)$$

$\frac{d^3y}{dt^3}+\frac{d^2x}{dt^2}+\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2=1,$ (15)

with the initial conditions

$$x(0)=y'(0)=0, \qquad y(0)=x'(0)=1. \qquad (16)$$

The exact solutions for Eqs. (14), (15) subject to the initial conditions in Eq. (16) are

$$x(t)=\sin t, \quad y(t)=\cos t \qquad (17)$$

Suppose that the solution can be expressed by a set of basefunctions $\{t^n \mid n=0,1,2,3,\cdots\}$, I choose the initialapproximation as

$x_0(t)=t, \quad y_0(t)=1-\frac{t^2}{2}.$ *(18)*

Defining the nonlinear operator as

$$N\begin{pmatrix}\phi_1(t;\,q)\\ \phi_2(t;\,q)\end{pmatrix}=\begin{pmatrix}\frac{\partial^3\phi_1(t;q)}{\partial t^3}-\frac{\partial^2\phi_2(t;q)}{\partial t^2}+\left(\frac{\partial\phi_1(t;q)}{\partial t}\right)^2+\left(\frac{\partial\phi_2(t;q)}{\partial t}\right)^2\\ \frac{\partial^3\phi_2(t;q)}{\partial t^3}+\frac{\partial^2\phi_1(t;q)}{\partial t^2}+\left(\frac{\partial\phi_1(t;q)}{\partial t}\right)^2+\left(\frac{\partial\phi_2(t;q)}{\partial t}\right)^2\end{pmatrix}. (19)$$

Thus, the zeroth-order deformation equation can be written in the form

and themth −order deformation equation can be expressed as

From the initial conditions and the initial approximation, I have

$\sum_{j=0}^{m-1} y_j'(t)y_{m-1-j}'(t)-\,(1-\chi_m)\,,$ (23)

$R_{y,m}(y_m$

$\sum_{j=0}^{m-1} y_j'(t)y_{m-1-j}'(t)-\,(1-\chi_m)\,,$ (24)

From Eq. (21), it implies that

$$x_m(t)=\chi_m x_{m-1}(t)+\hbar\int_0^t\int_0^w\left(\int_0^r R_{x,m}(x_{m-1})\,ds\right)drdw+C_{1,m}\,t+C_{2,m} \qquad (25)$$

$$y_m(t)=\chi_m y_{m-1}(t)+\hbar\int_0^t\int_0^w\left(\int_0^r R_{y,m}(y_{m-1})\,ds\right)drdw+C_{3,m}\,t+C_{4,m} \qquad (26)$$

Then, I obtain

$x_1(t)=\hbar\left(\frac{t^3}{6}+\frac{t^5}{60}\right), \qquad y_1(t)=\hbar\left(\frac{t^5}{60}\right)$ (29)$R_{x,2}(x_1)=x_1'''(t)-y_1''(t)+2\,x_0'(t)\,x_1'(t)+2y_0'(t)y_1'(t)$

$$x_2(t)=\hbar\left(\frac{t^3}{6}+\frac{t^5}{60}\right)+\hbar^2\left(\frac{t^3}{6}+\frac{t^5}{30}-\frac{t^6}{360}+\frac{t^7}{1260}-\frac{t^8}{2016}\right), \qquad (32)$$

$$y_2(t)=\hbar\left(\frac{t^5}{60}\right)+\hbar^2\left(\frac{t^4}{24}+\frac{t^5}{30}+\frac{t^6}{360}+\frac{t^7}{1260}-\frac{t^8}{2016}\right), \qquad (33)$$

I now successively obtain the second-order analyticalapproximation by HAM as the following

$x(t)\approx$ t

$$t+\left(\frac{\hbar}{3}+\frac{\hbar^2}{6}\right)t^3+\left(\frac{\hbar}{30}+\frac{\hbar^2}{30}\right)t^5-\frac{\hbar^2}{360}t^6+\frac{\hbar^2}{1260}t^7-\frac{\hbar^2}{2016}t^8 \qquad (34)$$

$$y(t) \approx 1-\frac{t^2}{2}+\left(\frac{\hbar^2}{24}\right)t^4+\left(\frac{\hbar}{30}+\frac{\hbar^2}{30}\right)t^5+\frac{\hbar^2}{360}t^6+\frac{\hbar^2}{1260}t^7-\frac{\hbar^2}{2016}t^8 \quad (35)$$

Therefore the approximate solution is given by at $\hbar = -1$

$$x(t) \approx t+\left(-\frac{t^3}{6}-\frac{t^6}{360}+\frac{t^7}{1260}-\frac{t^8}{2016}\right), \quad (36)$$

$$y(t) \approx 1-\frac{t^2}{2}+\left(\frac{t^4}{24}+\frac{t^6}{360}+\frac{t^7}{1260}-\frac{t^8}{2016}\right). \quad (37)$$

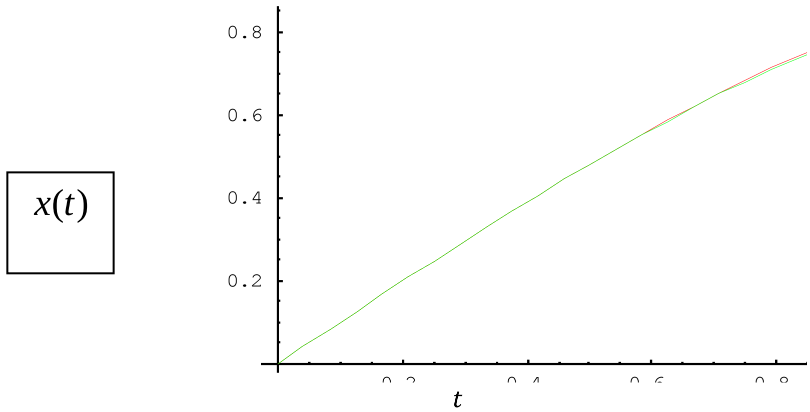

Fig. 1: Zoom For Comparison between Exact (17) and HAM (36) Solutions at $\hbar = -1$

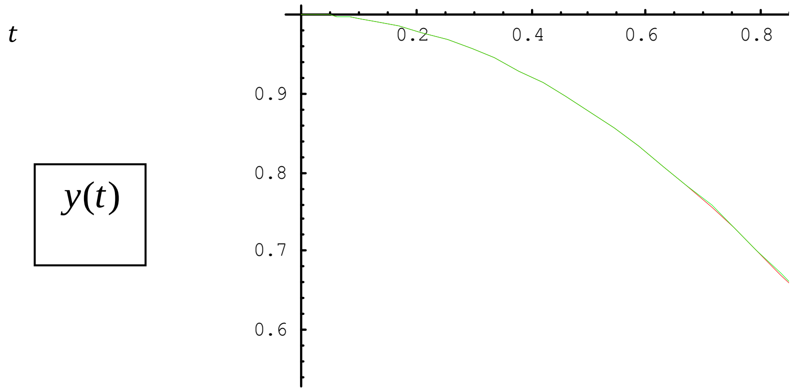

Fig. 2: Zoom For Comparison between Exact (17) and HAM (37) Solutions at $\hbar = -1$

II. Conclusion

In summary,homotopyanalysis approximate method is applied to obtain analytical approximation solution for nonlinear dynamical system. The fundamental idea of the method is essentially different from other existing analytical methods. The homotopy analysis approximate method provide an ingenious avenue for controlling the convergences of approximation series. The exact solutions of this example can be used to verify the accuracy of the method. Homotopy analysis method has been known as a powerful scheme for solving many functional equations such as algebraic equations, ordinary and partial differential equations, integral equations and so on. The HAM has been shown to solve effectively, easily and accurately a large class of nonlinear problems with the approximations which are rapidly convergent to the exact solution, In this work, the HAM has been successfully employed to obtain the approximate or analytical solution of the nonlinear dynamical system.The present techniques can also be further generalized to investigate more complicated nonlinear dynamical systems that can only be solved by numerical approaches.

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