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Neural Networks In Mathematical Model With A Derivation Of Fourth Order Runge Kutta Method

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Abstract: *In this papera new type of neural networks for the derivation of fourth order Runge-Kutta Method which involves tedious computation of many unknowns and its details.Its analysis can hardly be found in many literatures due to the vital role played by the method in the field of computation and applied science.*
Keywords: *Fourth order Rungekutta Method, Neural Network, Derivation, Analysis.*

I. Introdution

Runge–Kutta (RK) pairs are widely used for the numerical solution of ordinary differential equations [Lawrence, 1985]

$$\boldsymbol{y' = f(x, y) \quad , \alpha \le x \le \beta , y(\alpha) = y_0}$$

With a given step 'h' through the interval [α, β] successively producing approximations y_n , y_{n+1}. Here deal exclusively with the neural networks deviation and the stability analysis of the fourth order Runge-Kutta Method through coverage of the derivation and analysis the reader is referred to [1,2,3,4,5].

II. Mathematical Formulation

The function is defined as $y_{n+1} = y_n + h\, \emptyset\, (x, y, h)$
Where $\emptyset\, (x, y, h) = \sum_{i=1}^{s} \beta_i K_i$

$$K_1 = f(x, y)$$

$$K_i = f\left(x + \gamma_i h\, , y_n + h \sum_{j=1}^{i-1} \alpha_{ij} K_j\right), i = 2,3,4, \dots , i - 1.$$

$$\gamma_i = \sum_{j=1}^{i-1} \alpha_{ij}\, , \qquad when\; s = 4$$

$$y_{n+1} = y_n + h(\beta_1 K_1 + \beta_2 K_2 + \beta_3 K_3 + \beta_4 K_4)$$

$$K_1 = f(x, y)$$

$$K_2 = f(x + \gamma_2 h, y_n + h\alpha_{21} k_1)$$

$$k_3 = f(x + \gamma_3 h, y_n + h(\alpha_{31} k_1 + \alpha_{32} k_2))$$

$$k_4 = f(x + \gamma_3 h, y_n + h(\alpha_{41} k_1 + \alpha_{42} k_2 + \alpha_{43} k_3))$$

Now functions are expanded by using a Taylor series expansion for function of two variables. To get the unknowns, we use the fourth order coefficients of order 4.

$$\tau_1{}^{(1)} = \sum_i \beta_i - 1$$

$$\tau_1{}^{(2)} = \sum_i \beta_i \gamma_i - \frac{1}{2}$$

$$\tau_1{}^{(3)} = \frac{1}{2} \sum_i \beta_i \gamma^2{}_i - \frac{1}{6}$$

$$\tau_2{}^{(3)} = \sum_{ij} \beta_i \alpha_{ij} \gamma_j - \frac{1}{6}$$

$$\tau_1{}^{(4)} = \frac{1}{6} \sum_i \beta_i \gamma^2{}_i - \frac{1}{24}$$

$$\tau_2{}^{(2)} = \sum_{ij} \beta_i \gamma_i \alpha_{ij} \gamma_j - \frac{1}{8}$$

$$\tau_3{}^{(4)} = \frac{1}{2} \sum_{i} \beta_i \alpha_{ij} \gamma^2{}_j - \frac{1}{24}$$

$$\tau_4{}^{(4)} = \sum_{ij} \beta_i \alpha_{ij} \alpha_{jk} \gamma_k - \frac{1}{24}$$

Setting the coefficients to zero, we have

$\beta_1 + \beta_2 + \beta_3 + \beta_4 = 1$ (1)

$\beta_2\gamma_2 + \beta_3\gamma_3 + \beta_4\gamma_4 = \frac{1}{2}$ (2)

$\beta_2\gamma^2{}_2\beta_3\gamma^2{}_3 + \beta_4\gamma^2{}_4 = \frac{1}{3}$ (3)

$\beta_3\alpha_{32}\gamma_2 + \beta_4\alpha_{42}\gamma_2 + \beta_4\gamma_4\alpha_{43}\gamma_3 = \frac{1}{6}$ (4)

$\beta_2\gamma^3{}_2 + \beta_3\gamma^3{}_3 + \beta_4\gamma^3{}_4 = \frac{1}{4}$ (5)

$\beta_3\gamma_3\alpha_{32}\gamma_2 + \beta_4\gamma_4\alpha_{42}\gamma_2 + \beta_4\gamma_4\alpha_{43}\gamma_3 = \frac{1}{8}$ (6)

$\beta_3\alpha_{32}\gamma^2{}_2 + \beta_4\alpha_{42}\gamma^2{}_2 + \beta_4\alpha_{43}\gamma^2{}_3 = \frac{1}{12}$ (7)

$\beta_4\alpha_{43}\alpha_{32}\gamma_2 = \frac{1}{24}$ (8)

using the simplifying assumptions by Butcher [3].

$\sum_{i=1}^{s} \beta_i \alpha_{ij} = \beta_i(1-\gamma_j), \qquad j = 2,3,4$ (9)

Which affect the expression for$\tau_2{}^{(3)}, \tau_3{}^{(4)}$ *and* $\tau_4{}^{(4)}$.

Then $\tau_2{}^{(3)} = \tau_1{}^{(2)} - 2\tau_1{}^{(3)}$

$$\tau_3{}^{(4)} = \tau_1{}^{(3)} - 3\tau_1{}^{(4)}$$

$$\tau_4{}^{(4)} = \tau_1{}^{(2)} - 2\tau_1{}^{(3)} - \tau_2{}^{(4)}$$

Now using equation (9) for $j = 2,3$ *and* 4 we have

$\beta_3\alpha_{32} + \beta_4\alpha_{42} = \beta_2(1-\gamma_2)$ (i)

$\beta_4\alpha_{43} = \beta_3(1-\gamma_2)$ (ii)

$0 = \beta_4(1-\gamma_4)$respectively. (iii)

Nowwhen$j = 4$ in (iii),$\gamma_4 = 1$ *and* $\beta_4 \neq 0$ for a four stage method.

By substituting$\gamma_4 = 1$ in equations 2,3 and 5 and solve for β_2, β_3*and* β_4 simultaneously. Therefore equations 2,3 and 5 becomes.

$$\beta_2\gamma_2 + \beta_3\gamma_3 + \beta_4 = \frac{1}{2}$$

$$\beta_2\gamma^2{}_2\beta_3\gamma^2{}_3 + \beta_4 = \frac{1}{3}$$

$$\beta_2\gamma^3{}_2 + \beta_3\gamma^3{}_3 + \beta_4 = \frac{1}{4}$$

Using crammer's rule , we first find the determinant of the coefficient matrix

$$D = \begin{vmatrix} \gamma_2 & \gamma_3 & 1 \\ \gamma^2{}_2 & \gamma^2{}_3 & 1 \\ \gamma^3{}_2 & \gamma^3{}_3 & 1 \end{vmatrix} = -\gamma_2\gamma_3(\gamma_2 - 1)(\gamma_2 - \gamma_3)(\gamma_3 - 1)$$

To solve for β_2

$$D_{\beta_2} = \begin{vmatrix} \frac{1}{2} & \gamma_3 & 1 \\ \frac{1}{3} & \gamma^2{}_3 & 1 \\ \frac{1}{4} & \gamma^3{}_3 & 1 \end{vmatrix} = \frac{-\gamma_3(\gamma_3 - 1)(2\gamma_3 - 1)}{12}$$

$$\beta_2 = \frac{D_{\beta_2}}{D} = \frac{\frac{-\gamma_3(\gamma_3-1)(2\gamma_3-1)}{12}}{-\gamma_2\gamma_3(\gamma_2 - 1)(\gamma_2 - \gamma_3)(\gamma_3 - 1)} = \frac{1 - 2\gamma_3}{12\gamma_2(1-\gamma_2)(\gamma_3 - \gamma_2)}$$

To solve for β_3

$$D_{\beta_3} = \begin{vmatrix} \gamma_2 & \frac{1}{2} & 1 \\ {\gamma^2}_2 & \frac{1}{3} & 1 \\ {\gamma^3}_2 & \frac{1}{4} & 1 \end{vmatrix} = \frac{\gamma_2(\gamma_2 - 1)(2\gamma_2 - 1)}{12}$$

$$\beta_3 = \frac{D_{\beta_3}}{D} = \frac{\frac{\gamma_2(\gamma_2-1)(2\gamma_2-1)}{12}}{-\gamma_2\gamma_3(\gamma_2 - 1)(\gamma_2 - \gamma_3)(\gamma_3 - 1)} = \frac{1 - 2\gamma_2}{12\gamma_3(\gamma_3 - \gamma_2)(1 - \gamma_3)}$$

To solve for β_4

$$D_{\beta_4} = \begin{vmatrix} \gamma_2 & \gamma_3 & \frac{1}{2} \\ {\gamma^2}_2 & {\gamma^2}_3 & \frac{1}{3} \\ {\gamma^3}_2 & {\gamma^3}_3 & \frac{1}{4} \end{vmatrix} = \frac{-\gamma_2\gamma_3(\gamma_2 - \gamma_3)(3 - 4\gamma_2 - 4\gamma_3 + 6\gamma_2\gamma_3)}{12}$$

$$\beta_4 = \frac{D_{\beta_4}}{D} = \frac{\frac{-\gamma_2\gamma_3(\gamma_2-\gamma_3)(3-4\gamma_2-4\gamma_3+6\gamma_2\gamma_3)}{12}}{-\gamma_2\gamma_3(\gamma_2 - 1)(\gamma_2 - \gamma_3)(\gamma_3 - 1)} = \frac{6\gamma_2\gamma_3 - 4(\gamma_2 + \gamma_3) + 3}{12(1 - \gamma_2)(1 - \gamma_3)}$$

Now to solve for α_{43} , we use equation(ii) when $j = 3$

Hence , we have

$$\Rightarrow \alpha_{43} = \frac{\beta_3(1 - \gamma_3)}{\beta_4} = \frac{(1 - \gamma_3)(1 - \gamma_3)12(1 - \gamma_2)(1 - \gamma_3)}{12\gamma_3(\gamma_3 - \gamma_2)(1 - \gamma_3)6\gamma_2\gamma_3 - 4(\gamma_2 + \gamma_3) + 3}$$

$$\alpha_{43} = \frac{(1 - \gamma_3)(1 - \gamma_2)(2\gamma_2 - 1)}{\gamma_3(\gamma_2 - \gamma_3)(6\gamma_2\gamma_3 - 4(\gamma_3 + \gamma_2)) + 3}$$

To solve for α_{32} and α_{42}, we use equation (i) and (8) when $j = 2$

$\beta_3\alpha_{32} + \beta_4\alpha_{42} = \beta_2(1 - \gamma_2)$ (i)

$\beta_4\alpha_{43}\alpha_{32}\gamma_2 = \frac{1}{24}$ (8)

From equation (8) above,

$$\alpha_{32} = \frac{1}{24\gamma_2} \times \frac{1}{\beta_4} \times \frac{1}{\alpha_{43}} = \frac{1}{24\gamma_2} \times \frac{12(1 - \gamma_2)(1 - \gamma_3)}{6\gamma_2\gamma_3 - 4(\gamma_3 + \gamma_2) + 3} \times \frac{\gamma_3(\gamma_2 - \gamma_3)(6\gamma_2\gamma_3 - 4(\gamma_3 + \gamma_2)) + 3}{(1 - \gamma_3)(1 - \gamma_2)(2\gamma_2 - 1)}$$

$$\alpha_{32} = \frac{\gamma_3(\gamma_2 - \gamma_3)}{2\gamma_2(2\gamma_2 - 1)}$$

Substituting this value (i), we have

$$\alpha_{42} = \frac{\beta_2(1 - \gamma_2) - \beta_3\alpha_{32}}{\beta_4}$$

$$\alpha_{42} = \left[\frac{1 - 2\gamma_3}{12\gamma_2(1 - \gamma_2)(\gamma_3 - \gamma_2)} \times (1 - \gamma_2) - \frac{1 - 2\gamma_2}{12\gamma_3(1 - \gamma_3)(\gamma_3 - \gamma_2)} \times \frac{\gamma_3(\gamma_2 - \gamma_3)}{2\gamma_2(2\gamma_2 - 1)}\right] \times \frac{12(1 - \gamma_2)(1 - \gamma_3)}{6\gamma_2\gamma_3 - 4(\gamma_2 + \gamma_3) + 3}$$

$$\alpha_{42} = \frac{(1 - \gamma_2)\{2(1 - \gamma_3)(1 - 2\gamma_3) - (\gamma_2 - \gamma_3)\}}{2\gamma_2(\gamma_2 - \gamma_3)\{6\gamma_2\gamma_3 - 4(\gamma_2 + \gamma_3) + 3\}}$$

Let $\gamma_2 \neq 0,1$, $\gamma_3 \neq 0,1$ $\gamma_2 \neq \gamma_3, \gamma_2 \neq \frac{1}{2}$

By choosing two free parameters, $\gamma_2 = \frac{1}{3}$ *and* $\gamma_3 = \frac{2}{3}$

Substituting these values into β_4, β_3 *and* β_2 we have: $\beta_4 = \frac{6\left(\frac{1}{3}\right)\left(\frac{2}{3}\right) - 4\left(\frac{2}{3} + \frac{1}{3}\right) + 3}{12\left(1 - \frac{1}{3}\right)\left(1 - \frac{2}{3}\right)} = \frac{\frac{4}{3} - 1}{\frac{8}{3}} = \frac{1}{8}$

$$\beta_3 = \frac{1 - 2\left(\frac{1}{3}\right) + 3}{12\left(\frac{2}{3}\right)\left(1 - \frac{2}{3}\right)\left(\frac{2}{3} - \frac{1}{3}\right)} = \frac{\frac{1}{3}}{\frac{8}{9}} = \frac{3}{8}$$

$$\beta_2 = \frac{1 - 2\left(\frac{2}{3}\right)}{12\left(\frac{1}{3}\right)\left(1 - \frac{1}{3}\right)\left(\frac{1}{3} - \frac{2}{3}\right)} = \frac{-\frac{1}{3}}{-\frac{8}{9}} = \frac{3}{8}$$

Using equation (1) we get

$$\beta_1 + \beta_2 + \beta_3 + \beta_4 = 1$$

$\Rightarrow \beta_1 = 1 - \beta_2 - \beta_3 - \beta_4 = \frac{1}{8}$

$$k_1 = f[x_n, y_n]$$
$$k_2 = f[x_n + \gamma_2 h, y_n + h\alpha_{21} k_1] = f\left[x_n + \frac{h}{3}, y_n + \frac{hk_1}{3}\right]$$
$$k_3 = f[x_n + \gamma_3 h, y_n + h(\alpha_{31} k_1 + \alpha_{32} k_2)] = f\left[x_n + \frac{2h}{3}, y_n + h(-\frac{k_1}{3} + k_2)\right]$$

Also $\gamma_2 = \alpha_{21} = \frac{1}{3}$

Using equation (ii) when $j = 3$,

$$\beta_4 \alpha_{43} = \beta_3(1 - \gamma_3)$$
$$\alpha_{43} = \frac{\beta_3(1-\gamma_3)}{\beta_4} = \frac{3}{8} \times \left(1 - \frac{2}{3}\right) \times \frac{8}{1} = 1$$

Also

$$\alpha_{42} = \frac{(1-\gamma_2)\{2(1-\gamma_3)(1-2\gamma_3) - (\gamma_2 - \gamma_3)\}}{2\gamma_2(\gamma_2 - \gamma_3)\{6\gamma_2\gamma_3 - 4(\gamma_2 + \gamma_3) + 3\}} = -1$$

Using equation (2) we can obtain γ_4 as

$$\beta_4 \gamma_4 = \frac{1}{2} - \beta_2\gamma_2 - \beta_3\gamma_3$$

$\Rightarrow \gamma_4 = \frac{\frac{1}{2} - \beta_2\gamma_2 - \beta_3\gamma_3}{\frac{1}{8}} = 1$

Hence $\gamma_4 = \alpha_{41} + \alpha_{42} + \alpha_{43}$

$\Rightarrow \alpha_{41} = \gamma_4 - \alpha_{42} - \alpha_{43} = 1$

also

$$\alpha_{42} = \frac{\gamma_3(\gamma_2 - \gamma_3)}{2\gamma_2(2\gamma_2 - 1)} = 1$$

From $\gamma_3 = \alpha_{31} + \alpha_{32}$

$\Rightarrow \alpha_{31} = \gamma_3 - \alpha_{32} = -\frac{1}{3}$

Finally , we know that $\gamma_1 = \alpha_{11} = 0$

Therefore, we have determined all the unknowns in the method and can be written in Butcher's Tableu as

0	0	0	0	0
$\frac{1}{3}$	$\frac{1}{3}$	0	0	0
$\frac{2}{3}$	$\frac{-1}{3}$	1	0	0
	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Which has the form

$$y_{n+1} = y_n + \frac{h}{8}(k_1 + 3k_2 + 3k_3 + k_4)$$

Where

$$k_4 = f[x_n + \gamma_4 h, y_n + h(\alpha_{41} k_1 + \alpha_{42} k_2 + \alpha_{43} k_3] = f[x_n + h, y_n + h(k_1 - k_2 + k_3)]$$

Rk Neural Network

The literature combining numerical analysis of fourth order Runge-KuttaMethod and NNs is limited. Lagaris *et al.* [11] presented a Neural-Network approach of solving fourth order Runge-Kutta Method, but they do not give comparisons with the traditional multistep or RK methods. Multistep methods depending directly and linearly on a set of points give extremely accurate results. In [11] ten points are used and it seems theoretically difficult to compete multistep methods with minimizations requiring repeated calls of and evaluations or even inversions of Jacobians. Recent literature has answered for the most of the claimed there drawbacks of discrete methods. For example RK can be combined with continuous [13] or highly differentiable solution [12]. Perhaps their technique is promising in parallel computers or stiff systems where an ordinary differential equation has to be solved anyway. RecentlyWang and Lin [14] proposed the so called RK NNs. Their approach is from system identification point of view and they are interested in estimating the function by an NN. They used a classical RK method [10] of fourth order with constant step size because it is easier to prove some theoretical results. From practical consideration we might observe better results when using newer higher order methods with variable step size implementation. Perhaps some modification is needed for the learning

algorithms reported there, since the simplification of dealing with scalar problems does not work for RK of ordersexceeding 3 [14, p. 173].
In this paper, we neither indent to solve fourth order Runge-KuttaMethod nor to verify the function. I am interested in deriving better RK pairs of a prescribed order using stages. Thus I introduce a feed forward NN consisted from hidden layers and each one contains neurons.

Analysis Of The Method

The stability polynomial is given by $R(h) = 1 + \bar{h}\,\beta^T\left(I - \bar{h}A\right)^{-1} e$ and it is required that $R(h) < 1$ for absolute stability. Now for the Runge kutta fourth order method,

$$y_{n+1} = y_n + \frac{h}{8}(k_1 + 3k_2 + 3k_3 + k_4)$$

The Butcher's tableu is

$$\begin{array}{c|cccc} 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 \\ \frac{2}{3} & \frac{-1}{3} & 1 & 0 & 0 \\ \hline & \frac{1}{8} & \frac{3}{8} & \frac{3}{8} & \frac{1}{8} \end{array}$$

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 \\ -\frac{1}{3} & 1 & 0 & 0 \\ 1 & -1 & 1 & 0 \end{bmatrix}, \qquad I - \bar{h}A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\frac{\bar{h}}{3} & 1 & 0 & 0 \\ \frac{\bar{h}}{3} & -\bar{h} & 1 & 0 \\ \bar{h} & \bar{h} & -\bar{h} & 1 \end{bmatrix}$$

$$\bar{h}\beta^T = \bar{h}\left(\frac{1}{8}\frac{3}{8}\frac{3}{8}\frac{1}{8}\right) = \left(\frac{\bar{h}}{8}\frac{3\bar{h}}{8}\frac{3\bar{h}}{8}\frac{\bar{h}}{8}\right)$$

$$R(\bar{h}) = 1 + \bar{h}\beta^T\left(I - \bar{h}A\right)^{-1} e = 1 + \left(\frac{\bar{h}}{8}\frac{3\bar{h}}{8}\frac{3\bar{h}}{8}\frac{\bar{h}}{8}\right) \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\frac{\bar{h}}{3} & 1 & 0 & 0 \\ \frac{\bar{h}}{3} & -\bar{h} & 1 & 0 \\ \bar{h} & \bar{h} & -\bar{h} & 1 \end{bmatrix}^{-1} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

$$= 1 + \left(\frac{\bar{h}}{8}\frac{3\bar{h}}{8}\frac{3\bar{h}}{8}\frac{\bar{h}}{8}\right) \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{\bar{h}}{3} & 1 & 0 & 0 \\ -\frac{\bar{h}}{3} + \frac{\bar{h}^2}{3} & \bar{h} & 1 & 0 \\ \bar{h} - \frac{\overline{2h}^2}{3} - \frac{\bar{h}^3}{3} & \bar{h} + \bar{h}^2 & \bar{h} & 1 \end{bmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

$$= 1 + \begin{bmatrix} \frac{\bar{h}}{8} + \frac{\bar{h}^2}{8} + \frac{\overline{3h}}{8}\left(-\frac{\bar{h}}{3} + \frac{\bar{h}^2}{3}\right) + \frac{\bar{h}}{8}\left(\bar{h} - \frac{\overline{2h}^2}{3} + \frac{\bar{h}^3}{3}\right) \\ \frac{\overline{3h}}{8} + \frac{\overline{3h}^2}{8} + \frac{\bar{h}}{8}\left(-\bar{h} + \bar{h}^2\right) \\ \frac{\overline{3h}}{8} + \frac{\bar{h}^2}{8} \\ \frac{\bar{h}}{8} \end{bmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

$$= 1 + \begin{bmatrix} \frac{\bar{h}}{8} + \frac{\bar{h}^2}{8} - \frac{3\bar{h}^2}{24} + \frac{3\bar{h}^3}{24} + \frac{\bar{h}^2}{8} - \frac{\overline{2h}^3}{24} + \frac{\bar{h}^4}{24} \\ \frac{\overline{3h}}{8} + \frac{\overline{3h}^2}{8} - \frac{\bar{h}^2}{8} + \frac{\bar{h}^3}{8} \\ \frac{\overline{3h}}{8} + \frac{\bar{h}^2}{8} \\ \frac{\bar{h}}{8} \end{bmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = 1 + \begin{bmatrix} \frac{\bar{h}}{8} + \frac{\bar{h}^2}{8} + \frac{\bar{h}^3}{24} + \frac{\bar{h}^4}{24} \\ \frac{\overline{3h}}{8} + \frac{\bar{h}^2}{4} + \frac{\bar{h}^3}{8} \\ \frac{\overline{3h}}{8} + \frac{\bar{h}^2}{8} \\ \frac{\bar{h}}{8} \end{bmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

$$= 1 + \frac{\bar{h}}{8} + \frac{\bar{h}^2}{8} + \frac{\bar{h}^3}{24} + \frac{\bar{h}^4}{24} + \frac{\overline{3h}}{8} + \frac{\bar{h}^2}{4} + \frac{\bar{h}^3}{8} + \frac{\overline{3h}}{8} + \frac{\bar{h}^2}{8} + \frac{\bar{h}}{8} = 1 + \bar{h} + \frac{\bar{h}^2}{2} + \frac{\bar{h}^3}{6} + \frac{\bar{h}^4}{24}$$

For absolute stability $-1 < \left|1 + \bar{h} + \frac{\bar{h}^2}{2} + \frac{\bar{h}^3}{6} + \frac{\bar{h}^4}{24}\right| < 1$

Taking the R.H.S

$$\left|1 + \bar{h} + \frac{\bar{h}^2}{2} + \frac{\bar{h}^3}{6} + \frac{\bar{h}^4}{24}\right| < 1$$

$$\bar{h} + \frac{\bar{h}^2}{2} + \frac{\bar{h}^3}{6} + \frac{\bar{h}^4}{24} < 0$$

Consider 3 cases as in [1]

Case 1:

When λ is real and $\lambda < 0$,

The roots are -2.785 and 0

Hence the stability internal is $h \in (-2.7, 0)$.

Case 2:

When λh is pure and imaginary,

We set $\lambda = iy$ in the stability polynomial to get

$$\left|1 + i(yh) - \frac{(yh)^2}{2} - i\frac{(yh)^3}{6} + \frac{(yh)^4}{24}\right| < 1$$

$\Rightarrow$

$$\left|1 + i(yh) - \frac{(yh)^2}{2} - i\frac{(yh)^3}{6} + \frac{(yh)^4}{24}\right| < 1$$

Let $t = yh$ and take magnitude

$\Rightarrow \left(1 - \frac{t^2}{2} + \frac{t^4}{24}\right)^2 + \left(t - \frac{t^3}{6}\right) < 1$

$$\left(1 - \frac{t^2}{2} + \frac{t^4}{24} - \frac{t^2}{2} + \frac{t^4}{4} - \frac{t^6}{48} + \frac{t^4}{24} - \frac{t^6}{48} + \frac{t^8}{578}\right) + \left(t^2 - \frac{t^4}{6} - \frac{t^4}{6} + \frac{t^6}{36}\right) < 1$$

Simplifying, we get

$$1 - \frac{t^6}{72} + \frac{t^8}{576} < 1 \Rightarrow \frac{t^6}{72} + \frac{t^8}{576} < 0$$

The equation is satisfied for $|t| < 2.8$

i.e.$|t| < 2\sqrt{2}$

Hence the stability interval is $0 < \bar{h} < 2\sqrt{2}$ i.e.$\bar{h} \in \left(0, 2\sqrt{2}\right)$

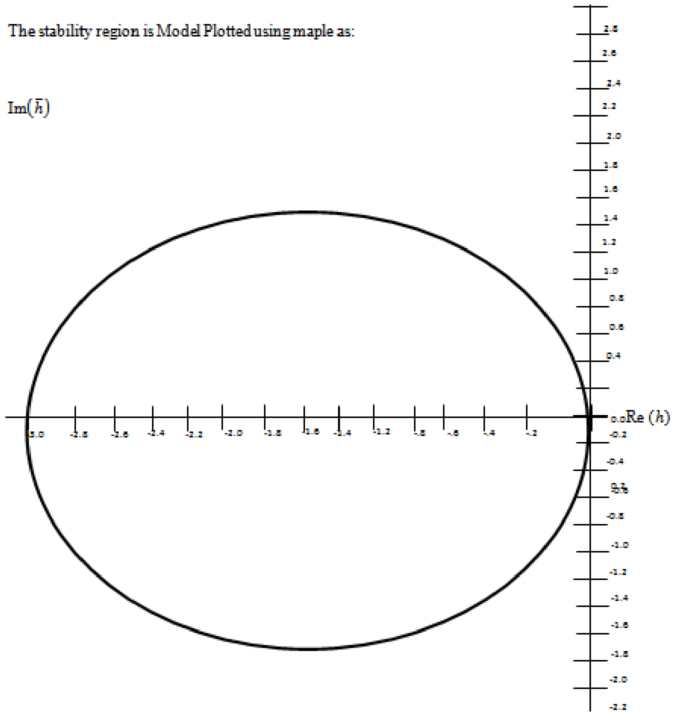
Case 3:

whenλ is complex with $\mathrm{Re}(\lambda) > 0$, we let $x + iy$ in

$$\left|1 + (\lambda h) + \frac{(\lambda h)^2}{2} + \frac{(\lambda h)^3}{6} + \frac{(\lambda h)^4}{24}\right| < 1$$

and plot the boundary of the region by plotting the real and imaginary parts.

Figure



III. Conclusion

In this paper,the use of fourth order Runge-Kutta Method in NeuralNetworks for ordinary differential equation to exist the value. We are interested in deriving better RK pairs of a prescribed order using cases. Thus we introduce a feedforward NN consisted from hidden layers and each one contains neurons. By simplifying ordinary differential equationin NeuralNetworks for the derivation and analysis of the fourth order Runge-Kutta method. It exists in the above furnishedModel Plot under its stability region. We also reduce the complexity of the method by proposing a step deviation approach for easy reference to students.

References

[1]. M.K. Jain,S .R. K.Iyengar, R. K. JainNumerical Methods for Scientific and Engineering computing,(2007).
[2]. J.D.Lambert , NumericalMethods for Ordinary Differential Systems, the initial value problem, John Wiley & Sons Ltd. , (1991).
[3]. J.C. Butcher, Numerical methods for Ordinary Differential Equations, John Wiley & Sons Ltd., (2003)
[4]. John R. Dorman , Numerical Methods for differential Equations, a Computational Approach, CRC Press, Inc. , (1996).
[5]. J .D. Lambert, Computational Methods in Ordinary Differential Equations, John Wiley & Sons Ltd., (1973).
[6]. Fudzia Ismail, Lecture Notes on Numerical Methods, University Putra Malaysia. (2010).
[7]. G. Byrne and Hindmarsh , RK Methods prove popular at IMA Conference on Numerical ODE's ,SIAM News, (1990),pp.14-15.
[8]. Lawrence F.Shamoine,Interpolation for Runge-Kutta Methods. SIAM Journal of numerical analysis.(1985),pp.1014-1027.
[9]. Fudzia Ismail, Lecture Notes on Numerical Methods, University Putra Malaysia. , (2010)
[10]. W.Kutta, "Beitrag zur nahenrungsweisen integration totaler differentialgleichungen,"ZAMP, vol. 46, 1901, pp. 435-453.
[11]. I. E. Lagaris, A. Likas, and D. I. Fotiadis, "Artificial neural networks for solving ordinary and partial differential equations," IEEE Trans. NeuralNetworks, vol. 9, 1998 pp. 987–1000.
[12]. S. N. Papakostas and C. Tsitouras, "Highly continuous interpolants for one-step ODE solvers and their application to Runge-Kutta methods,"SIAM J. Numer. Anal., vol. 34, 1997, pp. 22–47.
[13]. C. Tsitouras and G. Papageorgiou, "Runge Kutta interpolants based on values from two successive integration steps," Computing, vol. 43, 1990, pp. 255–266.
[14]. Y.J. Wang and C.T. Lin, "Runge-Kutta neural network for identification of dynamical systems in high accuracy," IEEE Trans. Neural Networks, vol. 9, 1998, pp. 294–307.