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Fuzzy Vertex Graceful Labeling On Wheel And Fan Graphs

R.Jebesty Shajila[1], S.Vimala[2],
[1]*Ph D Research Scholar, Mother Teresa Women's University, Kodaikanal*
[2]*Assistant Professor, Mother Teresa Women's University, Kodaikanal*,

Abstract:A labeling graph G which can be gracefully numbered is said to be graceful. A graph which admits a fuzzy graceful labeling is called a fuzzy graceful graph. In this paper we introduced fuzzy vertex gracefulness and discussed to wheel graphs and fan graphs.
***Key words**: Graceful labeling, Fuzzy graceful labelling*

I. Introduction

Zadeh introduced the fuzzy set as a class of object with a continuum of grades of membership. In contrast to classical crisp sets where a set is defined by either membership or non-membership, the fuzzy approach relates to a grade of membership between [0,1], defined in terms of the membership function of a fuzzy number. Fuzzy relation on a set was first defined by Zadeh in 1965, the first definition of a fuzzy graph was introduced by Kaufmann in 1973 and the structure of fuzzy graphs developed by Azriel Rosenfeld in 1975. Fuzzy graphs have many more applications in modeling real time systems where the level of information inherent in the system varies with different levels of precision.The concept of a graceful labeling has been introduced by Rosa in 1966.This note is a further contribution on fuzzy labeling. Fuzzy labeling for fuzzy wheel graph and fuzzy fan graph are called a fuzzy labeling wheel graph and fuzzy labeling fan graph respectively.

II. Preliminaries and Main Results

Definition:1

Let U and V be two sets. Then ρ is said to be a fuzzy relation from U into V if ρ is a fuzzy set of UxV.

Definition: 2

A fuzzy graph G =(σ, μ) is a pair of functions $\sigma : V \rightarrow [0,1]$ and $\mu : VxV \rightarrow [0,1]$, where for all u,v ϵ V, we have $\mu(u,v) \leq \sigma(u) \wedge \sigma(v)$.

Definition: 3

A labeling of a graph is an assignment of values to the vertices and edges of a graph.

Definition:4

A graceful labeling of a graph G with q edges is an injection f: $V(G) \rightarrow \{0,1,2,\ldots,q\}$ such that when each edge xy ϵ E(G) is assigned the label $|f(x) - f(y)|$, all of the edge labels are distinct.

Definition:5

A graph G =(σ, μ) is said to be a fuzzy labeling graph if $\sigma : V \rightarrow [0,1]$ and $\mu : VxV \rightarrow [0,1]$ is bijective such that the membership value of edges and vertices are distinct and $\mu(u,v) < \sigma(u) \wedge \sigma(v)$ for all u,v ϵ V.

Definition: 6

In the mathematical discipline of graph theory, a wheel graph Wn is a graph with n vertices ($n \geq 4$) formed by connecting a single vertex to all vertices of an n-1 cycle.

Definition :7

A wheel graph with fuzzy labeling is called a fuzzy wheel graph.

Definition:8

A fan graph $F_{m,n}$ is defined as the graph join $\overline{Km}$ +P_n

where $\overline{Km}$ is the empty graph on m nodes and P_n is the path graph on n nodes.

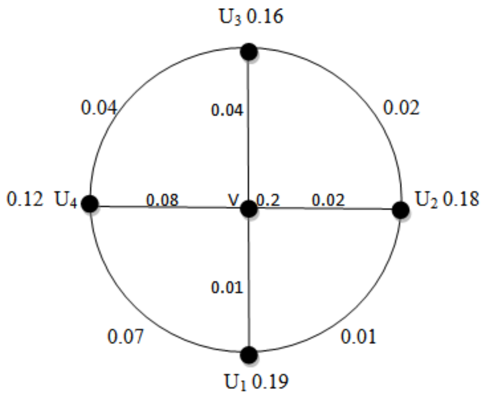
The case m=1 corresponds to the usual fan graphs, while m=2 corresponds to the double fan graphs, etc.,

Definition : 9

In a fuzzy wheel graph if all vertices are distinct then it is called fuzzy vertex graceful labeling wheel graph.

A wheel in a fuzzy graph consists of two node sets V and U with |V| = 1and |U| > 1, such that $\mu(v,u_i) > 0$, where i = 1 to n-1 and $\mu(u_i,u_{i+1}) > 0$ where i = 1 to n-2.

Example:



Fuzzy vertex graceful labeling wheel graph W_5

Definition :10

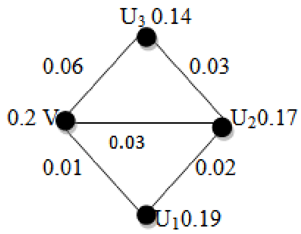
In a fuzzy fan graph if all vertices are distinct then it is called fuzzy vertex graceful labeling fan graph.

A fuzzy graceful fan graph $F_{1,n}$ consists of two node sets F and F_n with $|F| = 1$ and $|F_n| > 1$, such that $\mu(F,F_i) > 0$, where $i = 1$ to n and $\mu(F_i,F_{i+1}) > 0$ where $i = 1$ to n-1.

Note:

In the fuzzy fan graph $F_{1,n}$ the vertex labeling $\sigma:F\to[0,1]$ satisfies the condition that if the values of F starts only from n-1/10 , n/10 , n+1/10 , etc., then the fan graph is a fuzzy vertex graceful labeling fan graph.

Example:



Fuzzy vertex graceful labeling fan graph $F_{1,3}$

Fuzzy vertex graceful labeling fan graph $F_{1,3}$

Preposition: 1

For some $n\geq4$, the wheel graph Wn is a fuzzy vertex graceful wheel graph, where the central vertex $\sigma:v\to[0,1]$ and the vertices from the outer cycle $\sigma:u_i\to[0,1]$ and $\mu(v,u_i) = (0.01)x2^{i-1}$, where i = 1 to n-1.

Proof:

A wheel graph W_n is a graph with n vertices if only $n\geq4$.

A wheel in a fuzzy graph consists of two node sets V and U with $|V| = 1$ and $|U| > 1$, such that $\mu(v,u_i) > 0$, where $i = 1$ to n-1 and $\mu(u_i,u_{i+1}) > 0$ where $i = 1$ to n-2.

In the wheel graph v is the central vertex , u_i denotes the vertices in the outer cycle.

Here when $\sigma:v\to[0,1]$ and $\sigma:u_i\to[0,1]$ defined by

$\sigma(u_i) = \sigma(v) - \mu(v,u_i)$, where $\mu(v,u_i) = (0.01)x2^{i-1}$, i = 1 to n-1 and

$\mu(u_i,u_{i+1}) = \mu(v,u_i)$ where i = 1 to n-2.

(or)

$\mu(u_{n-2},u_{n-1}) = \mu(v,u_{n-2})$

but $\mu(u_{n-1},u_1) = \mu(v,u_{n-1}) - \mu(v,u_1)$

Example:1

Case (i) when $\sigma(v)$ starts from n-3/10 to n+3/10

Here when $\sigma:v\to[0,1]$ and $\sigma:u_i\to[0,1]$,

$\sigma(u_1) = \sigma(v) - 0.01$

$\sigma(u_2) = \sigma(v) - 0.02$

$\sigma(u_3) = \sigma(v) - 0.04$

$\sigma(u_4) = \sigma(v) - 0.08$ etc.,

$\sigma(u_i) = \sigma(v) - (0.01)x2^{i-1}$, where i = 1 to n-1.

ie, $\sigma(u_i) = \sigma(v) - \mu(v,u_i)$, where $\mu(v,u_i) = (0.01)x2^{i-1}$, i = 1 to n-1.

Also

$\mu(u_1,u_2) = \mu(v,u_1)$

$\mu(u_2,u_3) = \mu(v,u_2)$

$\mu(u_3,u_4) = \mu(v,u_3)$, etc.,

$\mu(u_i,u_{i+1}) = \mu(v,u_i)$ where $i = 1$ to $n-2$.

(or)

$\mu(u_{n-2},u_{n-1}) = \mu(v,u_{n-2})$

While $\mu(u_{n-1},u_1) = \mu(v,u_{n-1}) – 0.01$

ie, $\mu(u_{n-1},u_1) = \mu(v,u_{n-1}) – \mu(v,u_1)$

Case (ii) when $\sigma(v)$ starts from n-3/100 to n+3/100

When σ:v⟶[0,1] and σ:u_i⟶[0,1]

$\sigma(u_1) = \sigma(v) – 0.001$

$\sigma(u_2) = \sigma(v) – 0.002$

$\sigma(u_3) = \sigma(v) – 0.004$

$\sigma(u_4) = \sigma(v) – 0.008$ etc.,

$\sigma(u_i) = \sigma(v) – (0.001)\text{x}2^{i-1}$, where i = 1 to n-1.

ie, $\sigma(u_i) = \sigma(v) – \mu(v,u_i)$, where $\mu(v,u_i) ==(0.001)\text{x}2^{i-1}$, i = 1 to n-1.

Also

$\mu(u_1,u_2) = \mu(v,u_1)$

$\mu(u_2,u_3) = \mu(v,u_2)$

$\mu(u_3,u_4) = \mu(v,u_3)$, etc.,

$\mu(u_i,u_{i+1}) = \mu(v,u_i)$ where $i = 1$ to $n-2$.

(or)

$\mu(u_{n-2},u_{n-1}) = \mu(v,u_{n-2})$

While $\mu(u_{n-1},u_1) = \mu(v,u_{n-1}) – 0.001$

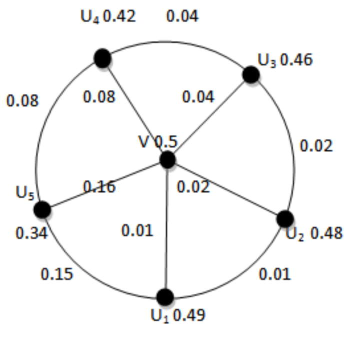
ie, $\mu(u_{n-1},u_1) = \mu(v,u_{n-1}) – \mu(v,u_1)$

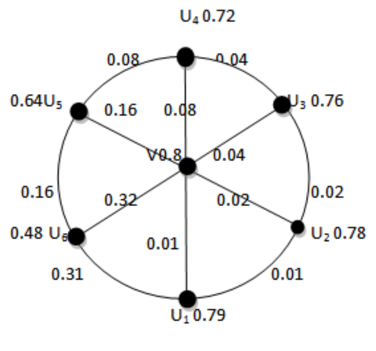
Like this for any values of σ: v⟶ [0, 1], the labeling of all vertices in the outer cycle u_i are distinct where i = 1 to n-1.

ie, $\sigma(u_i) = \sigma(v) – \mu(v,u_i)$, where i = 1 to n-1.

Therefore according to this condition the wheel graph W_n is a fuzzy vertex graceful wheel graph.

Examples:2





Fuzzy vertex graceful labeling wheel graph W_6 Fuzzy vertex graceful labeling wheel graph W_7

Preposition:2

Every fuzzy fan graph $F_{1,n}$ is a fuzzy vertex graceful fan graph.

Proof:

Consider the usual fan graph $F_{1,n}$ when m=1.

If we take F as the central vertex (m=1) and F_n be the other vertices in the fan graph with n nodes.

A fuzzy graceful fan graph $F_{1,n}$ consists of two node sets F and F_n with $|F| = 1$and $|F_n| > 1$, such that $\mu(F,F_i) > 0$, where $i = 1$ to n and $\mu(F_i,F_{i+1}) > 0$ where $i = 1$ to $n-1$.

If the fuzzy fan graph is a vertex graceful then

$\mu(F,F_{n+1}) - \mu(F,F_n) = \mu(F_n,F_{n+1})$, where n= 1,2,3,...

Example : 3

Case (i) The value of F starts from n-1/10

Here

$\mu(F,F_1) = 0.01$

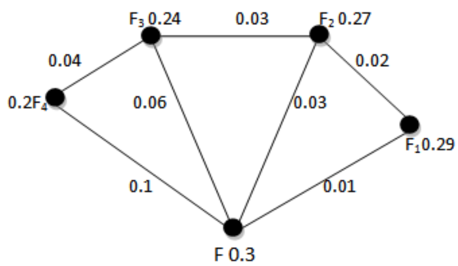
$\mu(F,F_2) = 0.03 = \mu(F,F_1) + 0.02$

$\mu(F,F_3) = 0.06 = \mu(F,F_2) + 0.03$
$\mu(F,F_4) = 0.1 = \mu(F,F_3) + 0.04$ etc.,
Therefore $\mu(F,F_{n+1}) = \mu(F,F_n) + (0.01)(n+1)$, where n= 1,2,3,...
ie, $\mu(F,F_{n+1}) - \mu(F,F_n) = (0.01)(n+1)$, where n= 1,2,3,...
Similarly,
$\mu(F_1,F_2) = 0.02$
$\mu(F_2,F_3) = 0.03$
$\mu(F_3,F_4) = 0.04$ etc.,
Therefore $\mu(F_n,F_{n+1}) =(0.01)(n+1)$, where n= 1,2,3,...
So $\mu(F,F_{n+1}) - \mu(F,F_n) = \mu(F_n,F_{n+1})$, where n= 1,2,3,...
Case (ii) The value of F starts from n-1/100
Here
$\mu(F,F_1) = 0.001$
$\mu(F,F_2) = 0.003 = \mu(F,F_1) + 0.002$
$\mu(F,F_3) = 0.006 = \mu(F,F_2) + 0.003$
$\mu(F,F_4) = 0.01 = \mu(F,F_3) + 0.004$ etc.,
Therefore $\mu(F,F_{n+1}) = \mu(F,F_n) + (0.001)(n+1)$, where n= 1,2,3,...
ie, $\mu(F,F_{n+1}) - \mu(F,F_n) = (0.001)(n+1)$, where n= 1,2,3,...
Similarly,
$\mu(F_1,F_2) = 0.002$
$\mu(F_2,F_3) = 0.003$
$\mu(F_3,F_4) = 0.004$ etc.,
Therefore $\mu(F_n,F_{n+1}) =(0.001)(n+1)$, where n= 1,2,3,...
So $\mu(F,F_{n+1}) - \mu(F,F_n) = \mu(F_n,F_{n+1})$, where n= 1,2,3,...

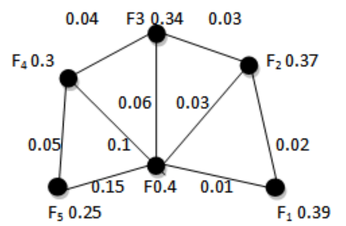
Since all edge values $\mu(F,F_i) > 0$, where i = 1 to n and $\mu(F_i,F_{i+1}) > 0$ where i = 1 to n-1 and which are less than the values of vertices, this fan graph is a fuzzy graceful fan graph as well as since all vertices are distinct, this graph is a fuzzy vertex graceful labeling fan graph.

According to these conditions we can come to the conclusion every fan graph $F_{1,n}$ is a fuzzy vertex graceful fan graph.

Examples:4



Fuzzy vertex graceful labeling fan graph $F_{1,4}$



Fuzzy vertex graceful labeling fan graph $F_{1,5}$

III. Concluding remarks

Fuzzy graph theory is finding an increasing number of applications in modelling real time systems where the level of information inherent in the system varies with different levels of precision. Fuzzy models are becoming useful because of their aim in reducing the differences between the traditional numerical models used in engineering and sciences and the symbolic models used in export systems. In this paper, the concept of fuzzy graceful labeling and fuzzy vertex graceful wheel graph, fuzzy vertex graceful fan graph has been introduced.

We are extending on fuzzy vertex graceful labeling in double wheel and double fan.

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