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MHD Boundary Layer Flow and Heat Transfer of Nanofluids over a Nonlinear Stretching Sheet in a Porous Medium.

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Abstract: *The MHD laminar boundary layer flow with heat and mass transfer of an electrically conducting water based nanofluid over a nonlinear stretching sheet with viscous dissipation effect is investigated numerically. This is the extension of previous study on flow and heat transfer with porous medium is presented in this paper. The sheet is assumed to be permeable. The governing partial differential equations are transformed into coupled nonlinear ordinary differential equations by using suitable similarity transformations. The transformed equations are then solved numerically using the well known explicit finite difference scheme known as the Keller Box method. A detailed parametric study is performed to access the influence of the physical parameters on velocity, temperature and nanoparticle concentration, fraction profiles as well as the local skin-friction coefficient, local Nusselt number and the local Sherwood number and then, the results are presented in both graphical and tabular forms.*
Keywords: *MHD, Nanofluids, Viscous dissipation, Stretching Sheet, Porous Medium.*

I. Introduction:

Heat transfer in porous media has numerous engineering applications such as ground water pollution, geothermal energy recovery, flow filtering media, thermal energy storage and crude oil extraction [1].

A nanofluid is a fluid which contains nanometer-sized solid particles. Nanometer-size solid particles have unique chemical and physical properties. Since suspending nanometer-size particles to the conventional heat transfer fluids leads a better heat transfer, nano fluids are proposed to be employed in several applications such as transportation, nuclear reactors and electronics [2].

Evaluating natural convection heat transfer from a horizontal plate embedded in a saturated porous medium has been conducted in some literatures. Cheng and Chang [3] investigated buoyancy induced flows in a saturated porous medium adjacent to impermeable horizontal surfaces. Nield and Bejan [4] investigated natural convection heat transfer from a horizontal plate in porous medium.

Nanofluids are used to enhance the thermal conductivity of base fluids like water, ethylene glycol, propylene glycol etc. They have several engineering and biomedical applications in cooling, cancer therapy and process industries. The enhancement of thermal conductivity of conventional heat transfer fluids through suspensions of solid particles is a relatively recent development in engineering technology. The resulting effect of these suspensions is to increase the coefficient of heat transfer. The suspended particles are able to increase the thermal conductivity and heat transfer performance since the thermal conductivity of solid metals is higher than base fluids. Major advantages of nanofluids are that they are more stable, have sufficient viscosity and better wetting, spreading and dispersion properties on solid surface even for modest nanoparticle concentrations [5]

The Keller-Box method introduced by Keller [6] is one of the best numerical method basically it's a mixed finite volume method which consists in taking the average of a conservation law and of the associated constitutive law at the level of the same mesh cell. Sarif [7] worked the numerical solution of the steady boundary layer flow and heat transfer over a stretching sheet with Newtonian heating by using Keller box method.

This paper provides the solution to the problem of flow and heat transfer of a nanofluid over a nonliner stretching porous sheet by considering the effect of radiation parameter by adopting the Keller Box method.

II. Mathematical Formulation

The Present steady of two-dimensional, incompressible viscous flow of a water-based nanofluid past over a nonlinear stretching surface. The sheet is extended with velocity $u_w(x)$=ax^n with fixed origin location, where n is a nonlinear stretching parameter, is a constant and x is the coordinate measured along the stretching surface. The nanofluid flows at y=0, where y is the coordinate normal to the surface. The fluid is electrically conducted due to an applied magnetic field $B(x)$normal to the stretching sheet. The magnetic Reynolds number is assumed small and so, the induced magnetic field can be considered to be negligible. The wall temperature T_w

and the nanoparticle fraction C_w are assumed constant at the stretching surface. When y tends to infinity, the ambient values of temperature and nanoparticle fraction are denoted by T_∞ and C_∞ respectively. The considered physical system is of importance in modern nano-technological fabrication and thermal materials processing. It is important to note that the constant temperature and nanoparticle fraction of the stretching surface T_w and C_w are assumed to be greater than the ambient temperature and nanoparticle fraction T_∞, C_∞, respectively. The coordinate system and the flow model are shown in Fig. 1. The governing equations of momentum, thermal energy and nanoparticles equations.

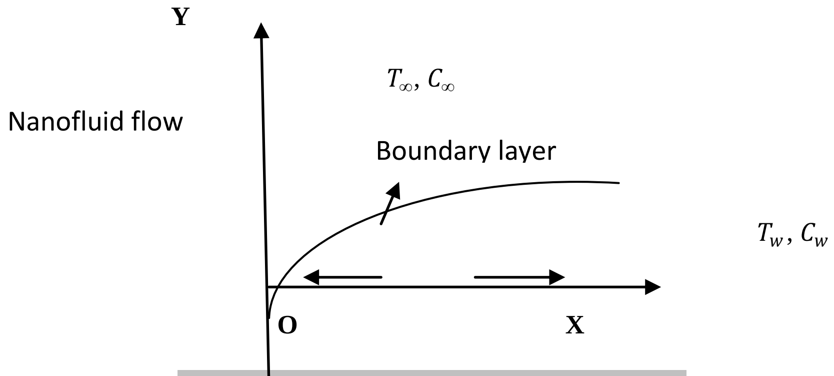

Fig 1: Physical Modal and coordinate system

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \ , \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B^2(x)}{\rho_f}u-\frac{\nu}{k}u, , \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\tau\left\{D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}\times(\frac{\partial T}{\partial y})^2\right\}+\frac{\upsilon}{c_p}(\frac{\partial u}{\partial y})^2-\frac{1}{(\rho_{cp})_f}\frac{\partial q_r}{\partial y} \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\left(\frac{D_T}{T_\infty}\right)\frac{\partial^2 T}{\partial y^2} \tag{4}$$

The boundary conditions for the velocity, temperature and nanoparticle fraction.

y = 0 : $u_w = ax^n$, v=0, T= T_w, C= C_w, (5)

y = ∞ : u = 0, v = 0, T= T_∞, C= C_∞, (6)

Here, u and v are the velocity components along the x and y axes, respectively. $\alpha = k/(\rho c)_f$ is the thermal diffusivity, σ is electrical conductivity, ν is the kinematic viscosity, ρ_f is the density
Of the base fluid, D_B is the Brownian diffusion coefficient and D_T is the thermo phoresis diffusion coefficient. $\tau = (\rho C)_p/(\rho C)_f$ is the ratio between the effective heat capacity of the nanoparticle material and heat capacity of the fluid, c is the volumetric volume coefficient, ρ_p is the density of the particles, and C is rescaled nanoparticle volume fraction. We assume that the variable magnetic field B(x) is of the form B(x) = $B_0 x^{(n-1)/2}$, K is permeability of the porous medium. Using Rossel and approximation for radiation we can write

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{6 a}$$

where k^* is the absorption coefficient, σ^* is the Stefan-Boltzman constant, Assuming the temperature difference within the flow is such that T^4 may be expanded in a Taylor series about T_∞ and neglecting higher orders we get $T^4 \approx 4T_\infty^3 T - 3T_\infty^4$.

Hence Eq. (6 a), becomes

$$\frac{\partial q_r}{\partial y} = -\frac{16\sigma^* T_\infty^3}{3(\rho_{cp})_f k^*}\frac{\partial^2 T}{\partial y^2} \tag{6 b}$$

$$\eta = y\sqrt{\frac{a(n+1)}{2v}}x^{\frac{(n-1)}{2}}, \quad u = ax^n f'(\eta), \qquad v = -\sqrt{\frac{a(n+1)}{2}}x^{\frac{(n-1)}{2}}\left(f(\eta) + \frac{n-1}{n+1}\eta f'(\eta) \right),$$

$$\theta(\eta) = (T - T_\infty) \big/ (T_W - T_\infty), \; \phi((\eta) = (C - C_\infty) \big/ (C_W - C_\infty). \tag{7}$$

Where ψ represents stream functions and is defined as $u = \frac{\partial\psi}{\partial y}$, $v = \frac{-\partial\psi}{\partial y}$ so that equation (1) is satisfied identical. The governing eqs (2) – (4) or reduced by eqs (7).

$$f''' + ff'' - \left(\frac{2n}{n+1}\right)f'^2 - (M+G)f' = 0 \tag{8}$$

$$(1 + \frac{4}{3}R)\theta'' + \Pr\left(f\theta' + Nb\,\theta'\phi' + Nt\,\theta'^2 + Ecf''^2\right) = 0 \tag{9}$$

$$\phi'' + Lef\phi' + \frac{Nt}{Nb}\theta'' = 0 \tag{10}$$

The transformed boundary conditions

$$f(0) = 0, \quad f'(0) = 1, \; \theta(0) = 1, \quad \phi(0) = 1,$$

$$f'(\infty) = 0, \; \theta(\infty) = 0, \quad \phi(\infty) = 0 \tag{11}$$

Where primes denote differentiation with respect to η, the involved physical parameters are defined as:

$$\Pr = \frac{v}{a}, \quad Le = \frac{v}{D_B}, \quad Nb = \frac{(\rho c)_p D_B (C_w - C_\infty)}{(\rho c)_f v}, \quad Nt = \frac{(\rho c)_p D_T (T_w - T_\infty)}{(\rho c)_f T_\infty v}, \quad M = \frac{2\sigma B_0^2}{a\rho f(n+1)},$$

$$Ec = \frac{u_w^2}{c_\rho (T_w - T_\infty)}, \; G = \frac{v}{k}, \; R = \frac{4\sigma^* T_\infty^3}{kk^*} \tag{12}$$

Here Pr, Le, Nb, Nt, M, Ec , and G denote the Prandtl number, the Lewis number, the Brownian motion parameter, the thermophoresis parameter, magnetic parameter, Eckert number and porous term respectively. This boundary value problem is reduced to the classical problem of flow and heat and mass transfer due to a stretching surface in a viscous fluid when n = 1 and Nb = Nt = 0 in eqs (9) and (10).

The quantities of practical interest, in this study, are the local skin friction C_{fx}, Nusselt number Nu_x and the Sherwood number Sh_x which are defined as

$$C_{fx} = \frac{\mu_f}{\rho u_w^2}\left(\frac{\partial u}{\partial y}\right)_{y=0}, \quad Nu_x = \frac{x q_w}{k(T_w - T_\infty)}, \quad Sh_x = \frac{x q_m}{D_B(C_w - C_\infty)} \tag{13}$$

Where k is the thermal conductivity of the nanofluid, and q_w, q_m are the heat and mass fluxes at the surface, respectively, given by

$$q_w = -\left[\frac{\partial T}{\partial y}\right]_{y=0}, \quad q_m = -D_B\left[\frac{\partial C}{\partial y}\right]_{y=0} \tag{14}$$

Substituting Eq (7) into Eqs (13) – (14), we obtain

$$\mathrm{Re}_x^{1/2} C_{fx} = \sqrt{\frac{n+1}{2}} f''(0), \qquad \mathrm{Re}_x^{-1/2} Nu_x = -\sqrt{\frac{n+1}{2}}\theta'(0), \qquad \mathrm{Re}_x^{-1/2} Sh_x = -\sqrt{\frac{n+1}{2}}\phi'(0),$$

Where $\mathrm{Re}_x = u_w \frac{x}{\nu}$ is the local Reynolds number.

III. Results And Discussion

The reduced Eqs. (8)–(11) are nonlinear and coupled, and thus their exact analytical solutions are not possible. They can be solved numerically using Keller – Box for different values of parameters such as magnetic parameter, Prandtl, Eckert and the Lewis numbers, the Brownian
motion parameter and the thermophoresis parameter. The effects of the emerging parameters on the dimensionless velocity, temperature, skin friction, the rates of heat and mass transfer are
Investigated.

The principal steps in using the Keller Box method are:

1) Reducing higher order ODEs (systems of ODEs) in to system of first order ODEs;
2) Writing the systems of first order ODEs into difference equations using central differencing scheme;
3) Linearizing the difference equations using Newton’s method and writing it in vector form;
4) Solving the system of equations using block eliminations method.

In order to solve the above differential equations numerically, we adopt Matlab software which is very efficient in using the well known Keller Box method.

To validate the present solution, comparisons have been made with previously published data in the literature for $-\theta'(0)$ and $-\phi'(0)$ in Table 1, and they are found to be in an excellent
Agreement. Effects of magnetic and viscous dissipation parameters on friction factor, Nusselt and Sherwood numbers are presented in Table 1, while keeping other parameter values preset from

Table 1, it is clear that the Nusselt number is a decreasing function of M, Ec, n Nt and Le, whereas Sherwood number is found to be a decreasing function of M, n and Nt, and an increasing function of Pr, Ec and Le. An increase in the Lewis number Le means that the fluid becomes more viscous. R, G an increase in the radiation parameter and Porous medium that the fluid becomes more viscous. Therefore, causes an increase in the rate of mass transfer. As expected, increasing the Lewis number reduces the rate of heat transfer. The values of the skin friction coefficient can be observed in an increasing manner for various values of M in Table 1

Table 1. Comparison of skin friction coefficient, Nusselt and Sherwood number for various values of Ec and M when Pr = 6.2; Le = 5; Nb = Nt = 0.1; n = 2; G=0;

Ec	M	Past Values			Present Values		
		$-f''(0)$	$-\theta'(0)$	$-\emptyset'(0)$	$-f''(0)$	$-\theta'(0)$	$-\emptyset'(0)$
0.0	0	1.1010	1.0671	1.0771	1.1014	1.0671	1.0768
0.1			0.8819	1.2234		0.8890	1.2233
0.2			0.7099	1.3707		0.7094	1.3708
0.3			0.5295	1.5191		0.5287	1.5194
0.5			0.1648	1.8193		0.1634	1.8200
0.0	0.5	1.3098	1.0436	1.0109	1.3099	1.0437	1.0109
0.1			0.8105	1.2060		0.8105	1.2060
0.2			0.5756	1.4027		0.5756	1.4028
0.3			0.3388	1.6011		0.3388	1.6012
0.5			0.1402	2.0028		0.1403	2.0029

Table 2 Showing results of $-f''(0), -\theta'(0), -\phi'(0)$ for the values of M, Pr, Nb, Nt, Ec, Le, n, R and G.

R	G	$-f''(0)$	$-\theta'(0)$	$-\emptyset'(0)$
0.0		1.3477	0.7960	1.2034
0.1			0.7842	1.2001
0.3			0.7549	1.2025
0.5			0.7236	1.2104
1.0			0.6501	1.2362
	0.0	1.3099	0.6613	1.2416
	0.1	1.3477	0.6501	1.2362
	0.3	1.4202	0.6286	1.2261
	0.5	1.4891	0.6082	1.2168
	1.0	1.6488	0.5610	1.1965

The effect of the magnetic parameter M is shown in Fig. 1. It is observed that the tangential velocity of the fluid decreases with the increase of the magnetic field number values. The decrease in the tangential velocity as the magnetic parameter M increases is because the presence of a magnetic field in an electrically conducting fluid introduces a force called the Lorentz force, which acts against the flow if the magnetic field is applied in the normal direction, as in the present study. This resistive force slows down the fluid velocity component as shown in Fig. 1

For different values of the magnetic parameter M the temperature profiles are plotted in Fig. 2. It is obvious that an increase in the Magnetic parameter M results in an increase in the temperature within the boundary layer.

Figs. 3 are prepared to show the influence of magnetic parameter M and nonlinear stretching parameter n on the dimensionless and nanoparticle concentration. The dimensionless

concentration profiles are clearly observed to be significantly enhanced with increasing magnetic parameter. As the Lorentz force is a resistive force which opposes the fluid motion, so heat is produced and as a result, the thermal boundary layer thickness and nanoparticle volume fraction boundary layer thickness become thicker for stronger magnetic field. From the same Fig. 3, it is also observed that the variation in the nanoparticle concentration due to nonlinear stretching parameter is slightly negligible for both positive and negative values of n.

For different values of the radiation parameter R the temperature profiles are plotted in Fig. 4. It is obvious that an increase in the radiation parameter R results in an increase in the temperature within the boundary layer.

Figure 5, show effect of porosity parameter G on the velocity profiles. It is observed that the presence of the porous medium is reduces the velocity profile. This is because the porous medium inhibits the fluid not to move freely through the boundary layer.

Figure 6 and Figure 7, show effect of porosity parameter G on the temperature and concentration profiles, respectively. It is observed that the presence of the porous medium. The temperature profile whereas is increases the concentration profile. This is because the porous medium inhibits the fluid not to move freely through the boundary layer. This leads the flow to increase thermal boundary layer thickness.

Figures 8 show the behavior of temperature for different values Prandtl number. The numerical results show that the effect of increasing values of Prandtl number results in a decreasing velocity. It is observed that an increase in the Prandtl number results a decrease of the thermal boundary layer thickness and in general lower average temperature within the boundary layer. The reason is that smaller values of Pr are equivalent to increase in the thermal conductivity of the fluid and therefore, heat is able to diffuse away from the heated surface more rapidly for higher values of Pr . Hence in the case of smaller Prandtl number as the thermal boundary later is thicker and the rate of heat transfer is reduced.

Figure 9 shows the effect of Le on the dimensionless concentration for fixed values of other parameters. It is observed that for larger values of Le suppress the concentration profile i.e. inhibit nanoparticle species diffusion, as observed. There will be a much greater reduction in the concentration boundary layer thickness.

For different values of the Eckert number Ec the temperature profiles are plotted in Fig. 10. It is obvious that an increase in the Eckert number Ec results in an increase in the temperature within the boundary layer.

Figures 11 show the behavior of temperature for different values Brownian motion parameter Nb. The numerical results show that the effect of increasing values of Brownian motion parameter Nb results in a decreasing velocity. It is observed that an increase in the Brownian motion parameter Nb results a decrease of the thermal boundary layer thickness and in general lower average temperature within the boundary layer. The reason is that smaller values of Nb are equivalent to increase in the thermal conductivity of the fluid and therefore, heat is able to diffuse away from the heated surface more rapidly for higher values of Nb . Hence in the case of smaller Brownian motion parameter Nb as the thermal boundary later is thicker and the rate of heat transfer is reduced.

Figure 12 shows the effect of Nb on the dimensionless concentration for fixed values of other parameters. It is observed that for larger values of Nb suppress the concentration profile i.e. inhibit nanoparticle species diffusion, as observed. There will be a much greater reduction in the concentration boundary layer thickness.

Figure 13, show effect of thermophoresis parameter Nt on the temperature and concentration profiles, respectively. It is observed that the presence of the thermophoresis parameter Nt . The temperature profile whereas is increases the concentration profile. This is because the thermophoresis parameter Nt inhibits the fluid not to move freely through the boundary layer. This leads the flow to increase thermal boundary layer thickness.

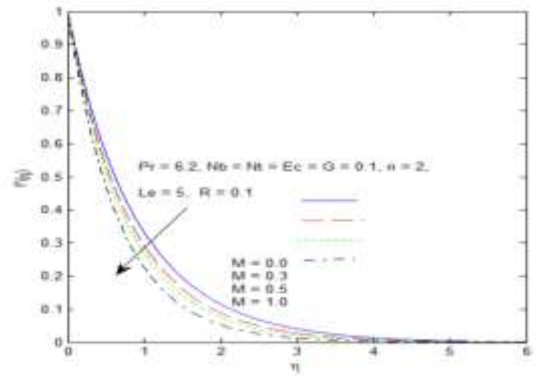

Figure 1 – Velocity profiles for M

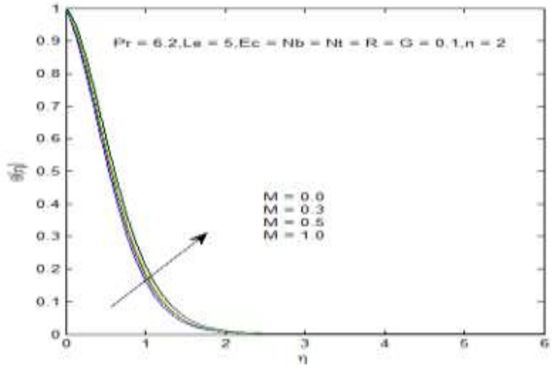

Figure 2 – Temperature profiles for M

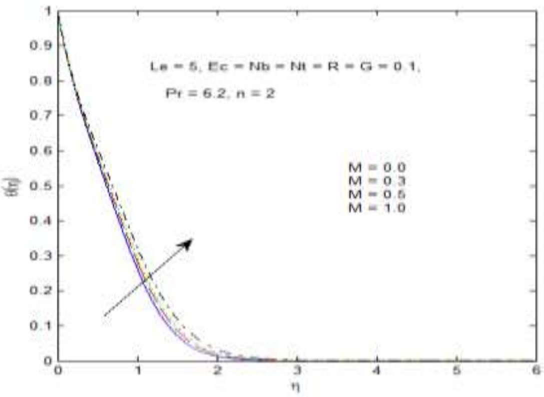

Figure 3 – Concentration profiles for M

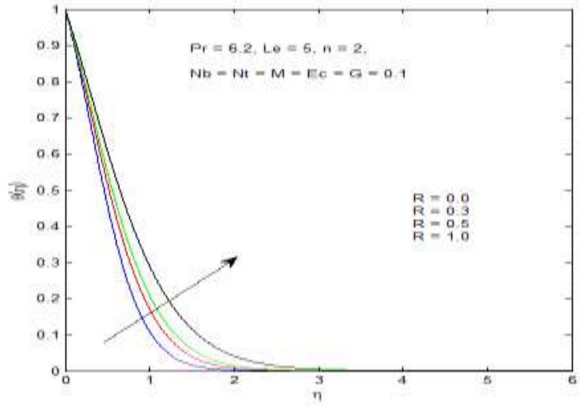

Figure 4 – Temperature profiles for R

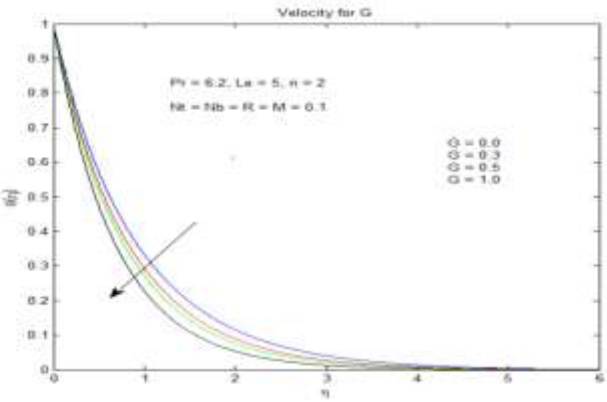

Figure 5 – Velocity profiles for G

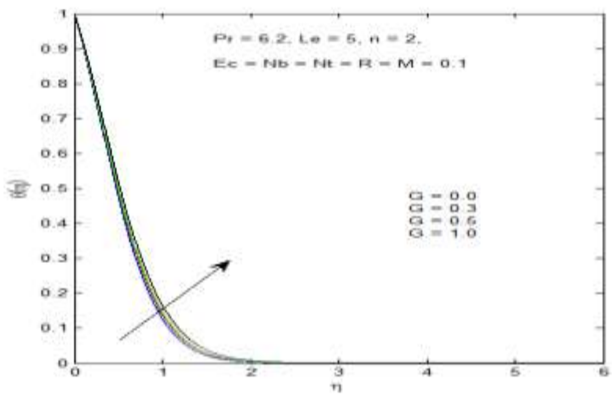

Figure 6 – Temperature profiles for G

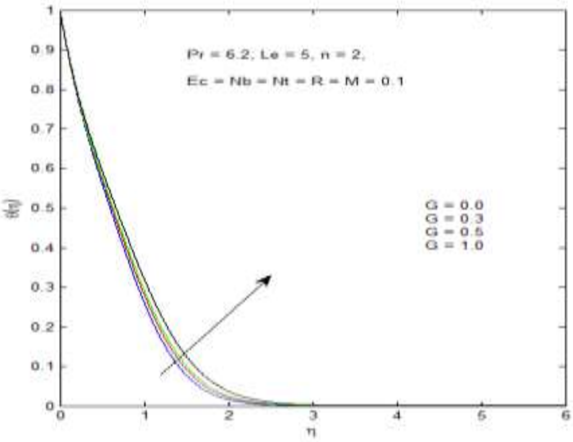

Figure 7 – Concentration profiles for G

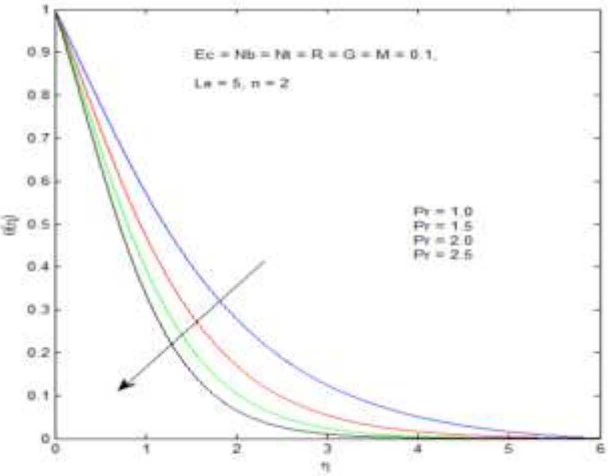

Figure 8 - Temperature profiles for Pr

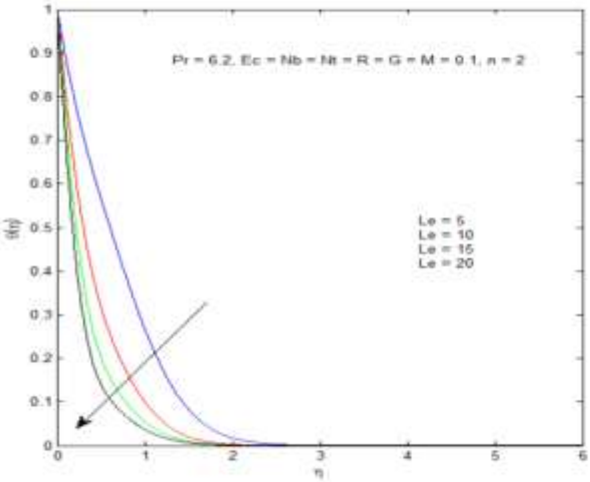

Figure 9 - Concentration profiles for Le

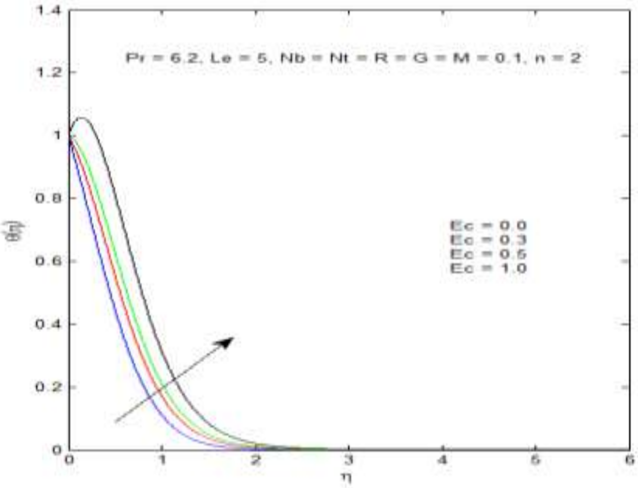

Figure 10 - Temperature profiles for Ec

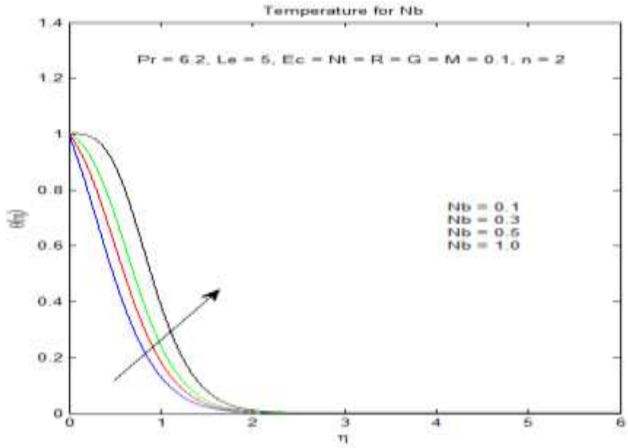

Figure 11 - Temperature profiles for Nb

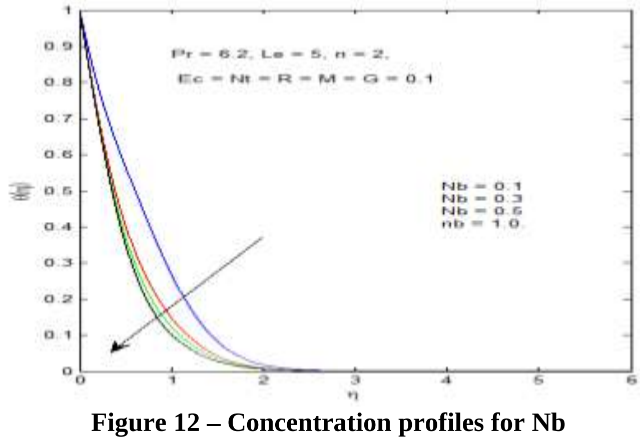

Figure 12 – Concentration profiles for Nb

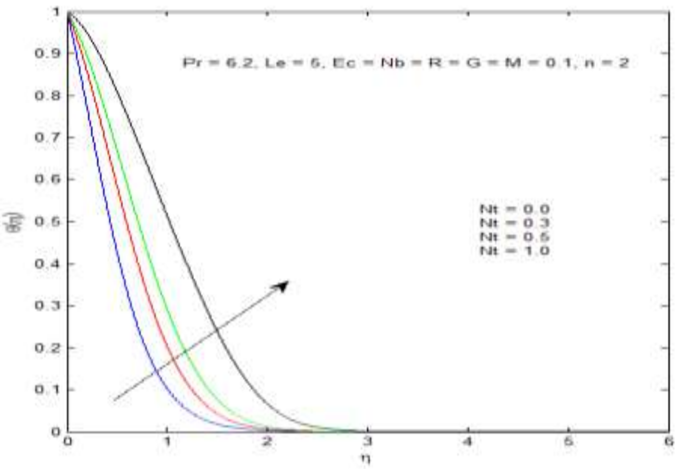

Figure 13 – Temperature profiles for Nt

IV. Conclusion

MHD Boundary Layer flow and Heat Transfer of Nanofluids over a Nonlinear Stretching Sheet in a Porous Medium. A similarity solution is presented which depends on Prandtl and Lewis numbers, magnetic, viscous dissipation, nonlinearity of stretching sheet, Brownian motion and thermophoresis parameters. The effects of governing parameters on the flow, concentration and heat transfer characteristics are presented graphically and quantitatively. The main observations of the present study are as follows:

- Increasing values of Prandtl number results in a decreasing velocity and temperature.
- An increase in the Eckert number Ec increases the temperature.
- The velocity decreases with an increase in magnetic parameter
- The velocity of porous medium decreases with an increases the temperature and concentration.
- An increase in the radiation parameter R increases the temperature.

Acknowledgements

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