

Indrajeet's Factorization for Quadratic

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Abstract

Step1: Find the value of $a \cdot c$.

Step2: Find the divisors of the value of $a \cdot c$, where not required all divisors of the value of $a \cdot c$, if you feel find all divisors, then you can find.

Step3: Choose two numbers from divisors of the value of $a \cdot c$ of which multiplication should be equal to the value of $a \cdot c$, and sum or difference should be equal to the value of b .

Step4: Change the sign of both chosen numbers.

Step5: Devide the both numbers by the value of a and write both numbers in this form $\frac{p}{q}$, where $q \neq 0$ (Let we wrote that numbers as $(\frac{A}{B}, \frac{M}{N})$)

Step6: Write both numbers as $(B \cdot x - A)(N \cdot x - M)$.

Keywords: Divisors and coefficients

I. Introduction

A quadratic is a polynomial that looks like $ax^2 + bx + c$, $a \neq 0$, where a , b and c are just numbers. As we know one rule for factorization of a quadratic or quadratic expression that is “we will find two numbers that will not only multiply to equal the constant term c , but also add up to equal to b , the coefficient of x -term i.e. $x^2 + 9x + 20$ I need to find factors of 20 that add up to 9. Since 20 can be written as the product of 4 and 5, and since $4+5=9$, then I will use 4 and 5. I know from multiplying polynomials that is quadratic is formed from multiplying two factors of the form $(x+p)(x+q)$, for some numbers p and q , then I will write in the two numbers that I found above $(x+4)(x+5)$ this is the factorization of $x^2 + 9x + 20$. I have also developed two new rules for factorise the quadratic or quadratic expression that are very good rule, because this both rule have some similarity and I have also developed a new rule for solve the quadratic equations “Indrajeet's rule for quadratic equation” this is already published by IOSR journal. “Indrajeet's factorization” and “Indrajeet's rule” both have some similarity, but work of both are different. “Indrajeet's rule” is a method to solve the quadratic equation and “Indrajeet's factorization” is also a method, but its work is different. “Indrajeet's factorization” is a rule for factorise the quadratic. This “Indrajeet's factorization” is a good rule for factorise the quadratic, because this rule is a short cut rule or time saving rule, when you will solve the problem of inequalities including quadratic, then you can use “Indrajeet's factorization” there this rule will good proof. One important point of this rule is you can apply its all steps in your mind and you can write direct as $(x+p)(x+q)$ or $(x-p)(x-q)$, so we can say that this is a good and time saving method.

II. Indrajeet's Factorization for Quadratic

Step1: Find the value of a·c.

Step2: Find the divisors of the value of a·c, where not required all divisors of the value of a·c, if you feel find all divisors, then you can find.

Step3: Choose two numbers from divisors of the value of a·c of which multiplication should be equal to the value of a·c, and sum or difference should be equal to the value of b.

Step4: Change the sign of both chosen numbers.

Step5: Devide the both numbers by the value of a and write both numbers in this form $\frac{p}{q}$, where $q \neq 0$ (Let we wrote that numbers as $\frac{A}{B}, \frac{M}{N}$).

Step6: Write both numbers as $(B \cdot x - A)(N \cdot x - M)$.

Keywords: Divisors and coefficients

Notes:

1: When two conjugate numbers are root of a given quadratic equation, then on factorising that “quadratic expression” by “Indrajeet's factorization for quadratic” I get a new result that is “constant× (quadratic expression)”.

2: When $b^2-4ac < 0$, then you should use another (or other) method to factorise the quadratic expression or quadratic, because in this case “Indrajeet's factorization” may be difficult.

3: When you feel difficult to factorise any question of quadratic expression or quadratic by using “Indrajeet's factorization”, then you should use SHREEDHARACHARYA'S rule or $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ and after it you should use step6 of rule 1.

III. Examples

Example1: Factorise the expression $x^2 - 6x + 8$.

Solution: Given expression is $x^2 - 6x + 8$

Here, $a=1, b=-6$ and $c=8$

On using step 1

$$a \cdot c = 1 \times 8 = 8$$

On using step 2

1, 2, 4, 8; -1, -2, -4, -8

On using step 3

-2, -4

On using step 4

2, 4

On using step 5

$$\frac{2}{1}, \frac{4}{1}$$

On using step 6

$$(x-2)(x-4) \text{ this is factorization of } x^2 - 6x + 8$$

Example 2: Factorise the expression $x^2 + 9x + 20$.

Solution: Given expression is $x^2 + 9x + 20$

Here, $a = 1$, $b = 9$ and $c = 20$

On using step 1

$$a \cdot c = 1 \times 20 = 20$$

On using step 2

1, 2, **4, 5**, ...

On using step 3

4, 5

On using step 4

-4, -5

On using step 5

$$\frac{-4}{1}, \frac{-5}{1}$$

On using step 6

$(x+4)(x+5)$ this is the factorization of $x^2 + 9x + 20$

Example 3: If $5x^2 - 6x - 2 = 0$, then find “constant \times (quadratic expression)”.

Solution: Given quadratic equation is $5x^2 - 6x - 2 = 0$

Here, $a = 5$, $b = -6$ and $c = -2$

$$\text{Or } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\text{Or } x = \frac{6 \pm \sqrt{(-6)^2 - 4 \times 5 \times (-2)}}{2 \times 5}$$

$$\text{Or } x = \frac{6 \pm \sqrt{36 + 40}}{10}$$

$$\text{Or } x = \frac{6 \pm \sqrt{76}}{10}$$

$$\text{Or } x = \frac{6 \pm 2\sqrt{19}}{10}$$

$$\text{Or } x = \frac{3 \pm \sqrt{19}}{5}$$

$$\text{Hence, } x = \frac{3 + \sqrt{19}}{5}, \frac{3 - \sqrt{19}}{5}$$

which are conjugate numbers, so I use step6 of “Indrajeet’s factorization for quadratic”, because these conjugate numbers are the value of step5.

$$\begin{aligned} &= (5x - 3 - \sqrt{19})(5x - 3 + \sqrt{19}) \\ &= 25x^2 - 30x - 10 \\ &= 5(5x^2 - 6x - 2) = \text{constant} \times (\text{quadratic expression}). \end{aligned}$$

Example 4: Solve the system of inequality $\begin{cases} x^2 + 6x - 27 > 0 \\ x^2 - 3x - 4 < 0 \end{cases}$

Solution: Given system is $\begin{cases} x^2 + 6x - 27 > 0 \\ x^2 - 3x - 4 < 0 \end{cases}$

On using step 1

Or $\begin{cases} a \times c = 1 \times (-27) = -27 > 0 \\ a \times c = 1 \times (-4) = -4 < 0 \end{cases}$ this is not wrong, if you feel difficult in understand, then you will understand after some steps.

On using step 2

$$\text{Or } \begin{cases} \pm 1, \pm 3, \pm 9, \pm 27 > 0 \\ \pm 1, \pm 2, \pm 4 < 0 \end{cases}$$

On using step 3

$$\text{Or } \begin{cases} -3, 9 > 0 \\ -4, 1 < 0 \end{cases}$$

On using step 4

$$\text{Or } \begin{cases} 3, -9 > 0 \\ 4, -1 < 0 \end{cases}$$

On using step 5

$$\text{Or } \begin{cases} \frac{3}{1}, \frac{-9}{1} > 0 \\ \frac{4}{1}, \frac{-1}{1} < 0 \end{cases}$$

On using step 6

$$\text{Or } \begin{cases} (x-3)\{x-(-9)\} > 0 \\ (x-4)\{x-(-1)\} < 0 \end{cases}$$

$$\text{Or } \begin{cases} x > 3 \text{ or } x < -9 \\ -1 < x < 4 \end{cases}$$

Thus, $x \in (3, 4)$ this is solution of the system.

If you want solve this question shortly, then you should apply all steps of “Indrajeet’s factorization” in mind and you should write direct as:

$$\text{Or } \begin{cases} (x-3)\{x-(-9)\} > 0 \\ (x-4)\{x-(-1)\} < 0 \end{cases}$$

$$\text{Or } \begin{cases} x > 3 \text{ or } x < -9 \\ -1 < x < 4 \end{cases}$$

Thus, $x \in (3, 4)$ this is solution of the system.

Example 5: Solve the system of inequality $\begin{cases} x^2 - x - 6 \leq 0 \\ x^2 - 6x + 5 > 0 \\ x^2 + 6x - 27 \leq 0 \end{cases}$

Solution: Given system is $\begin{cases} x^2 - x - 6 \leq 0 \\ x^2 - 6x + 5 > 0 \\ x^2 + 6x - 27 \leq 0 \end{cases}$

On using step 1

$$\text{Or } \begin{cases} a \times c = 1 \times (-6) = -6 \leq 0 \\ a \times c = 1 \times 5 = 5 > 0 \\ a \times c = 1 \times (-27) = -27 \leq 0 \end{cases}$$

On using step 2

$$\text{Or } \begin{cases} \pm 1, \pm 2, \pm 3, \pm 6 \leq 0 \\ \pm 1, \pm 5 > 0 \\ \pm 1, \pm 3, \pm 9, \pm 27 \leq 0 \end{cases}$$

On using step 3

$$\text{Or } \begin{cases} -3, 2 \leq 0 \\ -5, -1 > 0 \\ -3, 9 \leq 0 \end{cases}$$

On using step 4

$$\text{Or } \begin{cases} 3, -2 \leq 0 \\ 5, 1 > 0 \\ 3, -9 \leq 0 \end{cases}$$

On using step 5

$$\text{Or } \begin{cases} \frac{3}{1}, \frac{-2}{1} \leq 0 \\ \frac{5}{1}, \frac{1}{1} > 0 \\ \frac{3}{1}, \frac{-9}{1} \leq 0 \end{cases}$$

On using step 6

$$\text{Or } \begin{cases} (x - 3)\{x - (-2)\} \leq 0 \\ (x - 5)(x - 1) > 0 \\ (x - 3)\{x - (-9)\} \leq 0 \end{cases}$$

$$\text{Or } \begin{cases} -2 \leq x \leq 3 \\ x > 5 \text{ or } x < 1 \\ -9 \leq x \leq 3 \end{cases}$$

Thus, $x \in [-2, 1)$ this is required solution.

If you want solve this question shortly, then you should apply all steps of

“Indrajeet’s factorization” in mind and you should write direct as:

$$\text{Or } \begin{cases} (x - 3)\{x - (-2)\} \leq 0 \\ (x - 1)(x - 5) > 0 \\ (x - 3)\{x - (-9)\} \leq 0 \end{cases}$$

$$\text{Or } \begin{cases} -2 \leq x \leq 3 \\ x < 1 \text{ or } x > 5 \\ -9 \leq x \leq 3 \end{cases}$$

Thus, $x \in [-2, 1)$ this is required solution.

IV. Conclusion

“Indrajeet’s factorization” is a unique rule to factorise the “quadratic” and this rule is a time Saving rule, because much problems of inequality including quadratic will solve easily by using “Indrajeet’s factorization”, so we can say that this is a good rule for students, teachers and more. This rule will be a good rule for students, because students want easy rule for solve the any problem, so this rule will be a good rule for quadratic, we know already one rule for factorization of quadratic that is a good rule, but “Indrajeet’s factorization” is a unique method to factorise the quadratic expression or quadratic and this rule is also easy for remember, because in this rule I used language and steps, so this is a easy rule for remember and also easy for factorise. In this rule I used divisors and coefficient of x^2 and coefficient of x . These all are best points for make easy this method, when we factorise any quadratic, then we lost more time in remembering two numbers which will be applicable, but in “Indrajeet’s factorization” I used divisors of the value of multiplication of coefficient of x^2 and constant, which proof this rule is a time saving rule. Use of coefficient of x^2 and coefficient of x are also good in making time saving rule, use of steps and use of language also make it easy.

References

Book

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Chapter in book

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