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Truncated Life Test Acceptance Sampling Plans Assuring Percentile Life under Gompertz Distribution

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Abstract: *Truncated life test single and double acceptance sampling plans are designed by specifying two points on the operating characteristic curve which are associated with consumer's risk and producer's risk by assuring percentile life when the lifetime of the product follows Gompertz distribution. The quality levels are expressed by the ratio of true percentile life to the specified percentile life. The optimal parameters - the sample size(s) and the acceptance number(s) are determined according to the simultaneous satisfaction of producer's and consumer's quality levels and their confidence levels by incorporating the minimum average sample number (ASN) at consumer's quality level. Tables are prepared for various values of consumer's risk , producer's risk, test termination ratio and shape parameter. Results are explained numerically.*
Keyword*: Acceptance sampling plans, Consumer's risk, producer's risk, truncated life test, Gompertz distribution.*

I. Introduction

Acceptance sampling is one of the major areas of statistical quality control which deals with procedures by which decision to accept or reject the lot is made based on the results of the inspection of samples. If the quality characteristics is regarding the lifetime of products, then the acceptance sampling procedure is called a life test. Manufactured product units are highly reliable with the advent of computer aided designing and manufacturing. Therefore the truncated life test plans which are terminated at the prescheduled time have been developed for the purpose of saving test time and cost.

Epstein (1954), a pioneer in introducing life test plans, developed single acceptance sampling plans for truncated life test with exponential lifetime distribution. Following this several authors developed truncated life test plans based on several distributions such as Goode and Kao (1961) for weibull distribution, Gupta and Groll(1961) for gamma distribution, Kantam and Rosaiah (1998) for normal and log normal distributions, Kantam et al.(2001) for log-logistic distribution, Baklizi and El-Masri (2004) for Birnbaum-Saunders distribution , Rosaiah and Kantam (2005) for inverse Rayleigh distribution, Wenhao Gui and Shangli Zhang (2014) for Gompertz distribution .

All these authors proposed truncated life test plans by using only one point on the operating characteristic curve which is the consumer's risk at the given quality level. This approach may not always satisfy the producer since the producers want the probability of rejecting a good lot to be minimum. Several authors developed acceptance sampling plans by two-point approach for controlling non-conforming fraction . But these articles do not consider the life test aspect. This necessitated to develop a two-point method which guarantees simultaneous protection to producer and consumer in life testing. Aslam and Jun (2013) developed truncated life test sampling plan when the lifetime follows Weibull, Gamma and generalised Rayleigh distributions interms of mean. Balamurali et al. (2013) developed truncated life test plan assuring median life under Frechet distribution. Percentile is the most appropriate average for decision making rather than mean and median in case of skewed lifetime distributions and more generalized measure in case of symmetrical distributions. This motivated the researcher to develop truncated life test plans based on percentiles using two-point approach.

This paper proposes the designing of truncated life test sampling plans following two point approach by imposing the minimum average sample number at the consumer's quality level when the lifetime of the product follows Gompertz distribution by assuring percentiles. Description of Gompertz distribution is given in section 2. Method of developing truncated life test plans by following two-point approach is provided in section 3. Applications of plans are given in last section.

Gompertz Distribution

The two-parameter Gompertz distribution was introduced by Benjamin Gompertz (1825) as a suitable model to describe human mortality and to establish actuarial tables. It has found widespread applications in various areas of reliability analysis.

The probability density function of Gompertz distribution is given by

$$f(t;\theta,\sigma)=(\theta/\sigma)\,e^{t/\sigma}\exp(-\theta(e^{t/\sigma}-1)),\ t>0,\theta>0,\sigma>0 \qquad (1)$$

where σ is the scale parameter which governs the dispersion of the distribution and θ the shape parameter which governs the shape of the distribution.

The cumulative distribution function of Gompertz distribution is

$$F(t;\theta,\sigma)=1-\exp\left(-\theta\left(e^{t/\sigma}-1\right)\right),\ t>0,\theta>0,\sigma>0 \qquad (2)$$

Read(1983) studied the properties of Gompertz distribution. Pollard and Valkovics (1992) obtained the characteristics of Gompertz distribution and explained its applications. Kunimura(1998) derived estimation of parameters. The characterization of mixtures of Gompertz distribution with order statistics is given by Wu and Lee (1999). Saracoglu et al. (2009) compared the estimators for stress- strength reliability.

The mean of the Gompertz distribution is given by $\mu=\sigma e^{\theta}\Gamma(0,\theta)$

where $\Gamma(s,x)=\int_x^{\infty} t^{s-1}e^{-t}dt$ is known as the upper incomplete gamma function .

The median of the Gompertz distribution is $\sigma\ln\left[1+\ln 2^{1/\theta}\right]$.

The mode of this distribution is $-\sigma\ln\theta$ when $0<\theta<1$ and zero when $\theta\geq 1$.

In the present work we construct truncated life test acceptance sampling plans using $100q^{th}$ percentile which is defined by

$$t_q=\sigma\ln\left[1+\ln(1-q)^{-1/\theta}\right]$$

For any given shape parameter, $100q^{th}$ percentile is directly proportional to scale parameter and it is an increasing function for q>0.5 and decreasing function when q<0.5. t_q may be expressed as

$t_q=\sigma b$ where $b=\ln\left[1+\ln(1-q)^{-1/\theta}\right]$ This implies $\sigma=t_q/b$.

The probability of failure from equation (2) is given by

$$\begin{aligned}F(t)&=1-\exp\left(-\theta\left(e^{t/\sigma}-1\right)\right),\quad t>0\\&=1-\exp\left(-\theta\left(e^{t/(t_q/b)}-1\right)\right),\quad t>0\quad \text{by replacing } \sigma=t_q/b\\&=1-\exp\left(-\theta\left(e^{\delta b}-1\right)\right),\quad t>0\quad \text{where } \delta=t/t_q\end{aligned} \qquad (3)$$

This expression emphasizes the dependence of F(t) on δ.

Taking partial derivative with respect to δ we get $\dfrac{\theta b e^{\delta b}\left(e^{\delta b}-1\right)}{e^{\theta\left(e^{\delta b}-1\right)}}>0$.

F(t, δ) is a non-decreasing function of δ .Therefore $F(t,\delta)\leq F(t,\delta_0)\Leftrightarrow t_q\geq t_q^{0}$

Design of Truncated Life Test Single And Double Acceptance Sampling Plans

Assume that the quality of a product be represented by its percentile lifetime t_q .The lot will be accepted if the experimental data supports the null-hypothesis H_0: $t_q\geq t_q^0$ against the alternative hypothesis H_1: $t_q<t_q^0$, where t_q^0 is a specified $100q^{th}$ percentile lifetime. The significance level for the test is $1-P^*$,where P^* is the consumer's confidence level.

The operating procedure for truncated life test single acceptance sampling plan is

(i) Take a sample of size n from a lot of products and put them on test.

(ii) Accept the lot if the number of failures during the test time is lesser than c. Terminate the test before the test time if the number of observed failures exceeds (c+1) and reject the lot.

The operating procedure for truncated life test double acceptance sampling plan is

(i) Take a first random sample of size n_1 from the submitted lot and put on the test for pre-assigned experimental time. If there are c_1 or less failures occur during the experimental time accept the lot. Terminate the test if there are (c_2+1) failures and reject the lot.

(ii) If the number of failures in the experimental time is between (c_1+1) and c_2 then draw a second random sample of size n_2 and put them on the test. If the number of failures in combined sample is at most c_2 accept the lot. Otherwise, terminate the test and reject the lot.

Often it is convenient to set the termination time as a multiple of the specified lifetime, t_q^0 as $t = \delta_0 t_q^0$. For a fixed P*, proposed truncated life test single acceptance sampling plan is characterized by three parameters (n, c, δ_0) and truncated life test double sampling plan is characterized by five parameters (n_1, n_2, c_1, c_2, δ_0), $n_1 < n_2$ and $c_1 < c_2$,

Here we consider that the lot size is larger enough to apply binomial model to calculate the probability of acceptance of the lot. Under the proposed single and double acceptance sampling plan , the probability of acceptance of the lot is

$$L(p) = \sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i} \tag{3}$$

$$L(p) = \sum_{i=0}^{c_1} \binom{n_1}{i} p^i (1-p)^{n_1-i} + \sum_{x=c_1+i}^{c_2} \binom{n_1}{x} p^x (1-p)^{n_1-x} \left\{ \sum_{i=0}^{c_2-x} \binom{n_2}{i} p^i (1-p)^{n_2-i} \right\} \tag{4}$$

where p is the function of cumulative distribution function of underlying lifetime distribution interms of percentile which is expressed by

$$p = 1 - \exp\left(-\theta \left(e^{\delta_0 b / \left(t_q / t_q^0 \right)} - 1 \right) \right) \tag{5}$$

Let p_1 be the probability of failure corresponding to the producer's risk and p_2 be the probability of failure corresponding to the consumer's risk. Accordingly p_1 is a desirable quality level and p_2 is an undesirable quality level. These values of p_1 and p_2 are obtained by considering $t_q / t_q^0 > 1$ and values of p_2 with $t_q / t_q^0 = 1$ in equation (5).

The criteria in determining the plan parameters have two aspects – one is to minimize the sample size and the other is the desired operating characteristic value. The sample size is to be minimized since it is related to test cost and time. The operating characteristic which is a function of lot quality should satisfy the consumer and the producer. In the proposed two-point approach, the design parameters of truncated life test single acceptance sampling plan are to be obtained based on the above criteria that satisfy the following two inequalities

$$\sum_{i=0}^{c} \binom{n}{i} p_1^i (1-p_1)^{n-i} \geq 1-\alpha \tag{6}$$

$$\sum_{i=0}^{c} \binom{n}{i} p_2^i (1-p_2)^{n-i} \leq \beta \tag{7}$$

The design parameters of truncated life test double acceptance sampling plan are to be obtained that satisfy the following two inequalities

$$\sum_{i=0}^{c_1} \binom{n_1}{i} p_1^i (1-p_1)^{n_1-i} + \sum_{x=c_1+i}^{c_2} \binom{n_1}{x} p_1^x (1-p_1)^{n_1-x} \left\{ \sum_{i=0}^{c_2-x} \binom{n_2}{i} p_1^i (1-p_1)^{n_2-i} \right\} \geq 1-\alpha \tag{8}$$

$$\sum_{i=0}^{c_1} \binom{n_1}{i} p_2^i (1-p_2)^{n_1-i} + \sum_{x=c_1+i}^{c_2} \binom{n_1}{x} p_2^x (1-p_2)^{n_1-x} \left\{ \sum_{i=0}^{c_2-x} \binom{n_2}{i} p_2^i (1-p_2)^{n_2-i} \right\} \leq \beta \tag{9}$$

Multiple solutions exist for the plan parameters from equations (6), (7) and (8), (9) by introducing the concept of minimum average sample number, the optimal parameters are determined.

ASN for single acceptance sampling plan is n

and ASN for double acceptance sampling plan is $n_1P_1+(n_1+n_2)(1-P_1)$.

where P_1 is the probability of acceptance or rejection based on first sample, which is defined as

$$P_1 = 1 - \sum_{i=c_1+1}^{c_2} \binom{n_1}{i} p^i (1-p)^{n_1-i}$$

The determination of optimal parameters of the proposed truncated life test sampling plan reduces to the non-linear programming problem:

Minimize $ASN(p_2)$

subject to

$L(p_1) \geq 1-\alpha$

$L(p_2) \leq \beta$

$n_1 < n_2$; n, n_1 and n_2 are integers.

$c_1 < c_2$; $c, c_1, c_2 \geq 0$; c, c_1 and c_2 are integers. (10)

The minimum sample size(s) and the acceptance number(s) are obtained using MATLAB for the given producer's risk α (= 0.05), consumer's risk β(=0.25,0.1,0.05,0.01), percentile ratios t_q/t_q^0 (=4,5,6,7,8), and various values of shape parameter and termination ratios and tabulated in Tables 1 and 2.Numerical values presented in Tables 1 and 2 signify that for fixed consumer's risk and producer's risk.

(i) increase in termination ratio decreases the sample sizes
(ii) increase in shape parameter generally decreases the sample sizes
(iii) increase in ratio of true percentile to the specified percentile decreases the sample size.
(iv) ASN values at the consumer's quality level for truncated life test double acceptance sampling plan is lesser than the truncated life test single acceptance sampling plan

Selection of plans

Suppose that the manufacturer wants to determine the design parameters of truncated life test sampling plans for assuring the percentile life of the electronic devices. From the past records it is seen that the lifetime of electronic devices follows Gompertz distribution with shape parameter θ=1.5 but true percentile is not known. The consumer and producer are contracting to accept the lots of electronic devices if the true percentile life is atleast 1000 hours at the consumer risk of 0.10 and producer risk of 0.05 when the true percentile life is 4000 hours. For the test time of 800 hours it is required to derive the sampling plans using the tabulated values. The requirements in this example are defined by α = 0.05, β = 0.10, t_q/t_q^0=4 and $t/t_q^0(\delta_0)$=0.8.

From Table 1, it is found that for the above requirements the truncated life test single acceptance sampling plan is (98,4) with ASN=98 at the consumer's quality level. It says that a sample of 98 devices should be put on test for 800 hours and the number of failures should be recorded. Accept the lot if there is 4 or less failures otherwise terminate the test and reject the lot.

The truncated life test double acceptance sampling plan may be selected from Table 2 for the above requirements as (59,60,1,5) with ASN=96.19 at the consumer's quality level. It implies that a sample of 59 units is to be randomly selected from the lot of devices and should be put on test for 800 hours. Accept the lot if the number of failures is less than or equal to 1 and reject the lot if the number of failures is greater than 5 by using the first sample results. If not take a second random sample of size 60 and put them on test and count the number of failures. If the number of cumulative failures (total number of failures in first and second sample) exceeds 5 terminate the test and reject the lot otherwise accept the lot.

II. Conclusion

In this paper, we have developed truncated life test single and double acceptance sampling plans when the lifetime of the product follows Gompertz distribution by assuring percentile lifetime. The optimal design parameters of the proposed plan are determined using two points on the operating characteristic curve which protects the consumer as well as the producer simultaneously. The results from the assumed values show that truncated life test double acceptance sampling plan is more economical than the truncated life test single acceptance sampling plan in a given situation. The failure probability may be derived and design parameters may be determined for any given lifetime distribution following the proposed two point approach with minimum ASN.

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Table 1 Optimal parameters for truncated life tests single acceptance sampling plan assuring 10th percentile under Gompertz distribution when α=0.05

β	t_w/t_c°	θ=0.5					θ=1					θ=1.5				
		δ_0=0.6	δ_0=0.8	δ_0=1	δ_0=1.25	δ_0=1.5	δ_0=0.6	δ_0=0.8	δ_0=1	δ_0=1.25	δ_0=1.5	δ_0=0.6	δ_0=0.8	δ_0=1	δ_0=1.25	δ_0=1.5
0.25	4.0	86,3	64,3	51,3	40,3	33,3	84,3	64,3	51,3	40,3	34,3	84,3	63,3	51,3	40,3	34,3
	5.0	66,2	49,2	39,2	31,2	25,2	65,2	49,2	39,2	31,2	26,2	64,2	48,2	39,2	31,2	26,2
	6.0	66,2	49,2	39,2	31,2	25,2	65,2	49,2	39,2	31,2	26,2	64,2	48,2	39,2	31,2	26,2
	7.0	66,2	49,2	39,2	31,2	17,1	65,2	49,2	39,2	31,2	26,2	64,2	48,2	39,2	31,2	26,2
	8.0	45,1	34,1	27,1	21,1	17,1	44,1	34,1	27,1	21,1	18,1	44,1	33,1	27,1	21,1	18,1
0.10	4.0	134,4	99,4	78,4	62,4	51,4	132,4	98,4	78,4	62,4	52,4	152,5	98,4	78,4	64,4	52,4
	5.0	112,3	83,3	65,3	52,3	42,3	110,3	82,3	65,3	52,3	43,3	109,3	82,3	65,3	52,3	44,3
	6.0	112,3	83,3	65,3	41,2	34,2	110,3	82,3	65,3	41,2	43,3	109,3	82,3	65,3	52,3	44,3
	7.0	89,2	66,2	52,2	41,2	34,2	87,2	65,2	52,2	41,2	34,2	87,2	65,2	52,2	42,2	35,2
	8.0	89,2	66,2	52,2	41,2	34,2	87,2	65,2	52,2	41,2	34,2	87,2	65,2	52,2	42,2	35,2
0.05	4.0	176,5	130,5	1035	81,5	66,5	173,5	129,5	1035	82,5	68,5	172,5	129,5	1035	82,5	68,5
	5.0	153,4	113,4	89,4	70,4	58,4	150,4	112,4	89,4	71,4	59,4	149,4	112,4	89,4	71,4	59,4
	6.0	130,3	96,3	76,3	59,3	49,3	127,3	95,3	76,3	60,3	50,3	126,3	95,3	76,3	60,3	50,3
	7.0	130,3	96,3	76,3	48,2	39,2	127,3	95,3	76,3	60,3	50,3	126,3	95,3	76,3	60,3	50,3
	8.0	105,2	78,2	61,2	48,2	39,2	103,2	77,2	61,2	49,2	40,2	102,2	77,2	61,2	49,2	41,2
0.01	4.0	267,7	198,7	1567	123,7	91,6	262,7	196,7	1567	124,7	103,7	261,7	195,7	1567	125,7	104,7
	5.0	219,5	162,5	1275	100,5	82,5	215,5	160,5	1275	101,5	84,5	213,5	160,5	1275	102,5	85,5
	6.0	193,4	143,4	1134	88,4	72,4	190,4	142,4	1134	90,4	74,4	189,4	141,4	1134	90,4	75,4
	7.0	193,4	124,3	97,3	76,3	63,3	190,4	142,4	1134	77,3	64,3	189,4	141,4	1134	90,4	75,4
	8.0	167,3	124,3	97,3	76,3	63,3	164,3	122,3	97,3	77,3	64,3	163,3	122,3	97,3	78,3	65,3

Table 2 Optimal parametrs for truncated life tests double acceptance sampling plans assuring 10th percentile under Gompertz distribution when α = 0.05

Θ	β	t_q/t_q^0	δ_r=0.6 n_1,n_2,c_1,c_2	ASN	δ_r=0.8 n_1,n_2,c_1,c_2	ASN	δ_r=1.0 n_1,n_2,c_1,c_2	ASN	δ_r=1.25 n_1,n_2,c_1,c_2	ASN
1	0.25	4.0	43,44,0,3	72.61	32,33,0,3	54.42	29,31,1,3	43.64	20,21,0,3	34.63
		5.0	34,36,0,2	53.56	25,27,0,2	39.91	20,22,0,2	32.22	16,17,0,2	25.51
		6.0	34,36,0,2	53.56	25,27,0,2	39.91	20,22,0,2	32.22	16,17,0,2	25.51
		7.0	34,36,0,2	53.56	25,27,0,2	39.91	20,22,0,2	32.22	16,17,0,2	25.51
		8.0	27,28,0,1	36.07	20,21,0,1	26.89	16,17,0,1	21.60	12,15,0,1	17.18
	0.10	4.0	66,67,0,4	107.49	49,51,0,4	81.03	39,40,0,4	64.33	31,32,0,4	51.48
		5.0	56,58,0,3	86.90	42,43,0,3	64.93	33,35,0,3	52.11	26,28,0,3	41.61
		6.0	56,58,0,3	86.90	42,43,0,3	64.93	33,35,0,3	52.11	26,28,0,3	41.61
		7.0	47,49,0,2	66.77	35,36,0,2	49.66	28,29,0,2	39.80	22,23,0,2	31.59
		8.0	47,49,0,2	66.77	35,36,0,2	49.66	28,29,0,2	39.80	22,23,0,2	31.59
	0.05	4.0	86,88,0,5	137.15	64,66,0,5	102.86	51,52,0,5	81.84	40,42,0,5	65.72
		5.0	76,77,0,4	115.08	56,58,0,4	86.29	45,46,0,4	68.85	35,37,0,4	55.08
		6.0	66,67,0,3	93.99	49,50,0,3	70.22	39,40,0,3	56.09	31,32,0,3	44.80
		7.0	66,67,0,3	93.99	49,50,0,3	70.22	39,40,0,3	56.09	31,32,0,3	44.80
		8.0	56,59,0,2	74.13	42,43,0,2	55.15	33,35,0,2	44.02	26,28,0,2	35.01
	0.01	4.0	131,132,0,7	192.69	98,99,0,7	144.48	78,79,0,7	115.52	62,63,0,7	92.37
		5.0	108,109,0,5	147.56	80,82,0,5	110.60	64,65,0,5	88.15	51,52,0,5	70.42
		6.0	97,98,0,4	126.25	72,73,0,4	94.25	57,58,0,4	75.07	45,47,0,4	60.06
		7.0	97,98,0,4	126.25	72,73,0,4	94.25	57,58,0,4	75.07	45,47,0,4	60.06
		8.0	90,91,0,3	108.20	64,66,0,3	79.28	51,52,0,3	63.05	40,42,0,3	50.18
1.5	0.25	4.0	42,44,0,3	71.90	32,33,0,3	54.38	29,31,1,3	43.64	23,25,1,3	34.99
		5.0	34,35,0,2	52.95	25,27,0,2	39.89	20,22,0,2	32.22	16,17,0,2	25.53
		6.0	34,35,0,2	52.95	25,27,0,2	39.89	20,22,0,2	32.22	16,17,0,2	25.53
		7.0	34,35,0,2	52.95	25,27,0,2	39.89	20,22,0,2	32.22	16,17,0,2	25.53
		8.0	34,35,0,2	52.95	20,21,0,1	26.87	16,17,0,1	21.60	16,17,0,2	25.53
	0.10	4.0	79,80,1,5	127.99	59,60,1,5	96.19	39,40,0,4	64.33	28,36,0,4	53.61
		5.0	56,57,0,3	86.13	42,43,0,3	64.84	33,35,0,3	52.11	26,28,0,3	41.68
		6.0	56,57,0,3	86.13	42,43,0,3	64.84	33,35,0,3	52.11	26,28,0,3	41.68
		7.0	47,48,0,2	66.21	35,36,0,2	49.60	28,29,0,2	39.80	22,23,0,2	31.64
		8.0	47,48,0,2	66.21	35,36,0,2	49.60	28,29,0,2	39.80	22,23,0,2	31.64
	0.05	4.0	86,87,0,5	136.06	64,65,0,5	102.08	51,52,0,5	81.84	41,42,0,5	65.91
		5.0	75,77,0,4	114.52	56,58,0,4	86.13	45,46,0,4	68.85	36,37,0,4	55.31
		6.0	65,67,0,3	93.42	49,50,0,3	70.10	39,40,0,3	56.09	31,32,0,3	44.90
		7.0	65,67,0,3	93.42	49,50,0,3	70.10	39,40,0,3	56.09	31,32,0,3	44.90
		8.0	65,67,0,3	93.42	42,43,0,2	55.08	33,35,0,2	44.02	26,29,0,2	35.40
	0.01	4.0	130,131,0,7	191.40	97,98,0,7	143.81	78,79,0,7	115.52	62,63,0,7	92.66
		5.0	107,108,0,5	146.49	80,81,0,5	109.97	64,65,0,5	88.15	51,52,0,5	70.63
		6.0	96,97,0,4	125.26	72,73,0,4	94.06	57,58,0,4	75.07	45,47,0,4	60.22
		7.0	96,97,0,4	125.26	72,73,0,4	94.06	57,58,0,4	75.07	45,47,0,4	60.22
		8.0	86,87,0,3	105.55	64,65,0,3	78.91	51,52,0,3	63.05	40,43,0,3	50.54